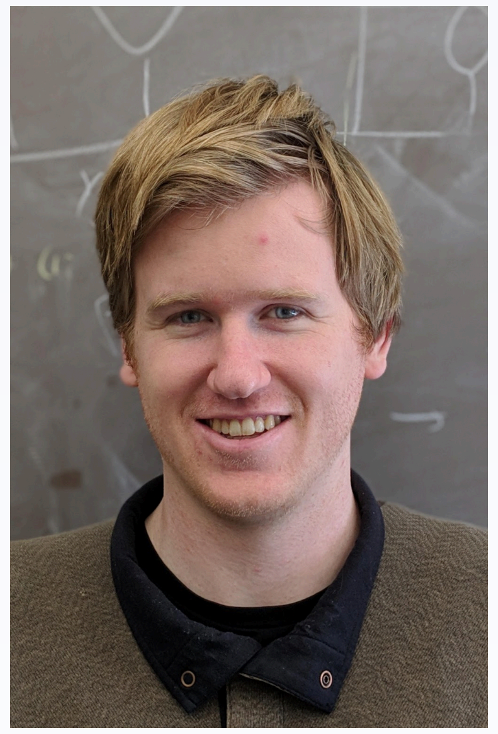
Probing *charming* and beautiful dynamics with energy correlators

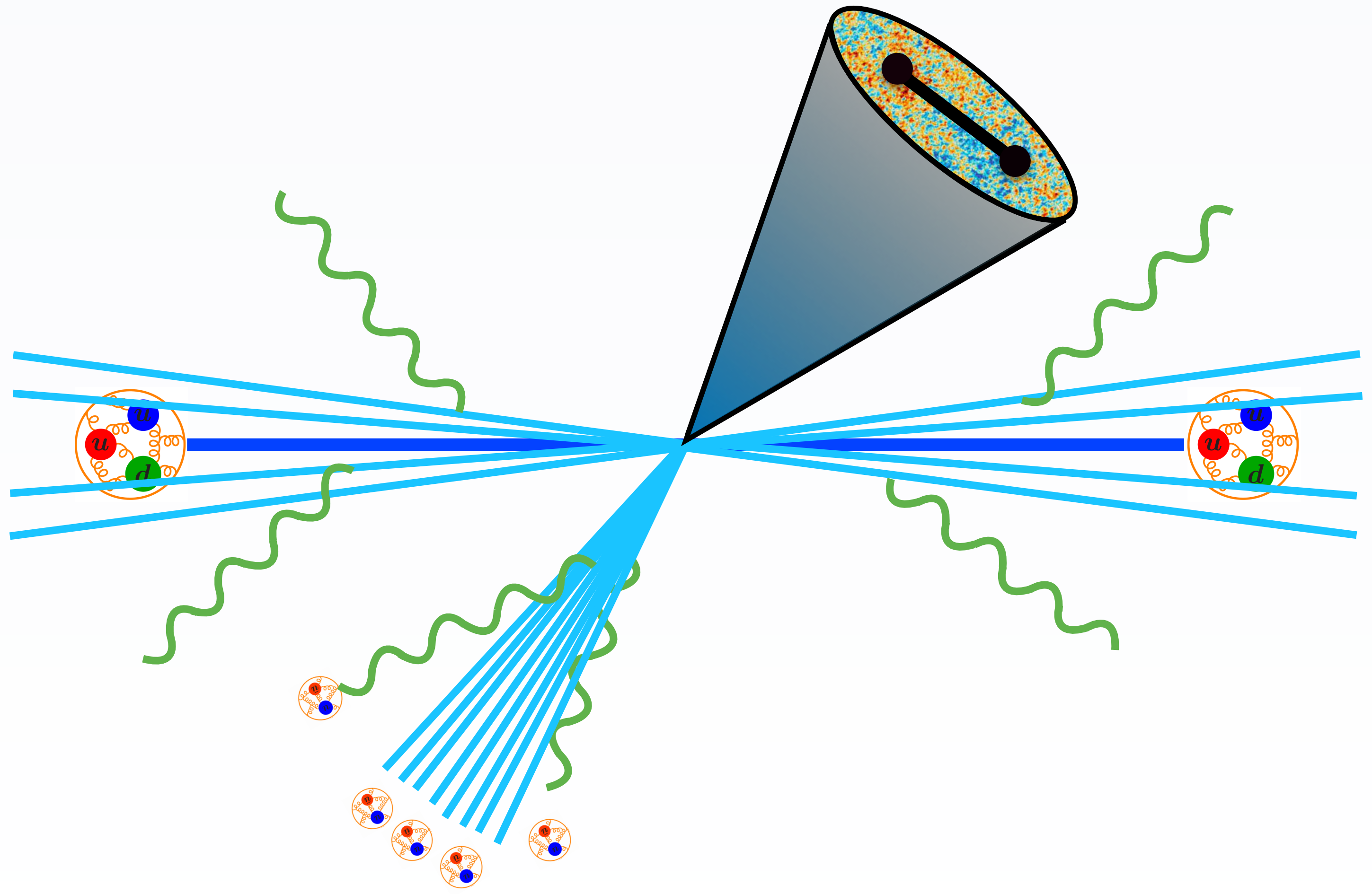
In collaboration with



Evan Craft, Mark Gonzalez, Bianka Meçaj, and Ian Moult

Yale



Kyle Lee

CTP, MIT

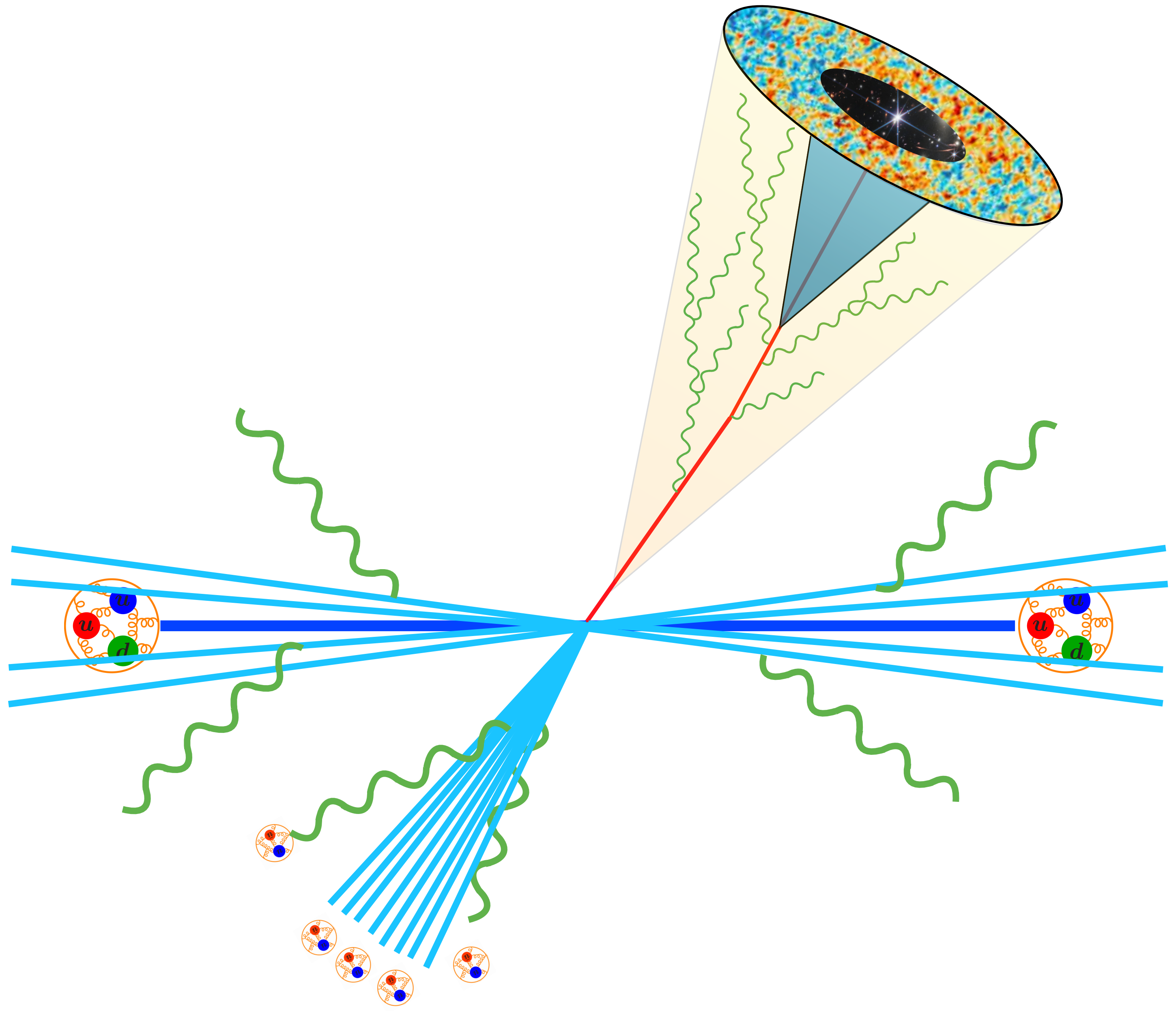
SCET 2023

Cosmology
Big Bang
Time
Exponential expansion
Dense spots in the universe
Decay
High Energy Collider
$\mathcal{E}(\hat{n}_1)$
χ
$\mathcal{E}(\hat{n}_2)$
$\langle \mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2)\rangle$

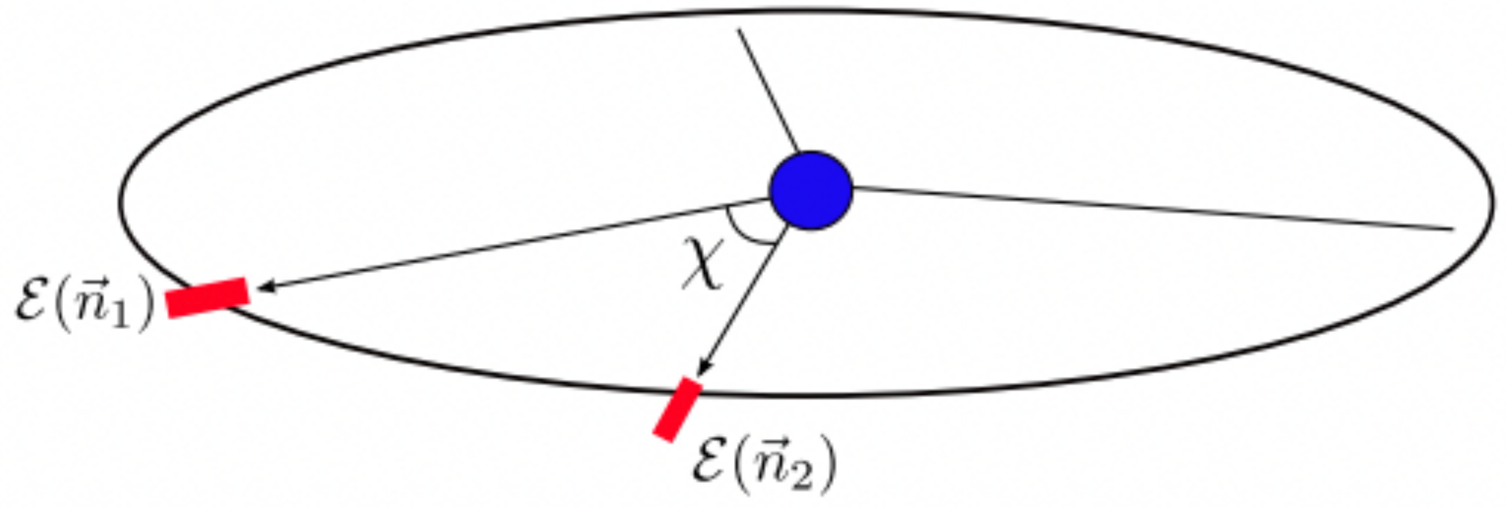
Energy Correlators mapping high energy collider events

Collinear limit

$\chi \to 0$

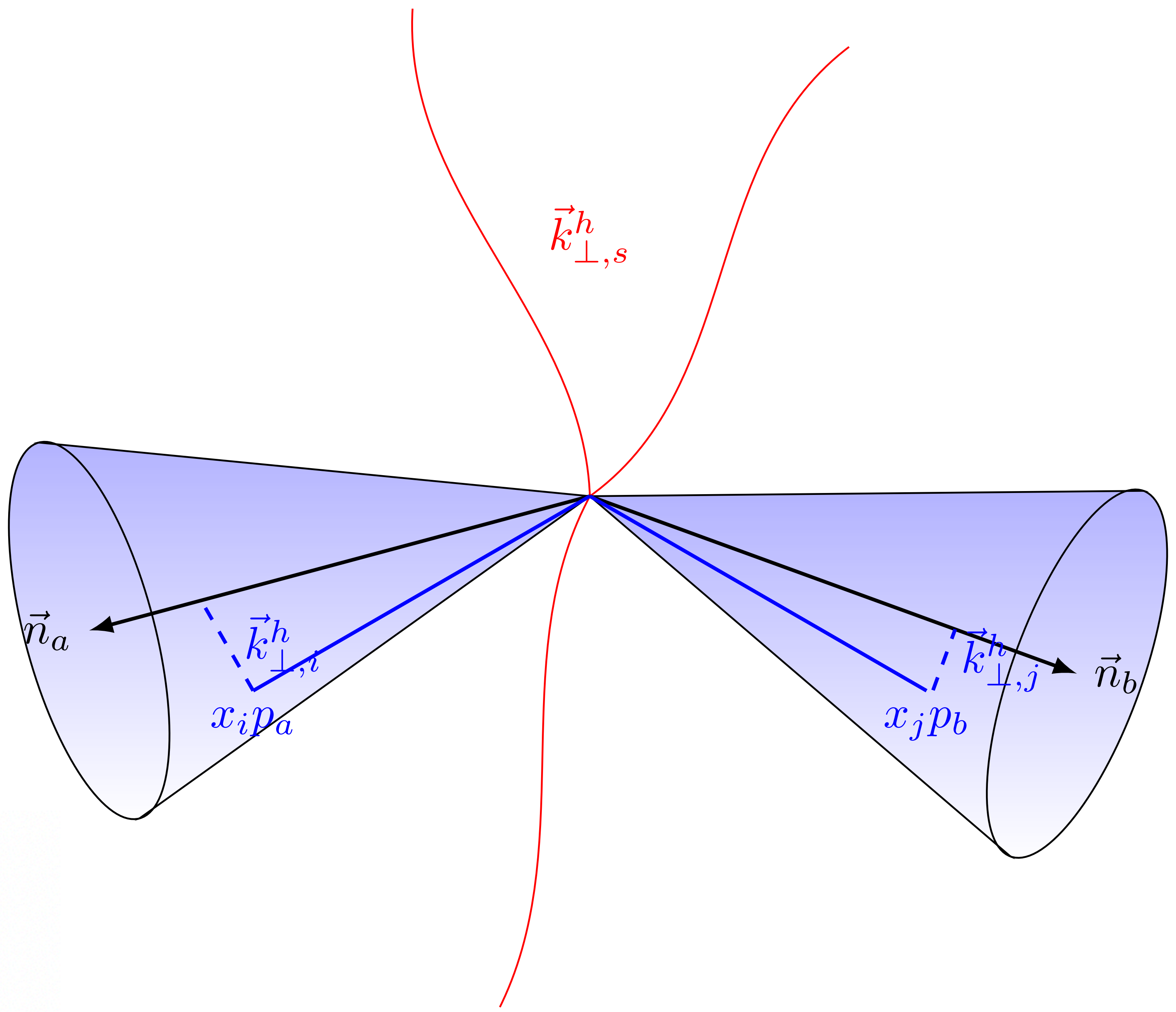


General angle region

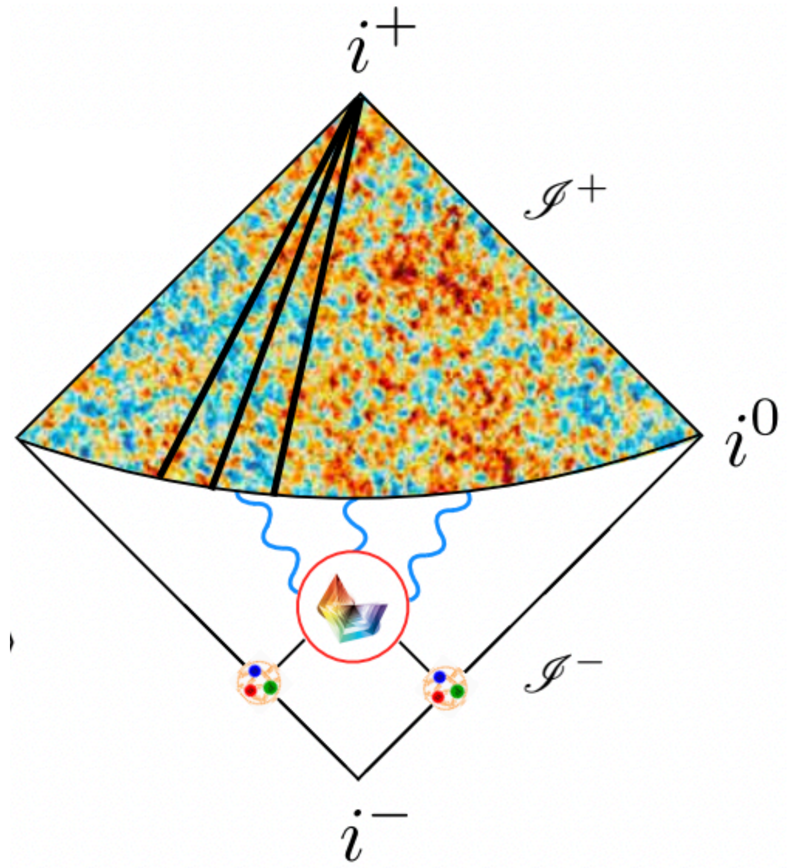


Back-to-back region

$\chi \to \pi$



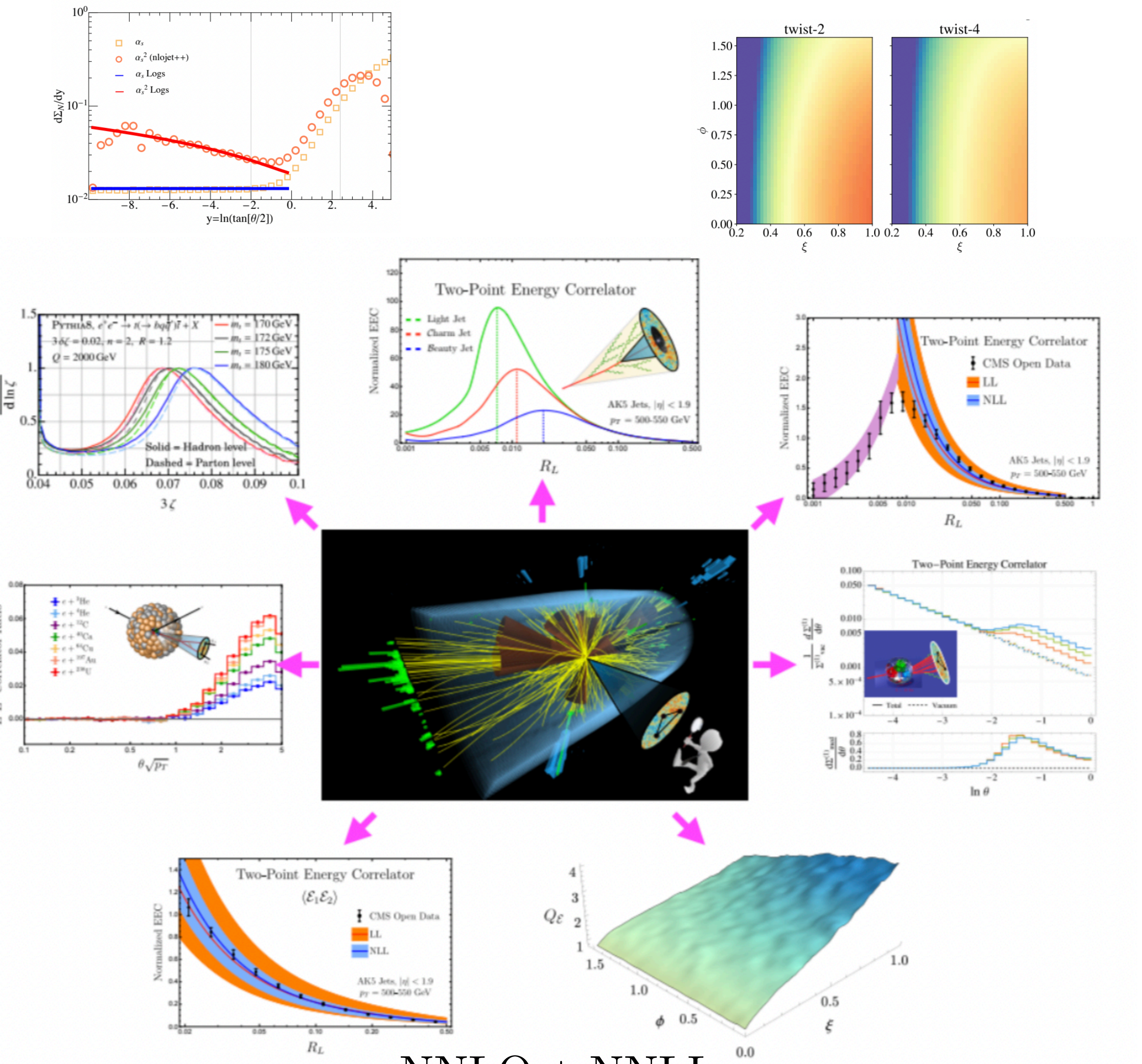
$$\mathcal{E}(\hat{n}) = \int_0^\infty dt \lim_{r\to\infty} r^2 n^i T_{0i}(t, r\hat{n}) \iff$$



Energy Correlators mapping high energy collider events

Collinear limit

[Haotian's talk] $\chi \to 0$

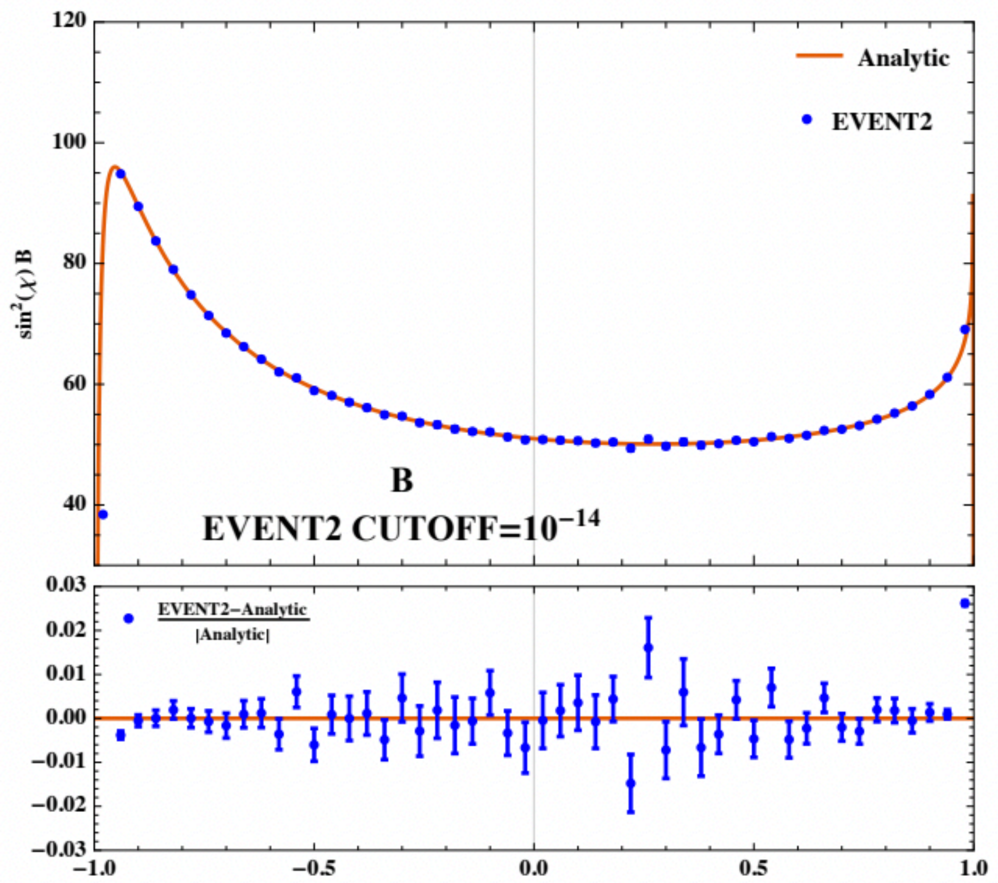


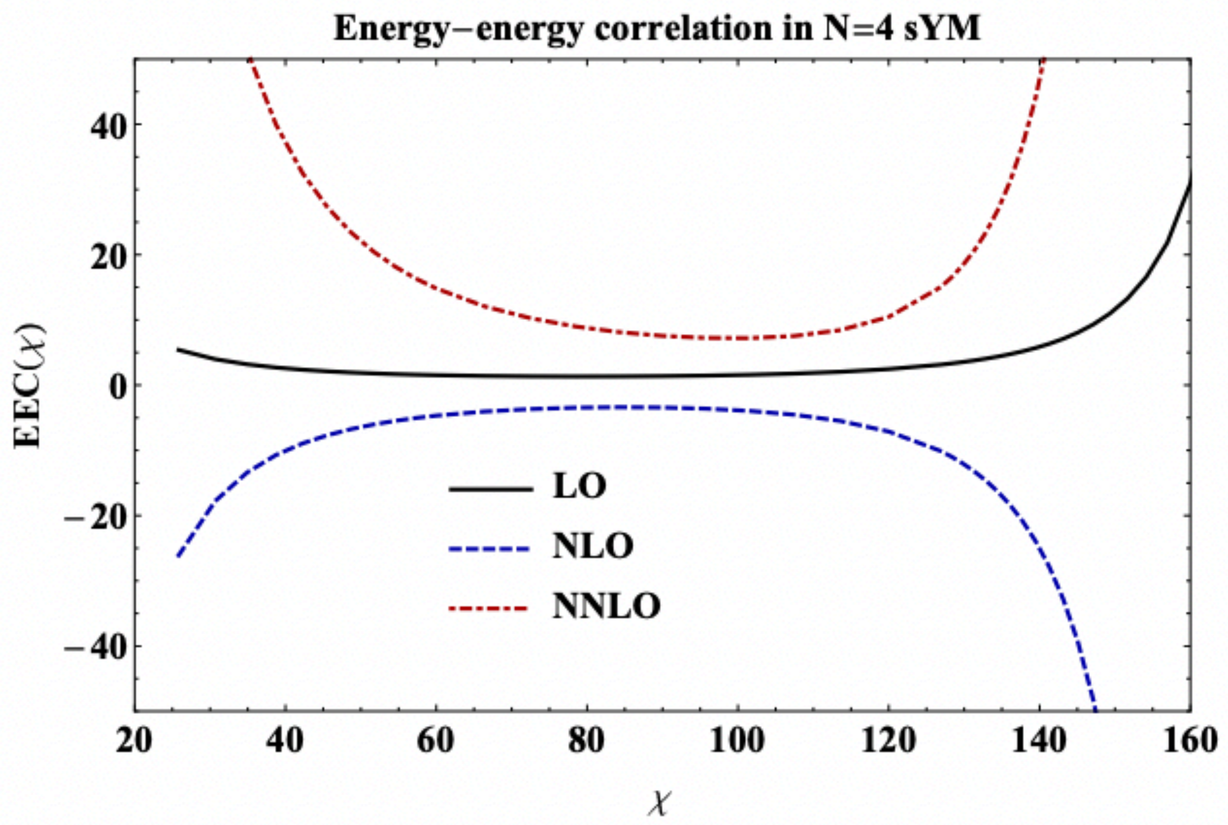
$\mathrm{NNLO} + \mathrm{NNLL}$

Dixon, Moult, Zhu, `19

General angle region

Dixon, Luo, Shtabovenko, Yang, Zhu, `18

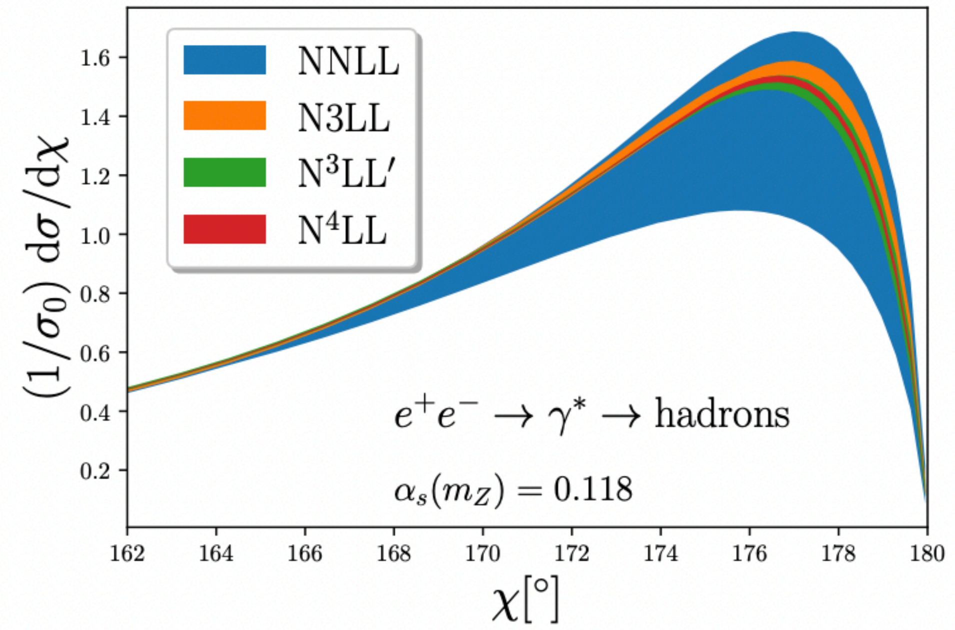




Henn, Sokatchev, Yan, Zhiboedov, `19

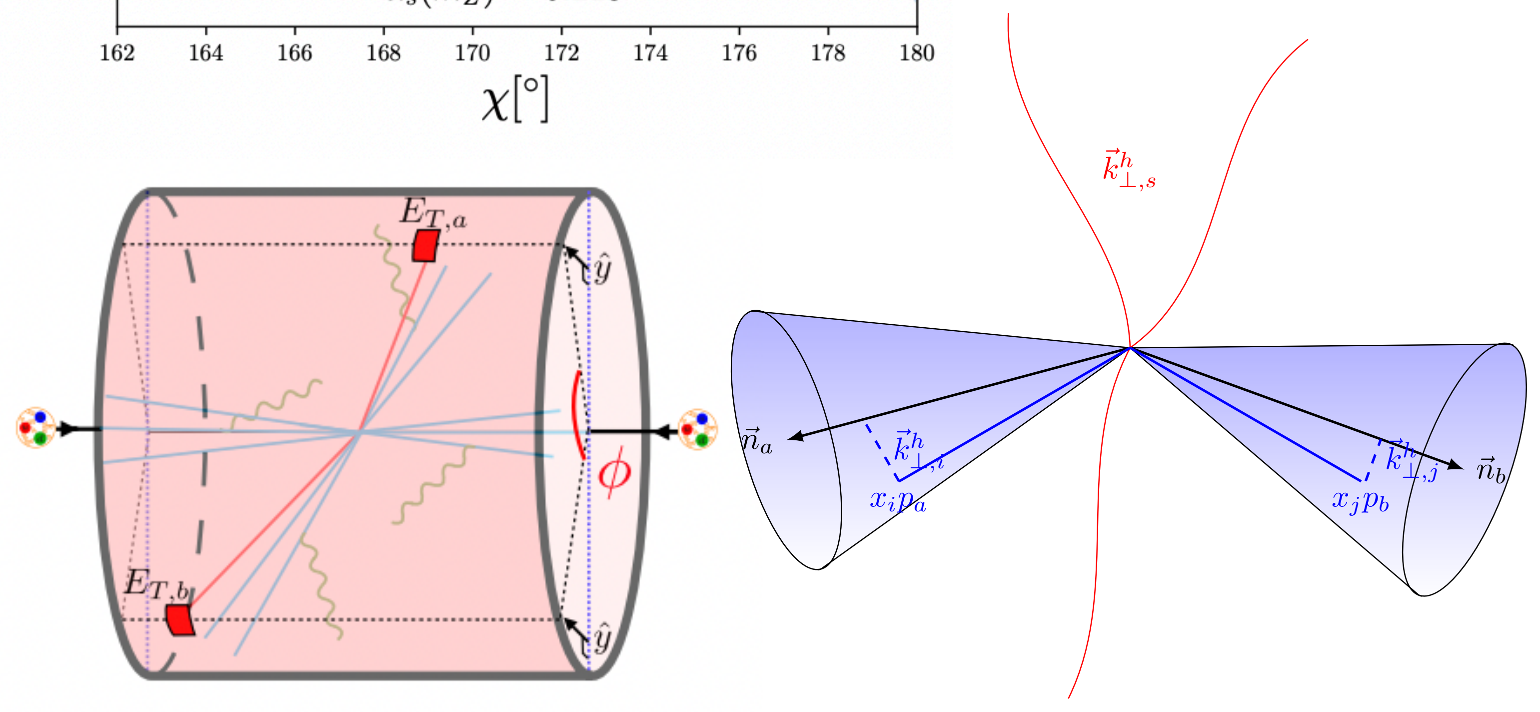
Back-to-back region

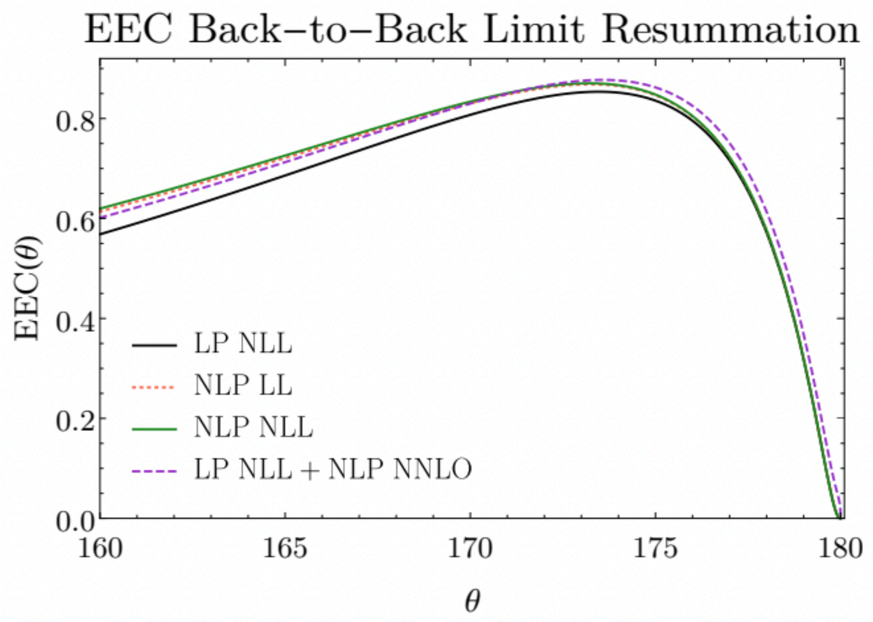
$\chi \to \pi$



$\mathrm{N^3LO} + \mathrm{N^4LL}$

Duhr, Mistlberger, Vita, `22





[Hua Xing's talk]

Energy Correlators mapping high energy collider events

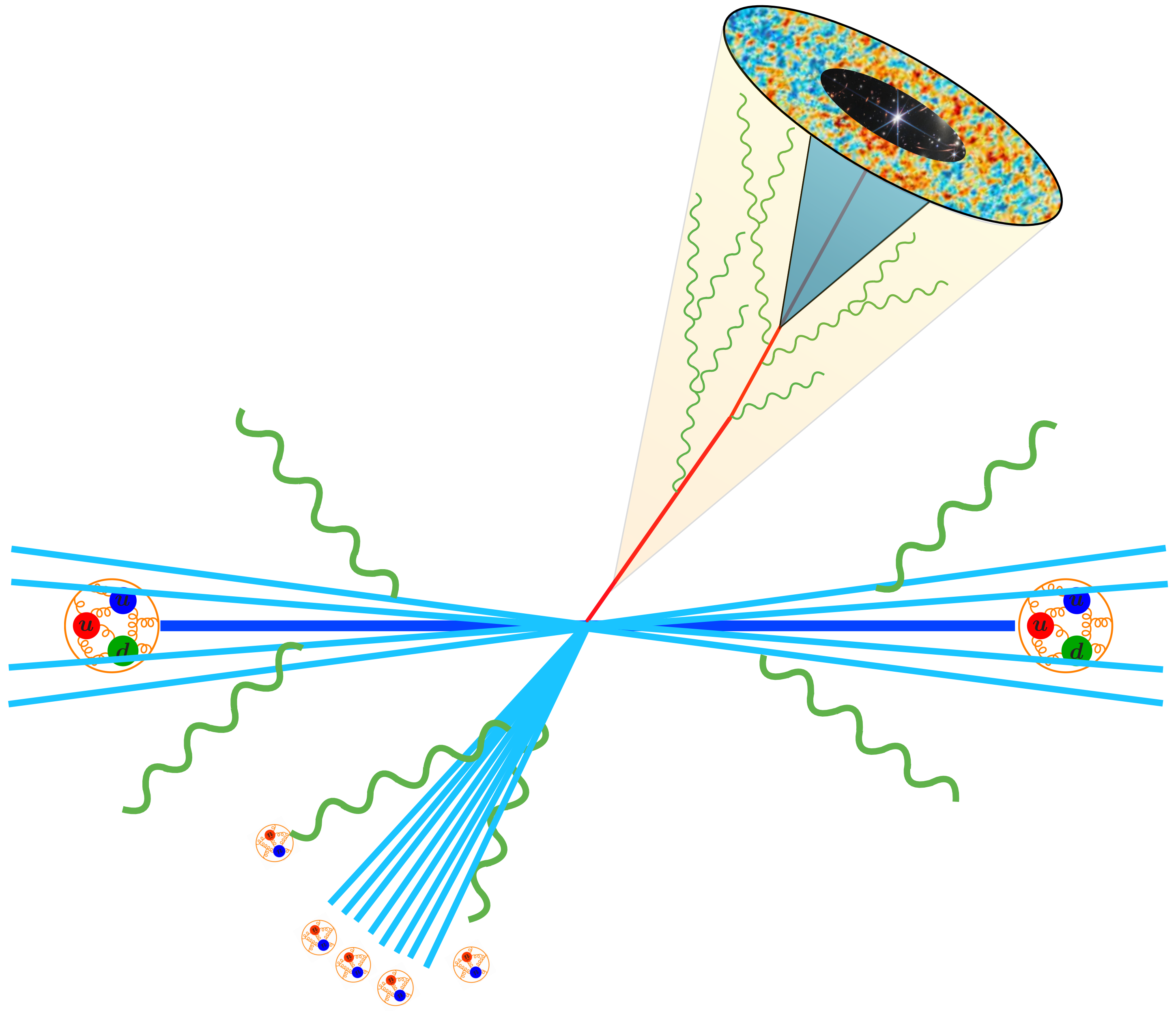
Collinear limit

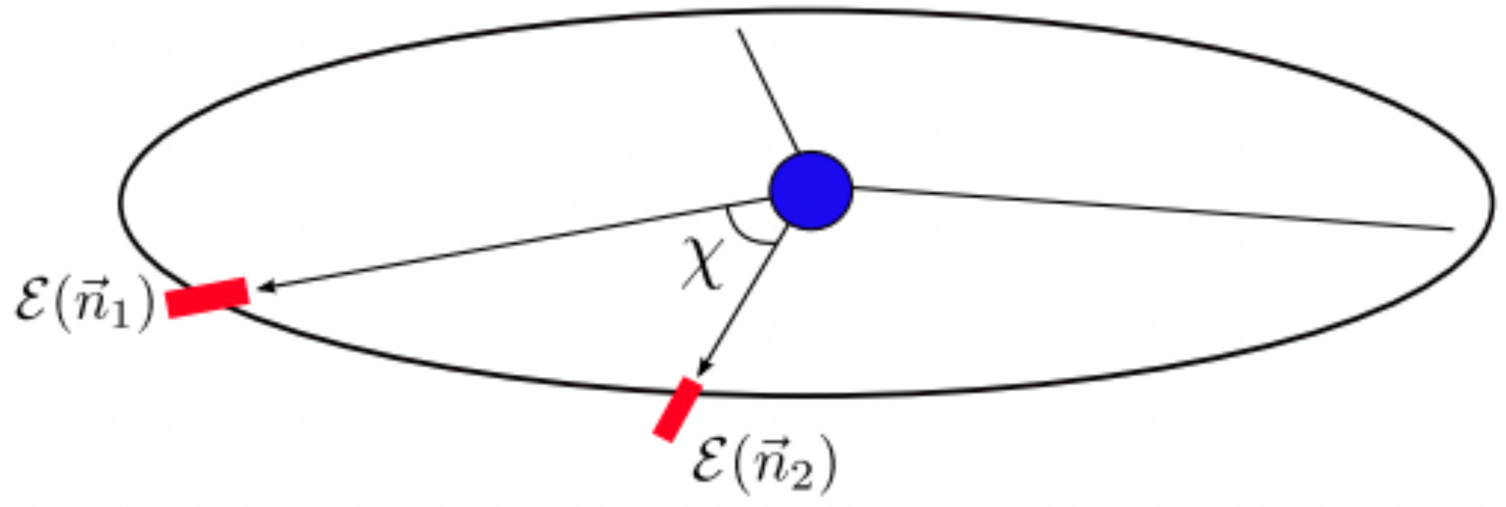
$\chi \to 0$

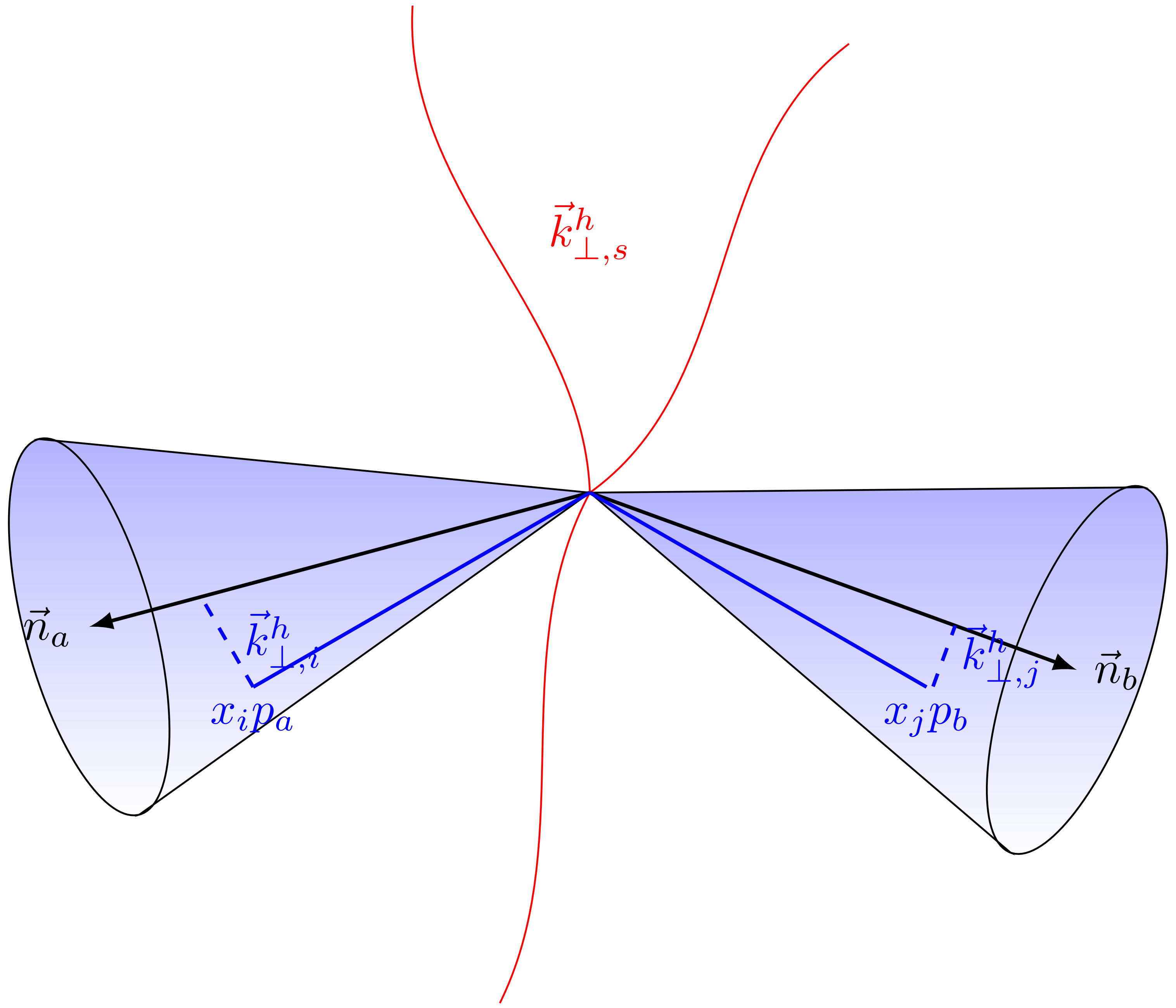
General angle region

Back-to-back region

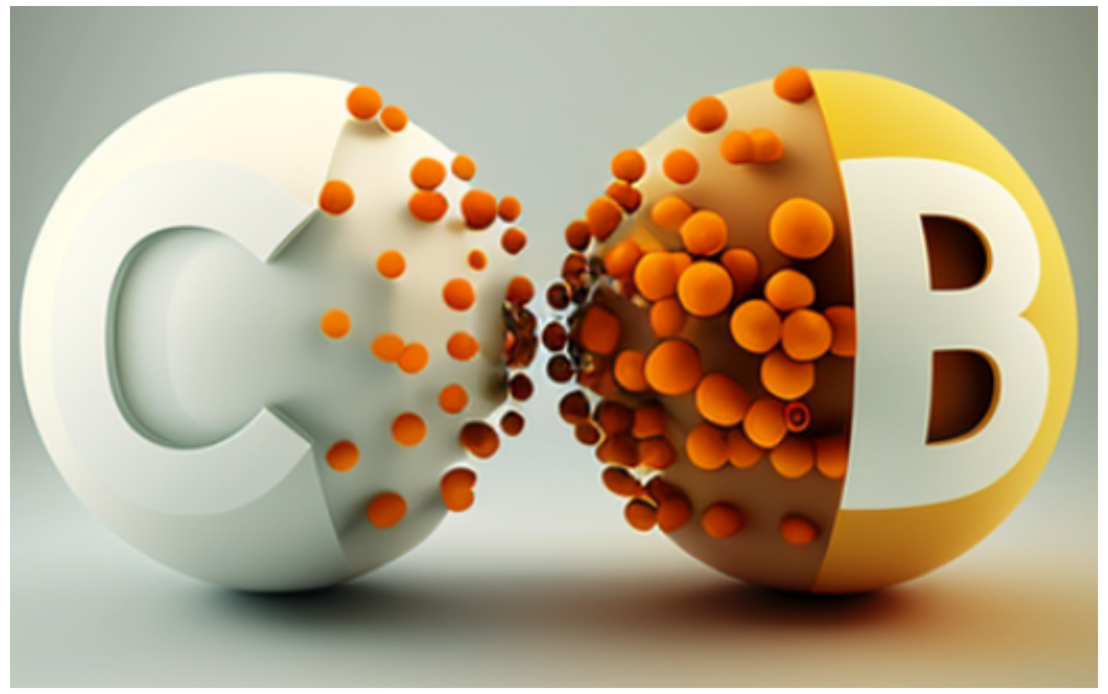
$\chi \to \pi$







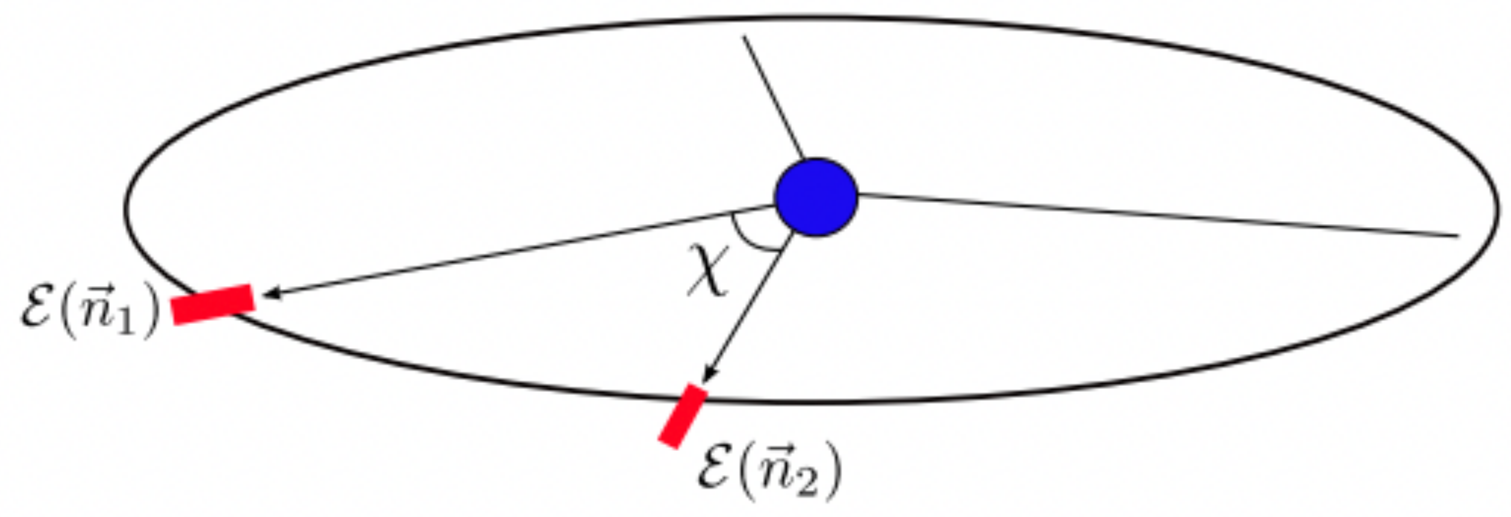
Charm



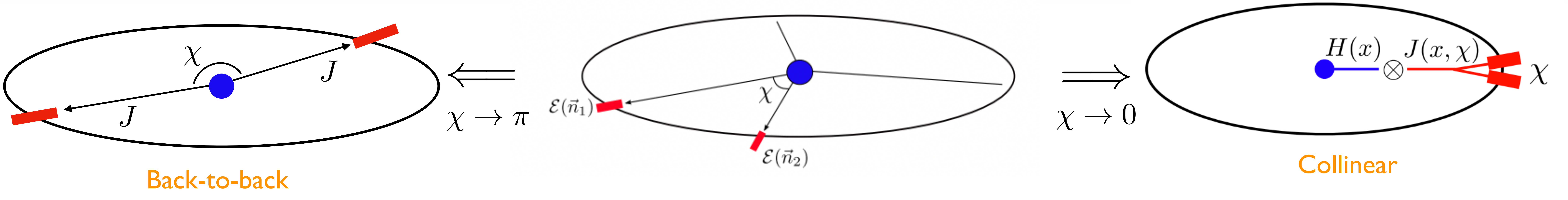
Beauty

How does intrinsic **heavy quark mass** affect each of these regions of particle collisions?

Energy-Energy Correlators at general angle with mass



Energy-Energy Correlators at general angle with mass

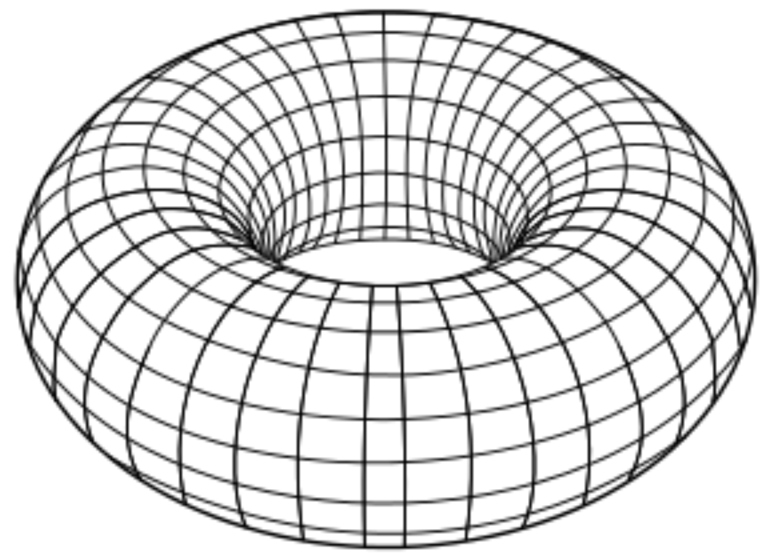


$$\frac{d\Sigma}{d\chi} = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta\left(\chi - \chi_{ij}\right) \quad \text{and} \quad \sum_{i,j} E_i E_j = Q^2 \quad \Longrightarrow \quad \int_0^\pi d\chi \frac{d\Sigma}{d\chi} = \sigma_{\text{tot}}$$

Sum rule

Dixon, Moult, Zhu, `19

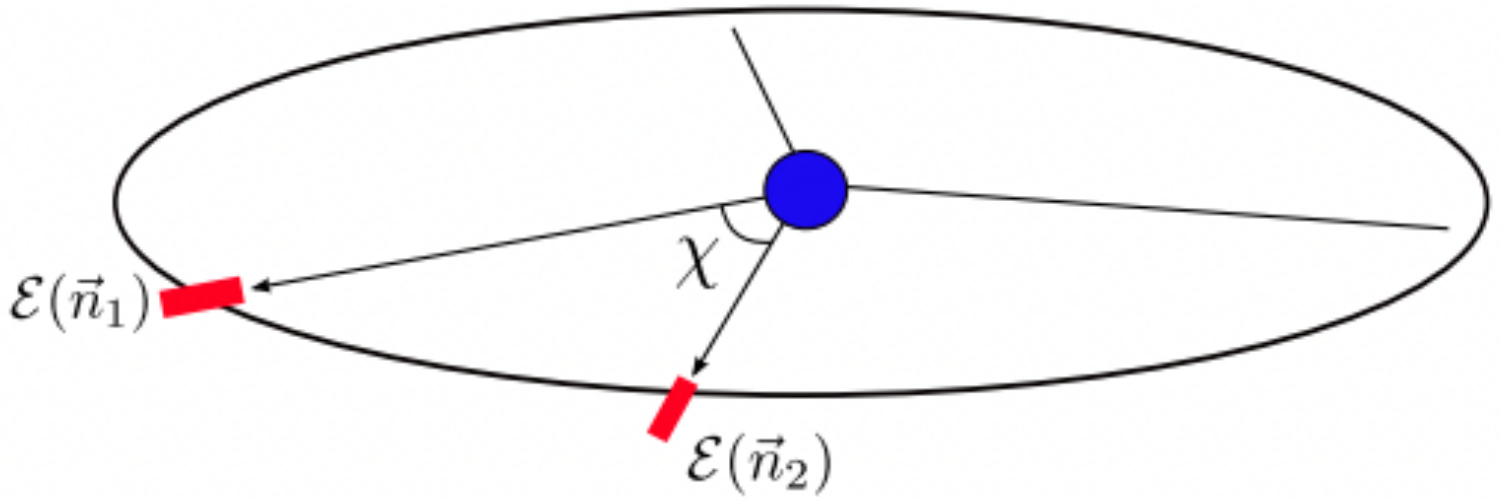
- Sum rule connects different singular regions
- Provides the means to study both collinear and back-to-back limits beyond leading power
  - Useful for developing / testing consistency of factorization formalism
- Opportunity to explore the function space of the observable, and also with e^+e^- data



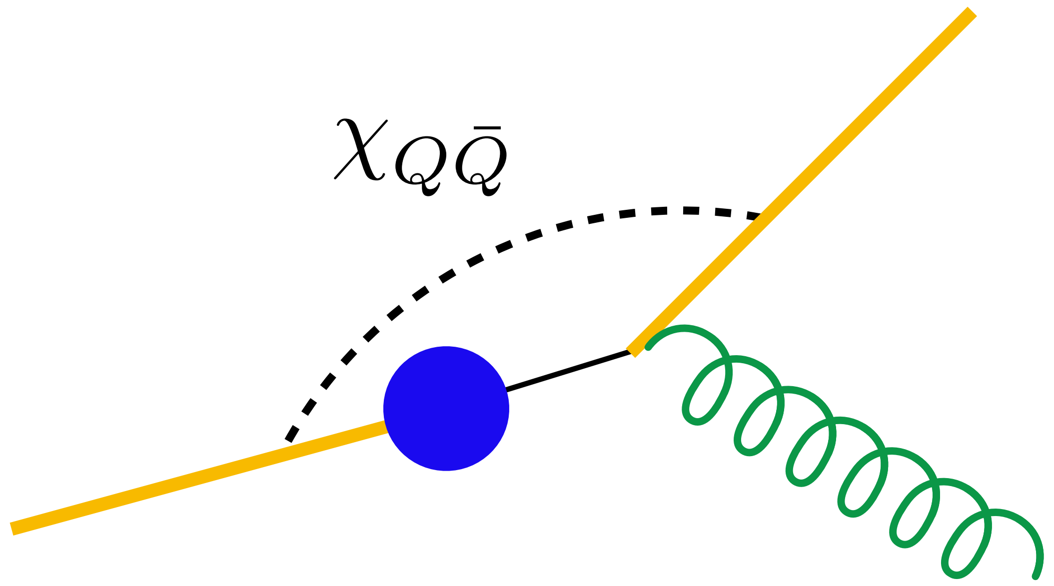
Energy-Energy Correlators at general angle with mass

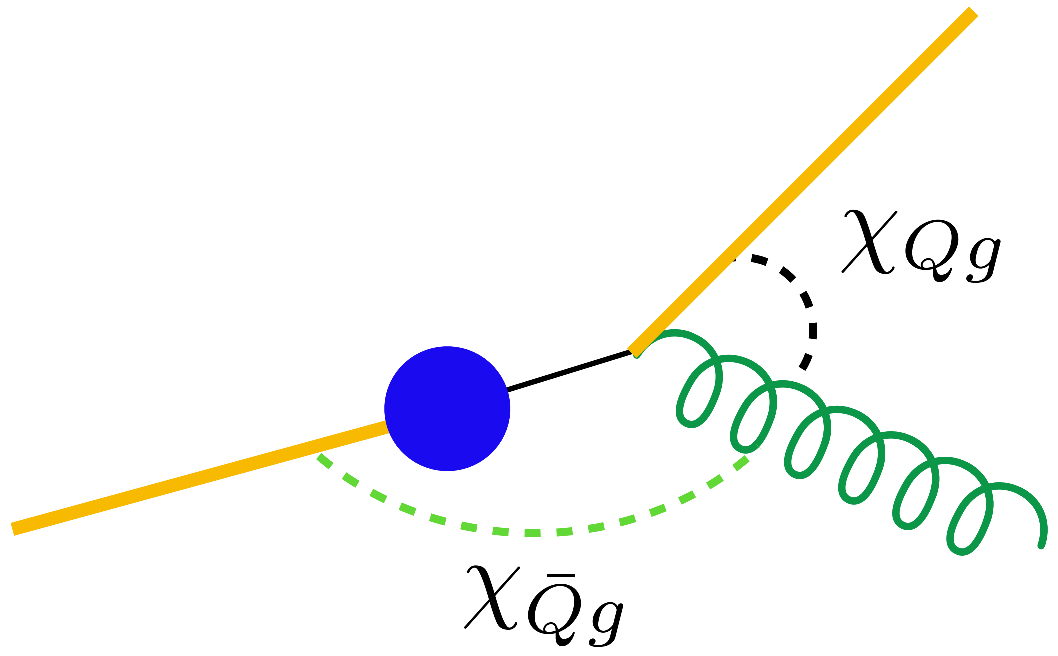
I   II   III   III

$$\left.\frac{d\Sigma_M^{(1)}}{d\chi}\right|_{e^+e^-} \sim \frac{\alpha_s C_F}{2\pi}\int dx_Q dx_{\bar{Q}} H\left(x_Q, x_{\bar{Q}}, y\right)\left[2x_Q x_{\bar{Q}}\delta\left(\chi_{Q\bar{Q}} - f_{Q\bar{Q}}\left(x_Q, x_{\bar{Q}}, y\right)\right) + 2x_Q x_g \delta\left(\chi_{Qg} - f_{Qg}\left(x_Q, x_{\bar{Q}}, y\right)\right) + \bar{Q}g\text{case}\right]$$

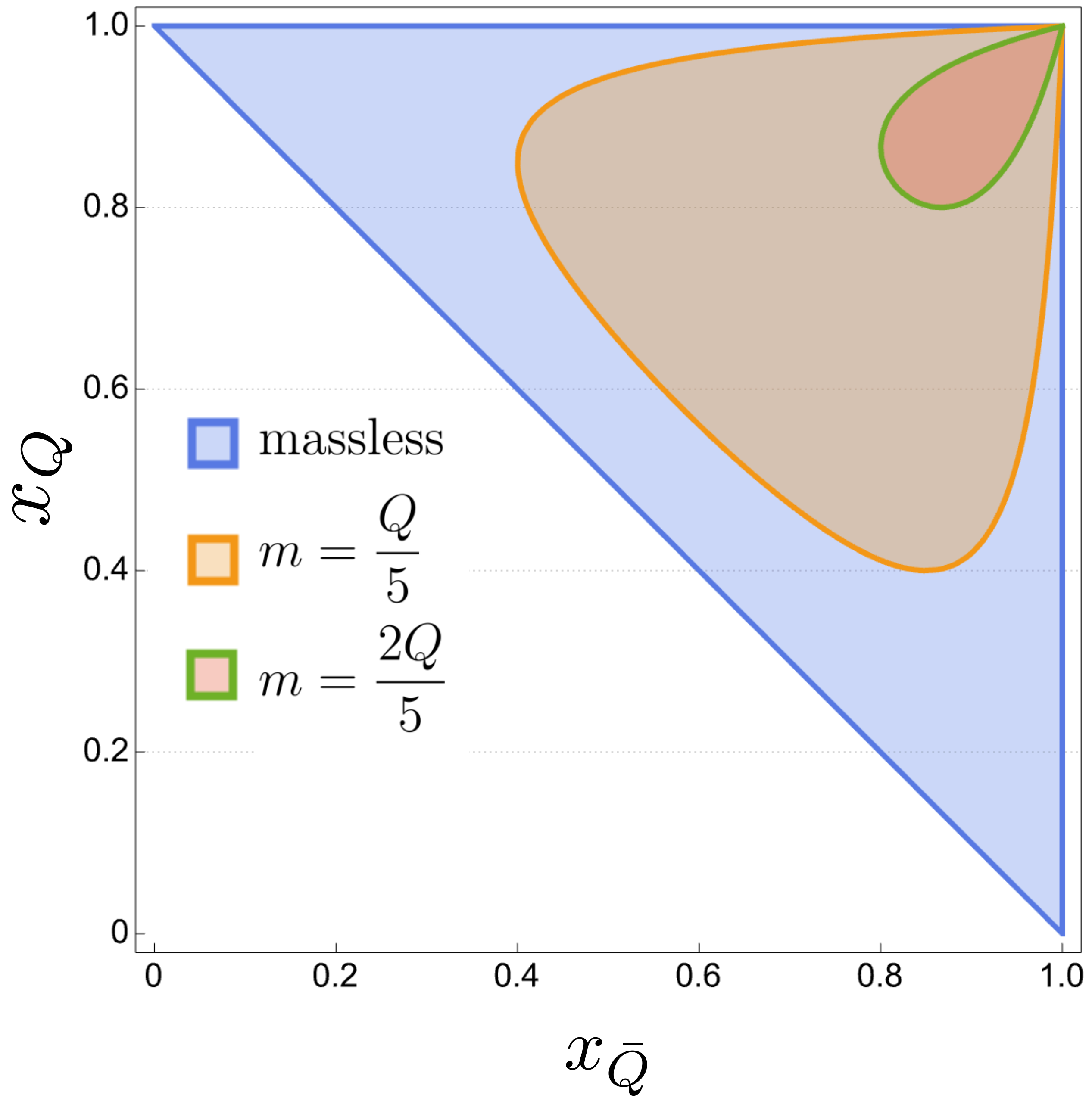


$$0 < y = \frac{2m}{Q} < 1$$





I Phase space

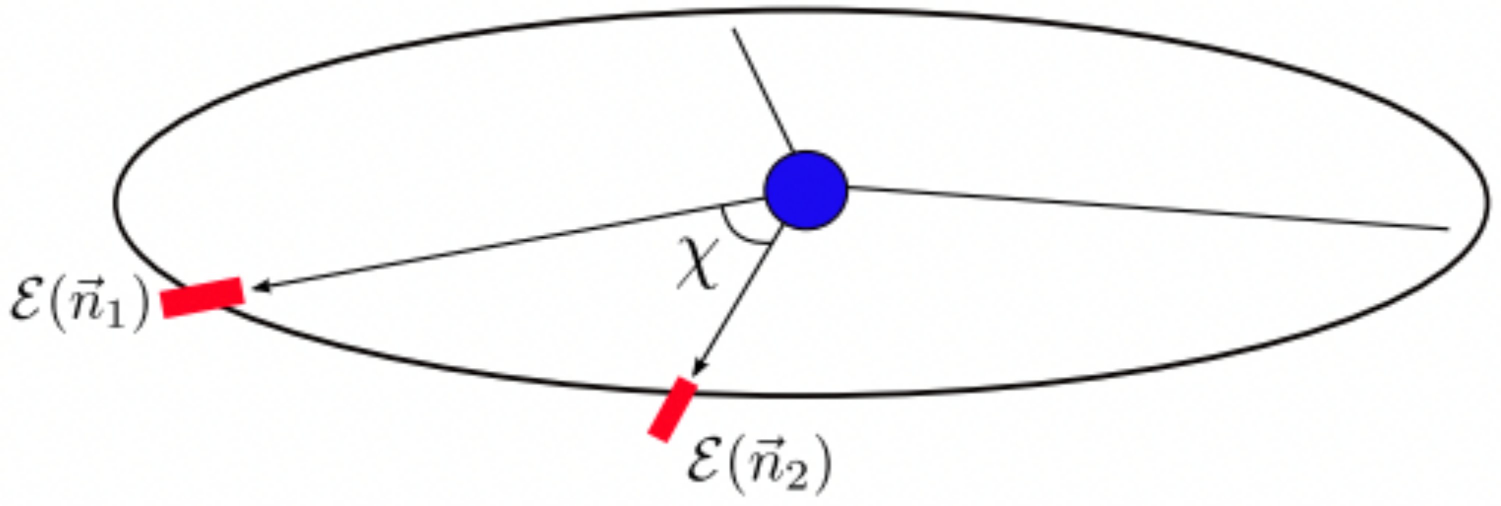


- Phase space suppression for massive final states

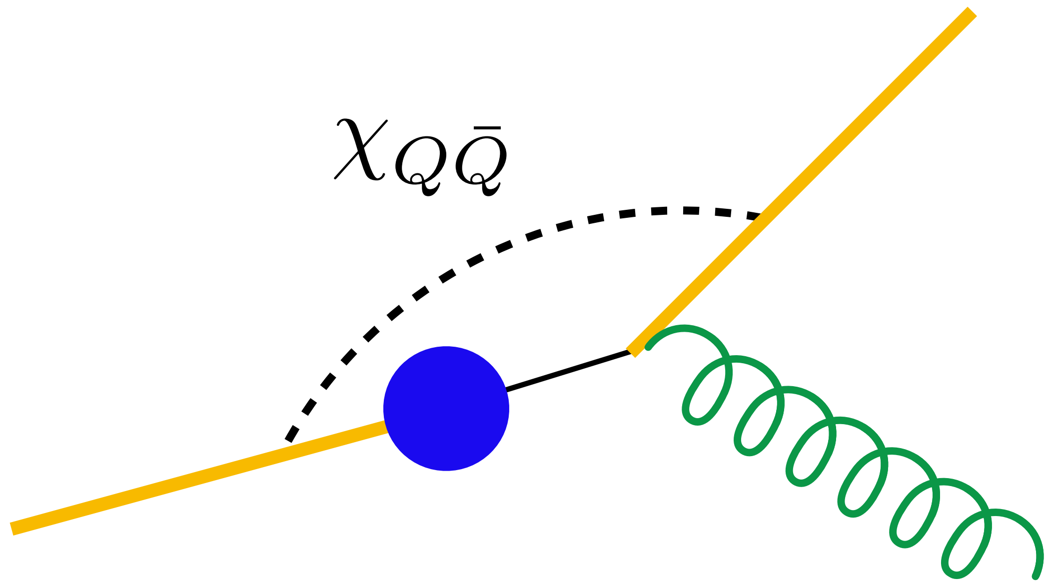
Energy-Energy Correlators at general angle with mass

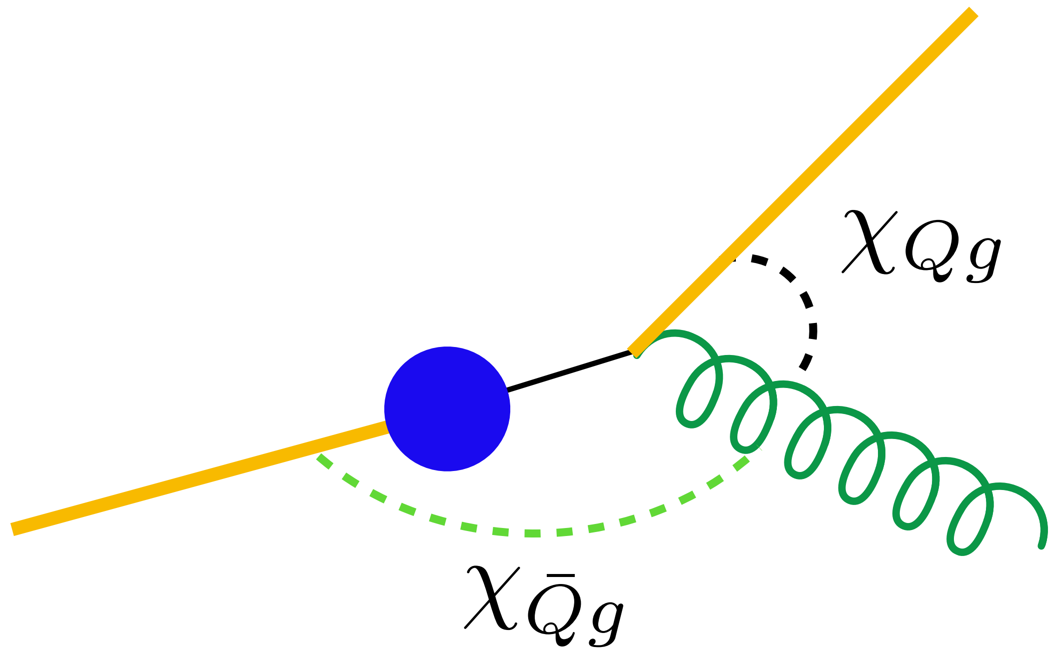
I II III III

$$\left.\frac{d\Sigma_M^{(1)}}{d\chi}\right|_{e^+e^-} \sim \frac{\alpha_s C_F}{2\pi}\int dx_Q dx_{\bar{Q}} H\left(x_Q, x_{\bar{Q}}, y\right)\left[2x_Q x_{\bar{Q}}\delta\left(\chi_{Q\bar{Q}} - f_{Q\bar{Q}}\left(x_Q, x_{\bar{Q}}, y\right)\right) + 2x_Q x_g \delta\left(\chi_{Qg} - f_{Qg}\left(x_Q, x_{\bar{Q}}, y\right)\right) + \bar{Q}g\text{case}\right]$$



$$0 < y = \frac{2m}{Q} < 1$$





II Matrix Elements

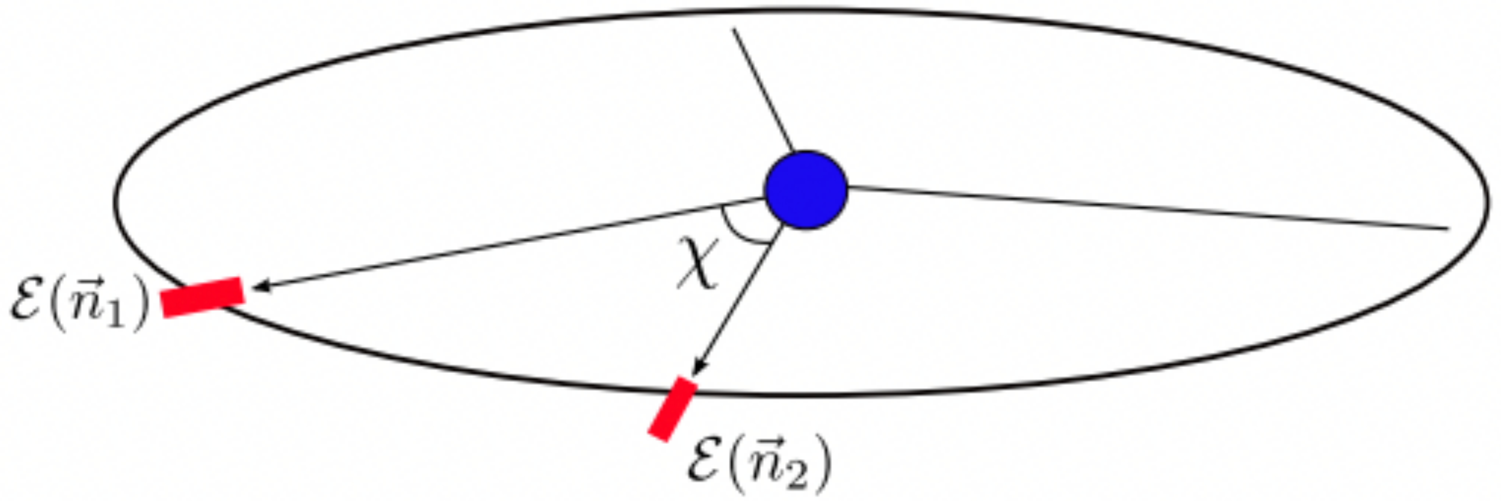
$$H(x_Q, x_{\bar{Q}}, y) = H_0(x_Q, x_{\bar{Q}}) + y^2 H_2(x_Q, x_{\bar{Q}}) + y^4 H_4(x_Q, x_{\bar{Q}})$$

The usual massless one $\quad H_0(x_Q, x_{\bar{Q}}) \sim \dfrac{x_Q^2 + x_{\bar{Q}}^2}{(1 - x_Q)(1 - x_{\bar{Q}})}$

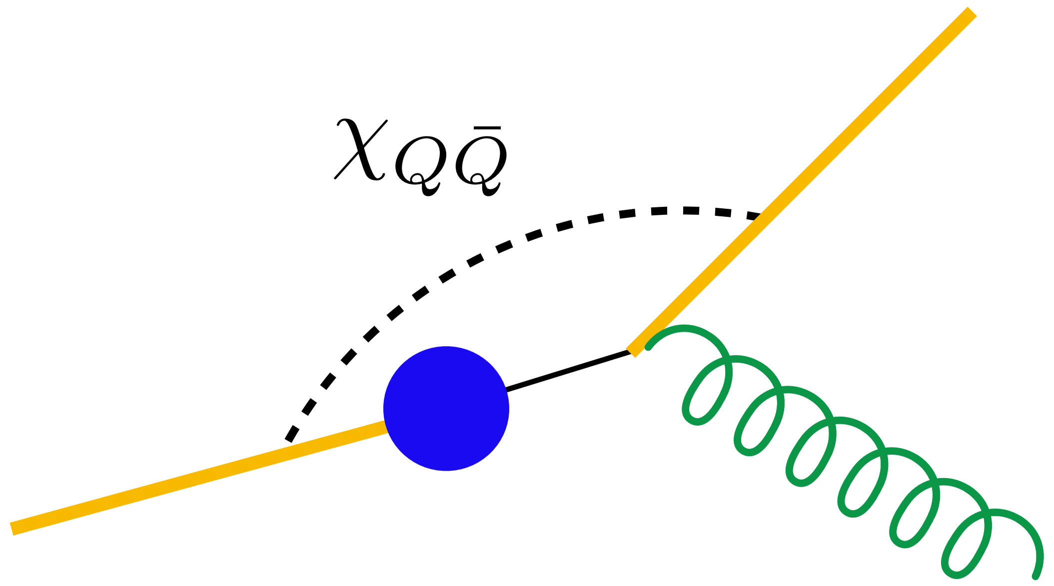
Energy-Energy Correlators at general angle with mass

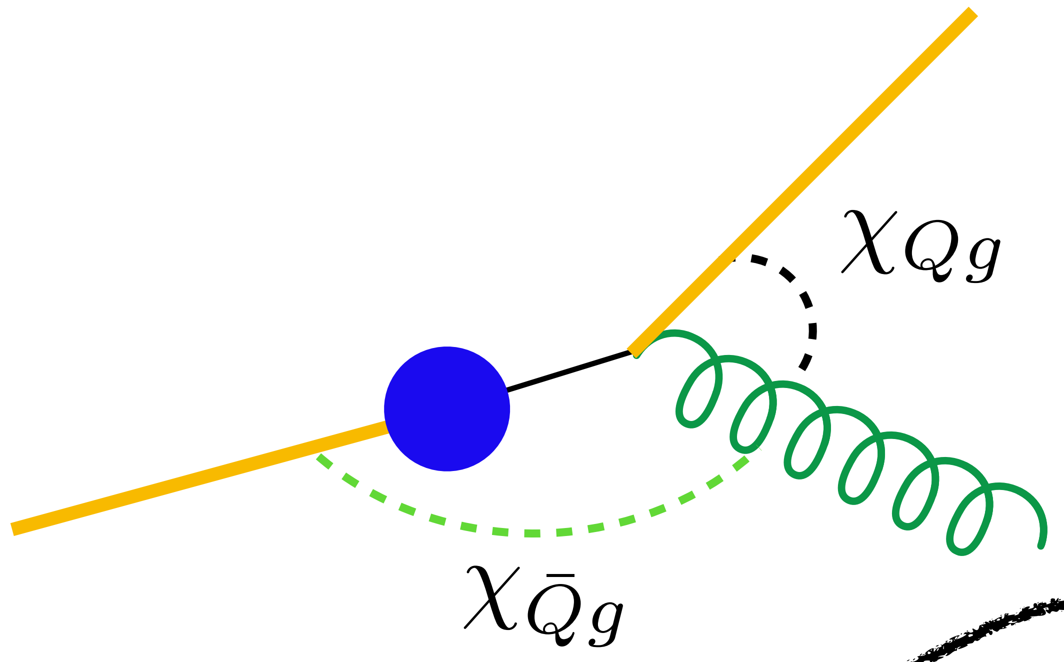
I II III III

$$\left.\frac{d\Sigma_M^{(1)}}{d\chi}\right|_{e^+e^-} \sim \frac{\alpha_s C_F}{2\pi}\int dx_Q dx_{\bar{Q}} H\left(x_Q, x_{\bar{Q}}, y\right)\left[2x_Q x_{\bar{Q}}\delta\left(\chi_{Q\bar{Q}} - f_{Q\bar{Q}}\left(x_Q, x_{\bar{Q}}, y\right)\right) + 2x_Q x_g \delta\left(\chi_{Qg} - f_{Qg}\left(x_Q, x_{\bar{Q}}, y\right)\right) + \bar{Q}g\text{case}\right]$$



$$0 < y = \frac{2m}{Q} < 1$$



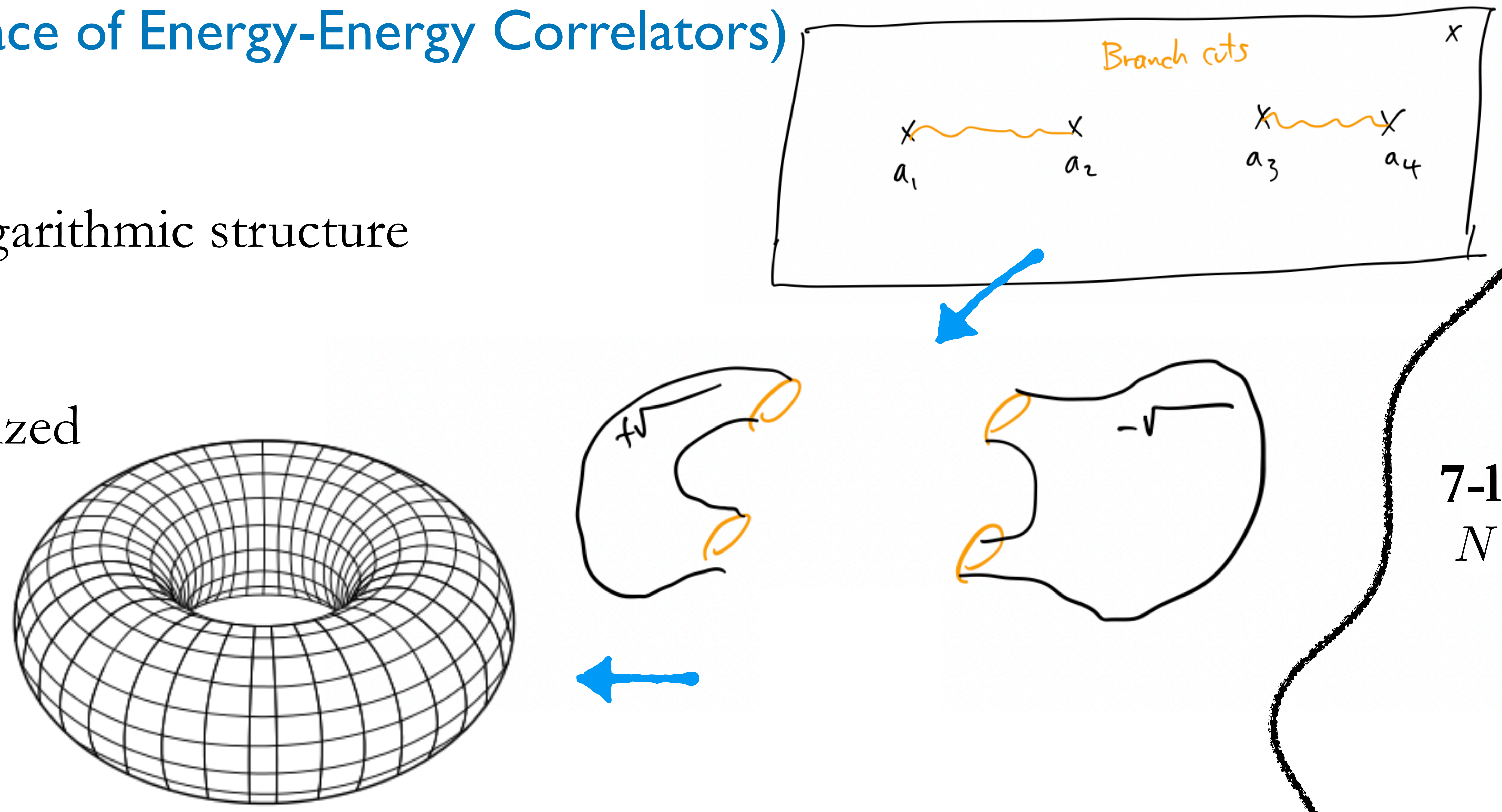


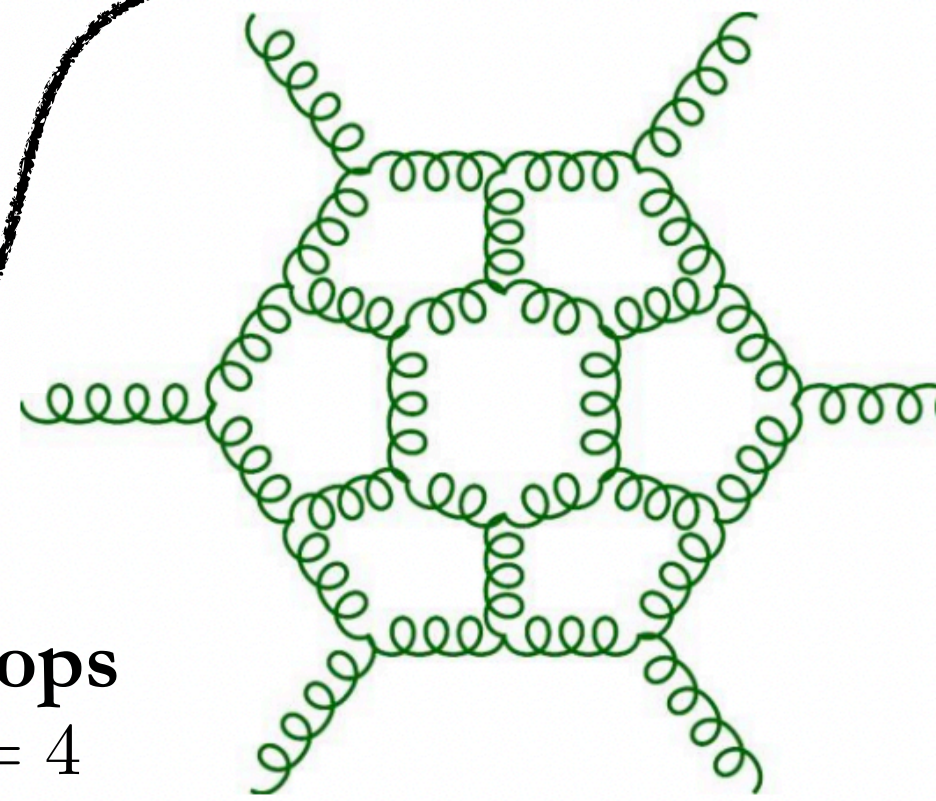
III Measurements (Study of function space of Energy-Energy Correlators)

$$f_{Qg} \sim \frac{1}{x_g\sqrt{x_Q^2 - y^2}}$$ can be rationalized, usual logarithmic structure

$$f_{Q\bar{Q}} \sim \frac{1}{\sqrt{x_Q^2 - y^2}\sqrt{x_{\bar{Q}}^2 - y^2}}$$ cannot be rationalized

$$\Longrightarrow \int dx^* \frac{1}{\sqrt{P_4(x^*)}}$$ **Elliptic integrals**



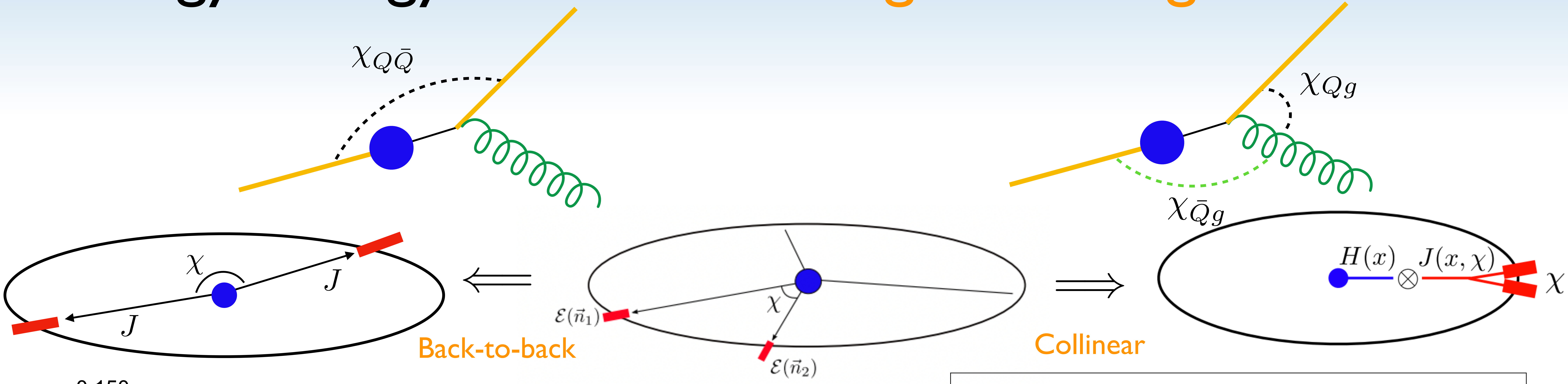


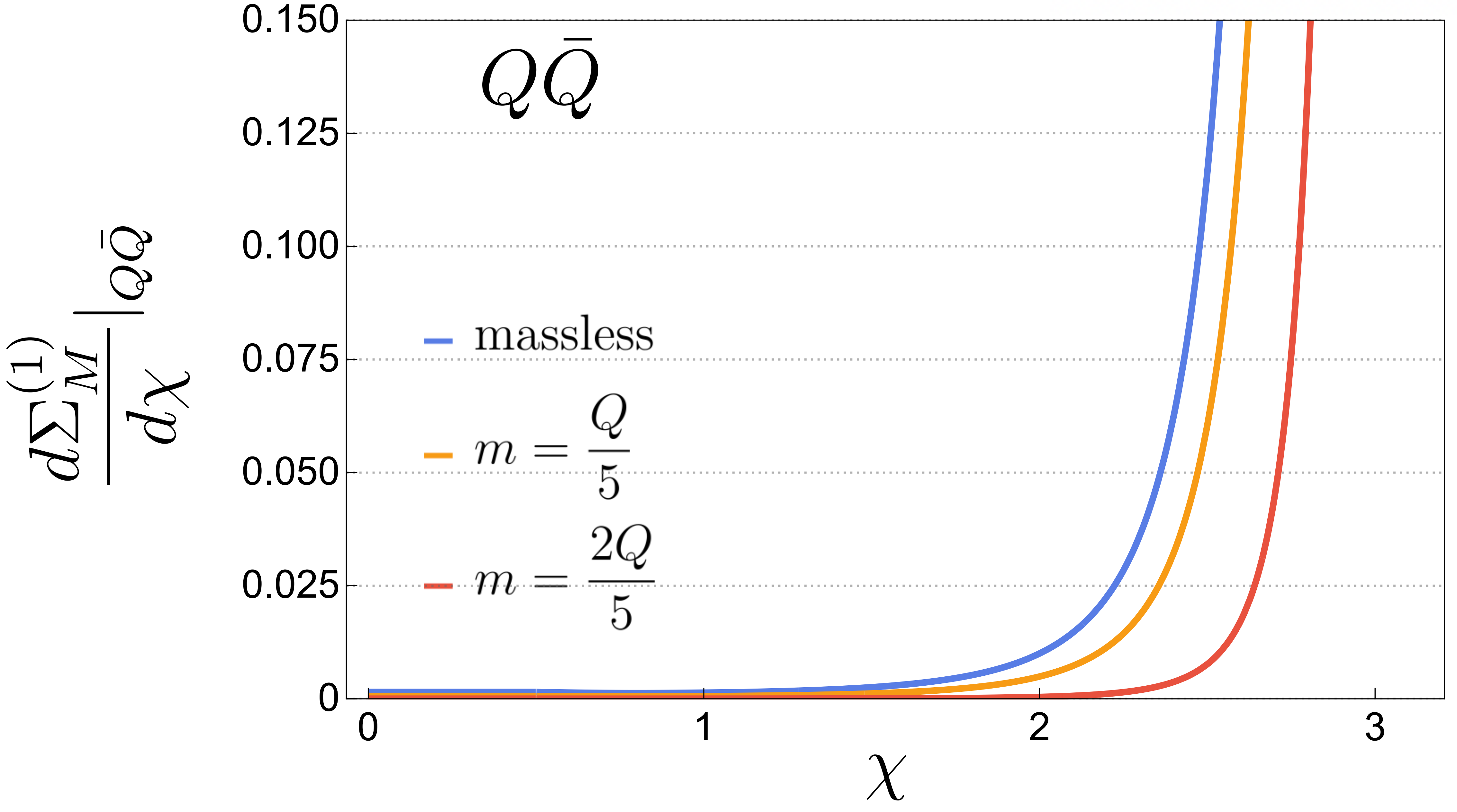
7-loops
$N = 4$

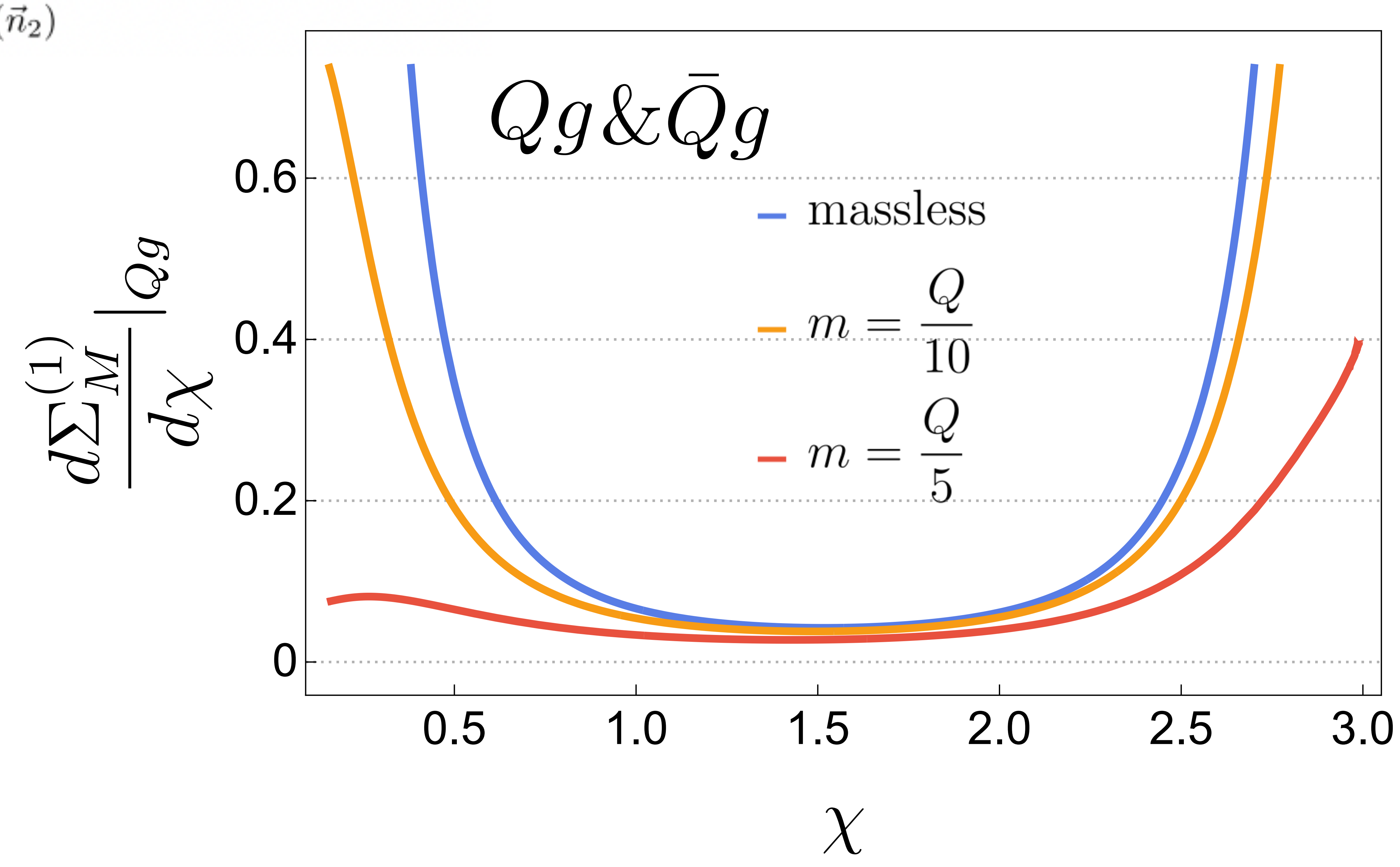
Amplitude bootstrap

Dixon et al, Henn et al, …

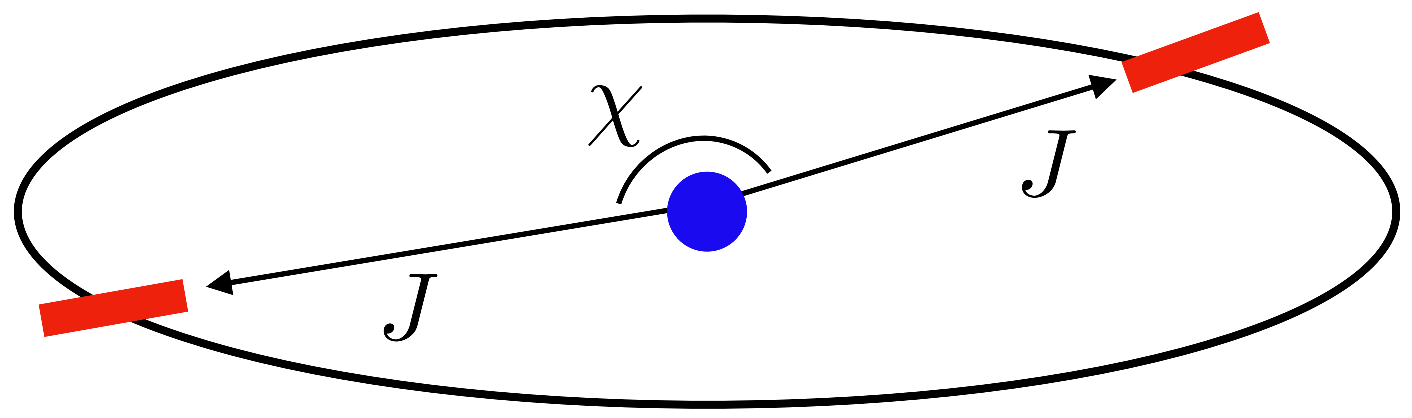
Energy-Energy Correlators at general angle with mass



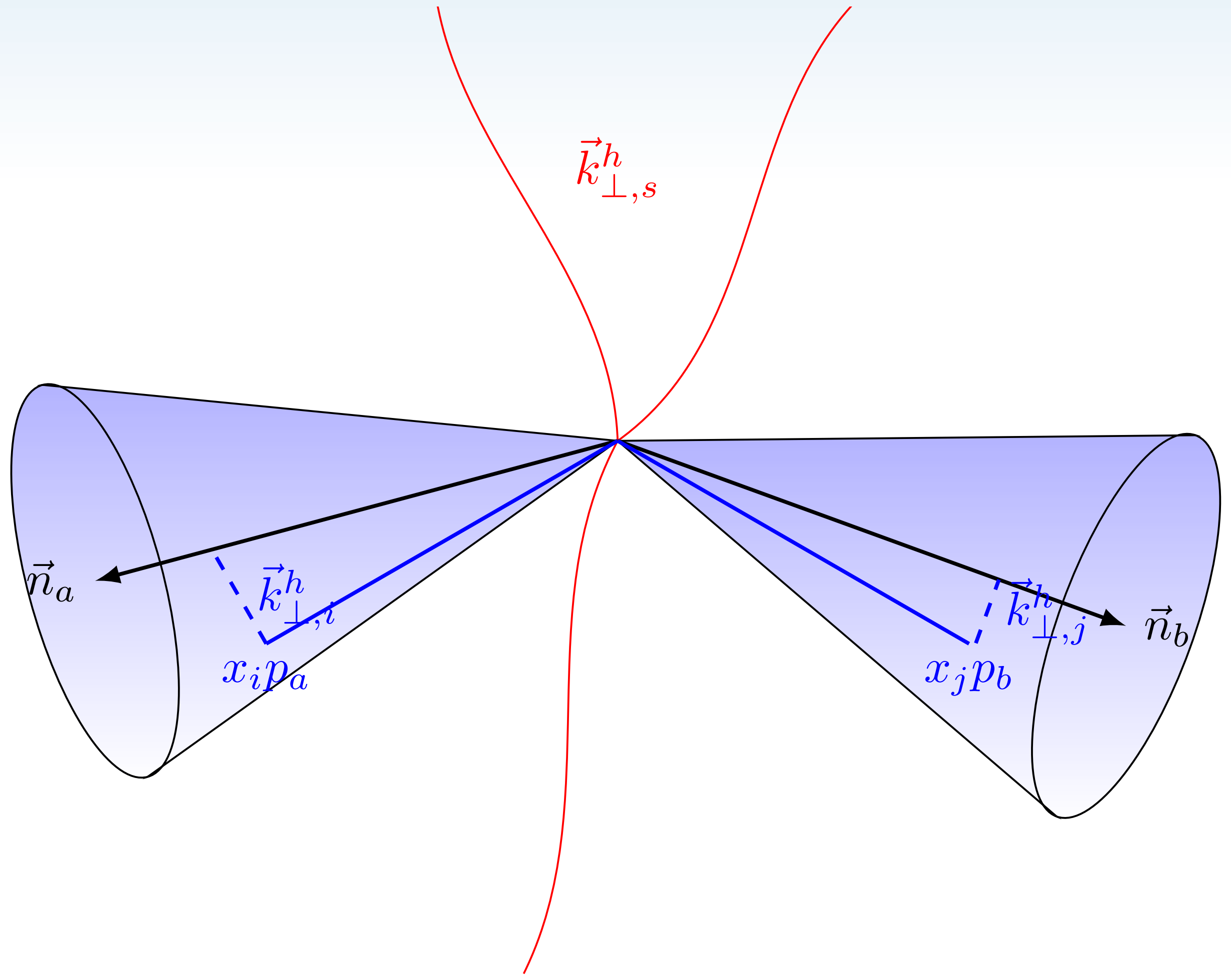




Back-to-back Energy-Energy Correlators with mass



Back-to-back Energy-Energy Correlators with mass



$$\frac{d\Sigma_M}{dz} = \frac{1}{2}\int d^2\vec{k}_\perp \int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\cdot\vec{k}_\perp} H(Q,\mu) J_n^M\left(\vec{b}_\perp, m,\mu,\nu\right) J_{\bar{n}}^M\left(\vec{b}_\perp, m,\mu,\nu\right) S^M_{\mathrm{EEC}}\left(\vec{b}_\perp, m,\mu,\nu\right)\delta\left(1-z-\frac{\vec{k}_\perp^2}{Q^2}\right)$$

where $z = \dfrac{1+\cos\chi}{2}$

Assuming $\Lambda^2_{\mathrm{QCD}} \ll Q^2(1-z) \sim m^2$, [See **Zhiquan**'s talk for NP matching]

von Kuk, Michel, Sun `23

$$J_n^M(b_\perp, m,\mu,\nu) = \sum_h \int dx\, x\, D_{h/Q}(x, b_\perp, m,\mu,\nu) = \int dx\, x\, d_{Q/Q}(x, b_\perp, m,\mu,\nu) \boxed{\sum_H \chi_H} = 1$$

Favored contribution

$$+\int dx\, x\, d_{\bar{Q}/Q}(x, b_\perp, m,\mu,\nu)\boxed{\sum_{\bar{H}} \chi_{\bar{H}}} = 1$$

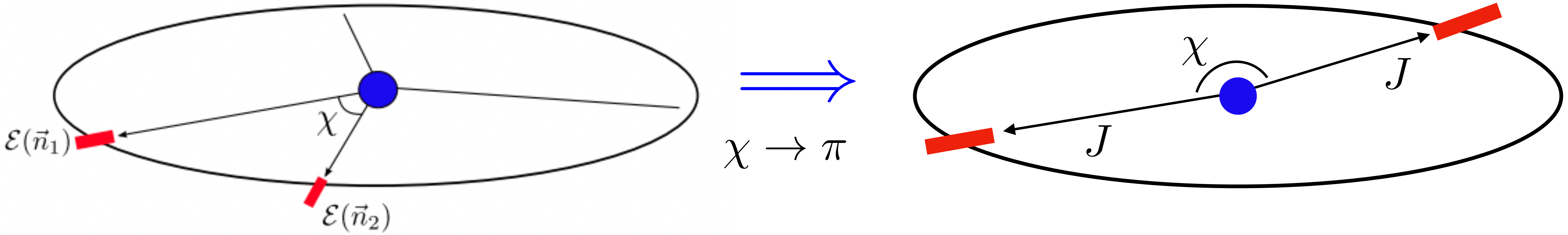
Disfavored contribution

$$+\int d\tau\,\tau \sum_j \mathcal{I}_{Qj}(\tau, b_\perp, m,\mu,\nu)\boxed{\sum_h \int dz\, z\, D_{j\to h}(z,\mu)} = 1$$

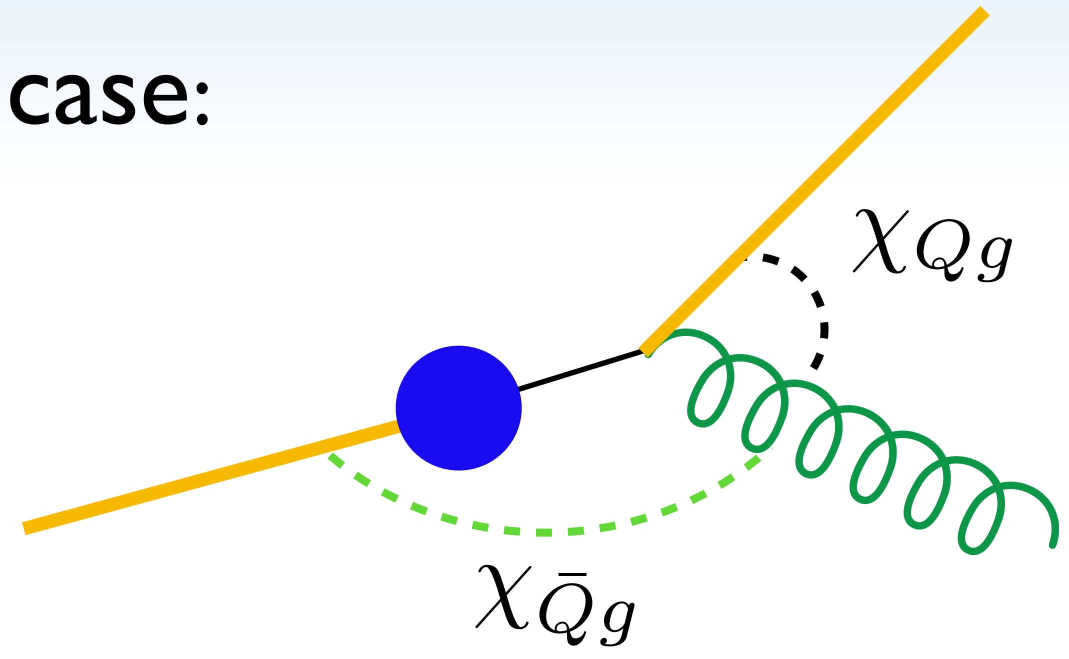
Light hadron contribution

$$+\,\mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{m}\right) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{Q\sqrt{1-z}}\right)$$

Back-to-back Energy-Energy Correlators with mass



Qg case:



Taking the back-to-back limit of the general angle result, $Q^2(1-z) \sim m^2$

$$\frac{d\Sigma_M}{dz}\bigg|^{z\to 1}_{Qg+\bar{Q}g} = \frac{\alpha_s C_F}{6\pi}\frac{Q^2}{m^2+Q^2(1-z)}\left(1+\frac{3Q^2(1-z)}{m^2+Q^2(1-z)}\right) \quad \textbf{LP}$$

$$+\frac{\alpha_s C_F}{8\pi}\left[\frac{31}{18}-\frac{Q^2(1-z)}{m^2+Q^2(1-z)}+\frac{4Q^4(1-z)^2}{(m^2+Q^2(1-z))^2}-\frac{2Q^6(1-z)^3}{(m^2+Q^2(1-z))^3}+12\ln\left(\frac{Q^2(1-z)+m^2}{Q^2}\right)\right]+\cdots$$

NLP

From the back-to-back factorization, at this order, we can simply look at $\mathcal{I}^{(1)}_{Qg}$

Assuming the reciprocity relation holds even with mass at this order using the known result from DY case, one finds

$$\frac{\alpha_s C_F}{6\pi}\frac{Q^2}{m^2+Q^2(1-z)}\left(6-\frac{2Q^2(1-z)}{m^2+Q^2(1-z)}\right)$$

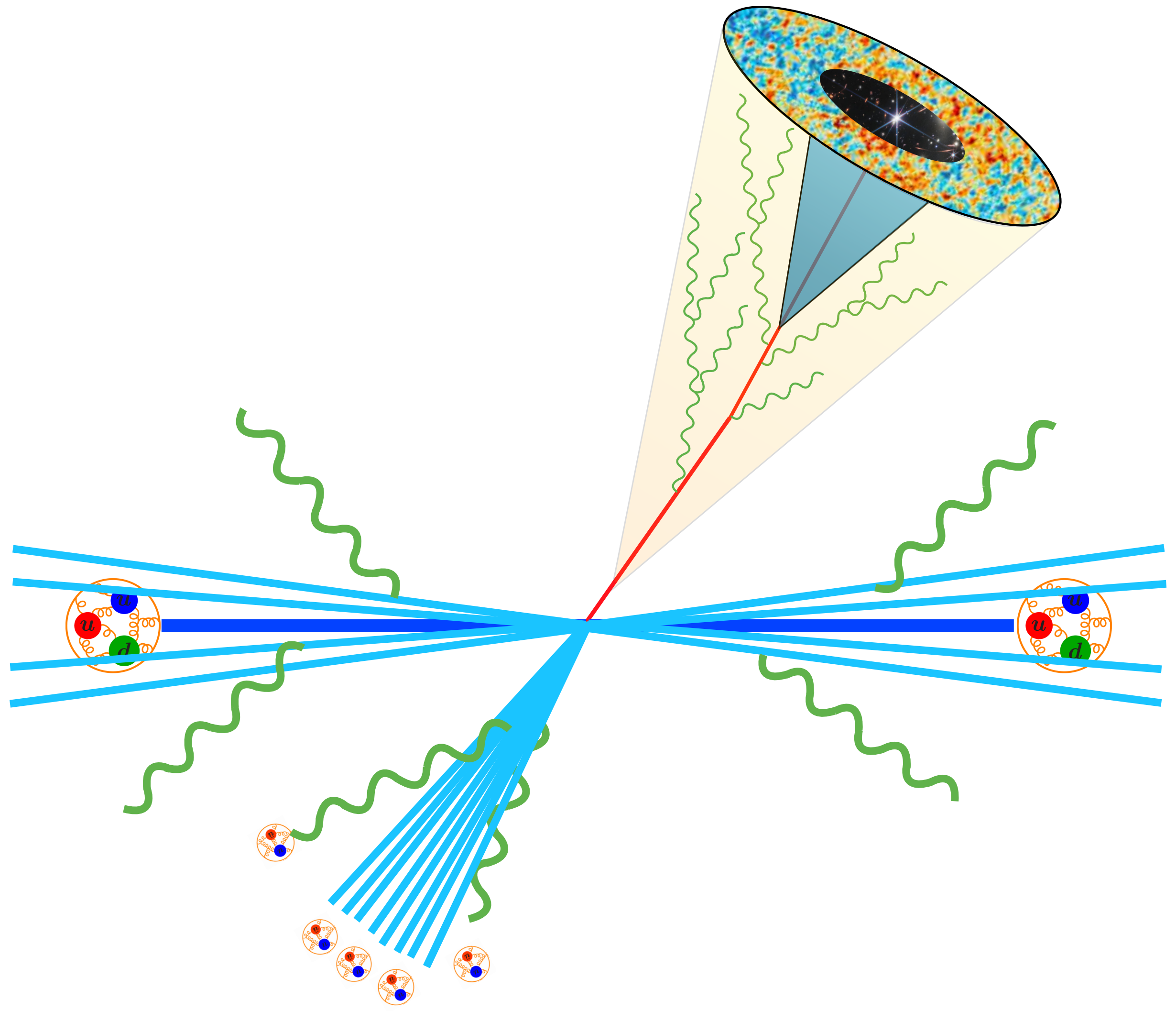
Reciprocity relation broken with mass!

Pietrulewicz, Samitz, Spiering, Tackmann `17

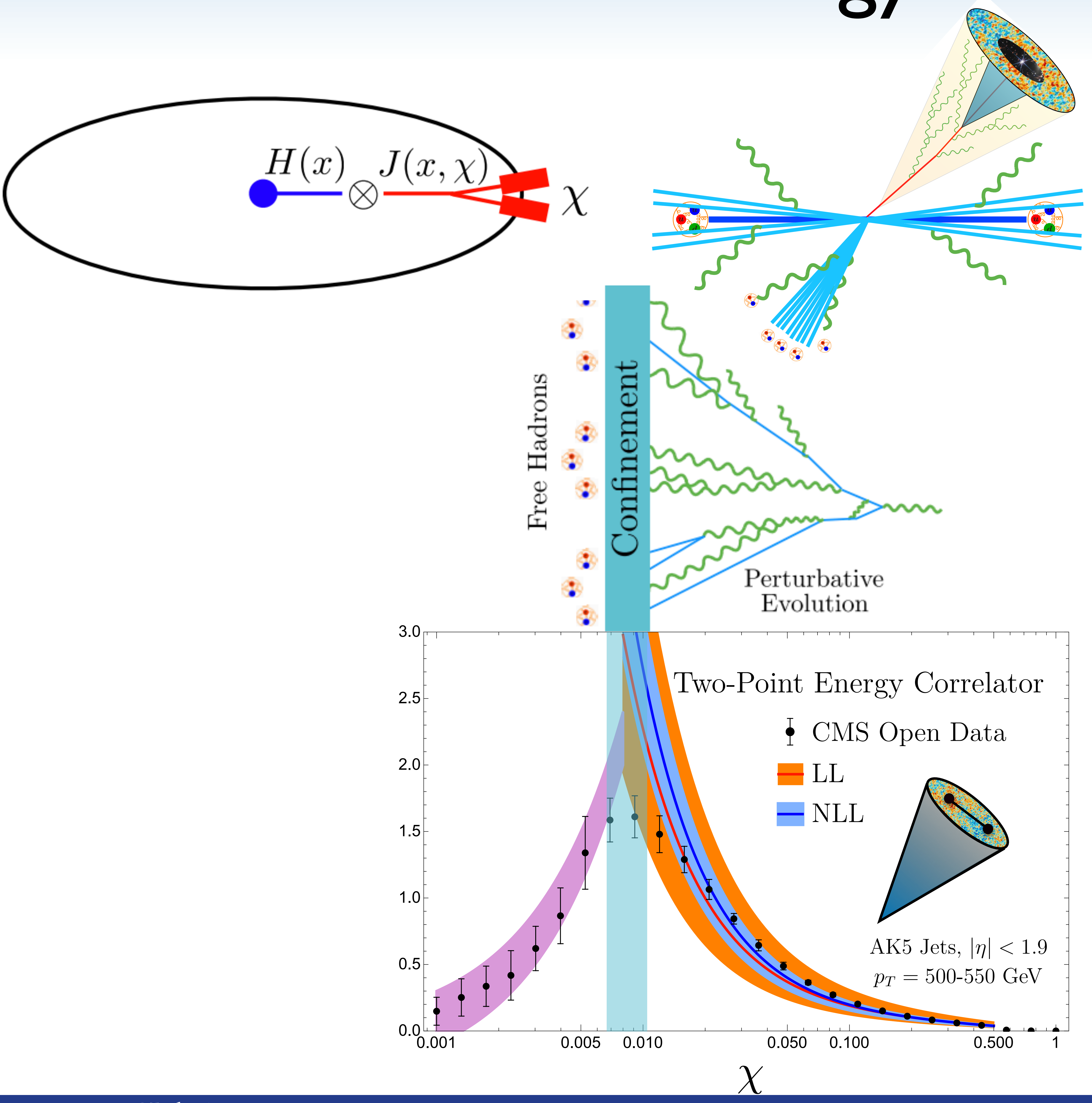
Gribov, Lipatov `72, Chen, Yang, Zhu, Zhu `20

- Expansion of the $Q\bar{Q}$ is less trivial, but is interesting how the elliptic structure is expected to simplify.

Collinear Energy-Energy Correlators with mass

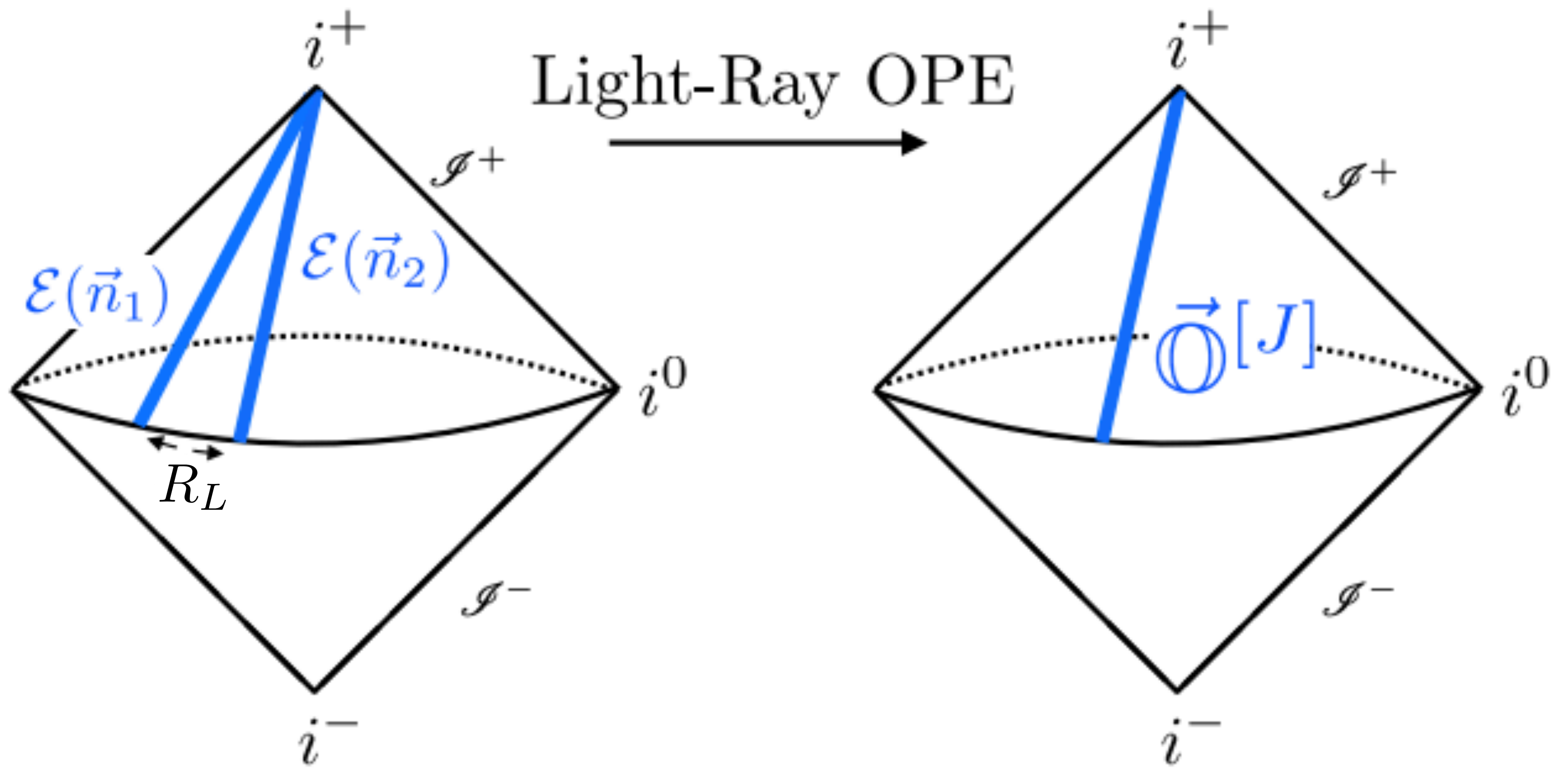


Collinear Energy-Energy Correlators with mass



$$\Sigma\left(\chi, p_T^2, m_Q, \mu\right) = \int_0^1 dx\, x^2 \vec{J}\left(\chi, x, m_Q, \mu\right) \cdot \vec{H}\left(x, p_T^2, \mu\right)$$

Collinear dynamics factorize identically for different collider environment

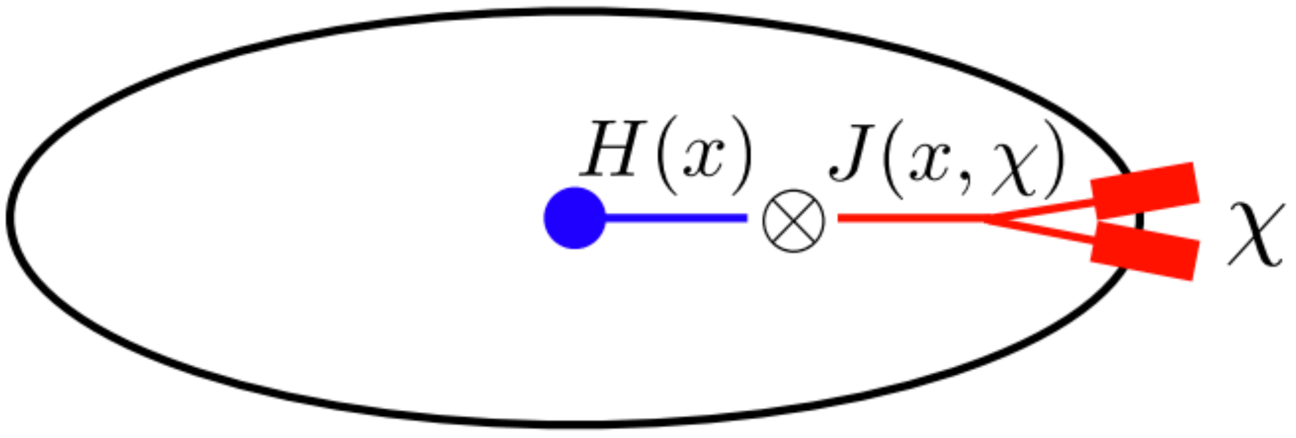


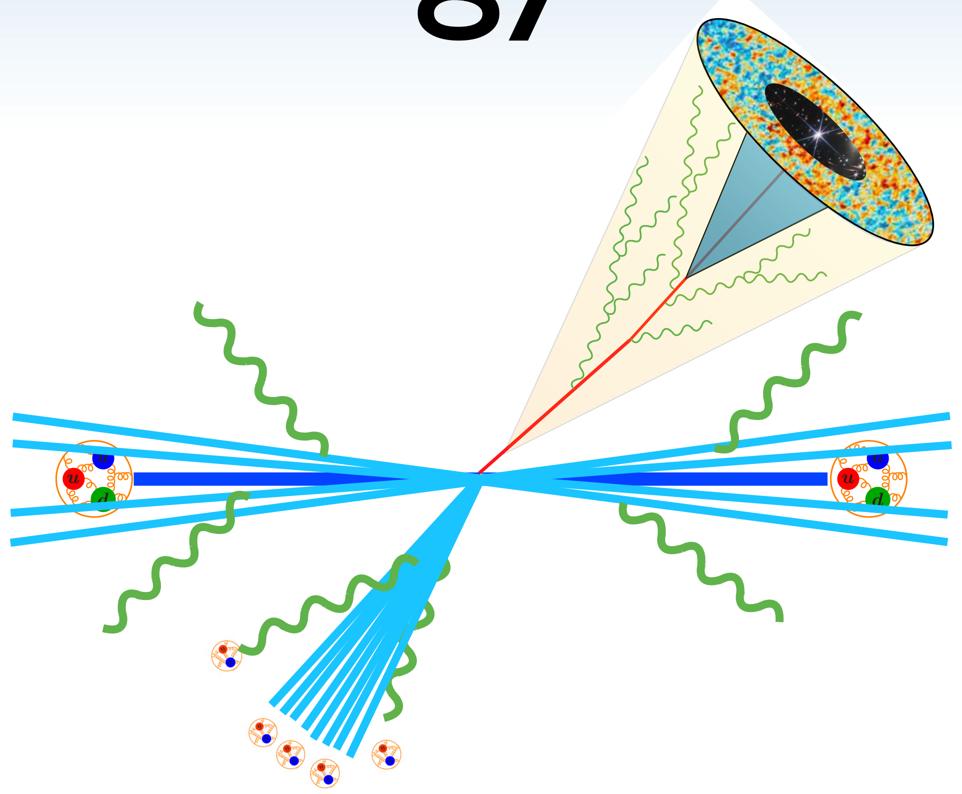
$\Longleftrightarrow$

$$\mathcal{E}\left(\hat{n}_1\right)\mathcal{E}\left(\hat{n}_2\right) = \frac{1}{\theta^{2(1-\gamma_3)}} \sum \vec{\mathbb{O}}_i^{J=3}\left(\hat{n}_1\right)$$

- The universal scaling behavior is given by the Light-Ray OPE in the conformal region.
- QCD isn't exactly conformal, we see sensitivity to the IR scale through confinement.

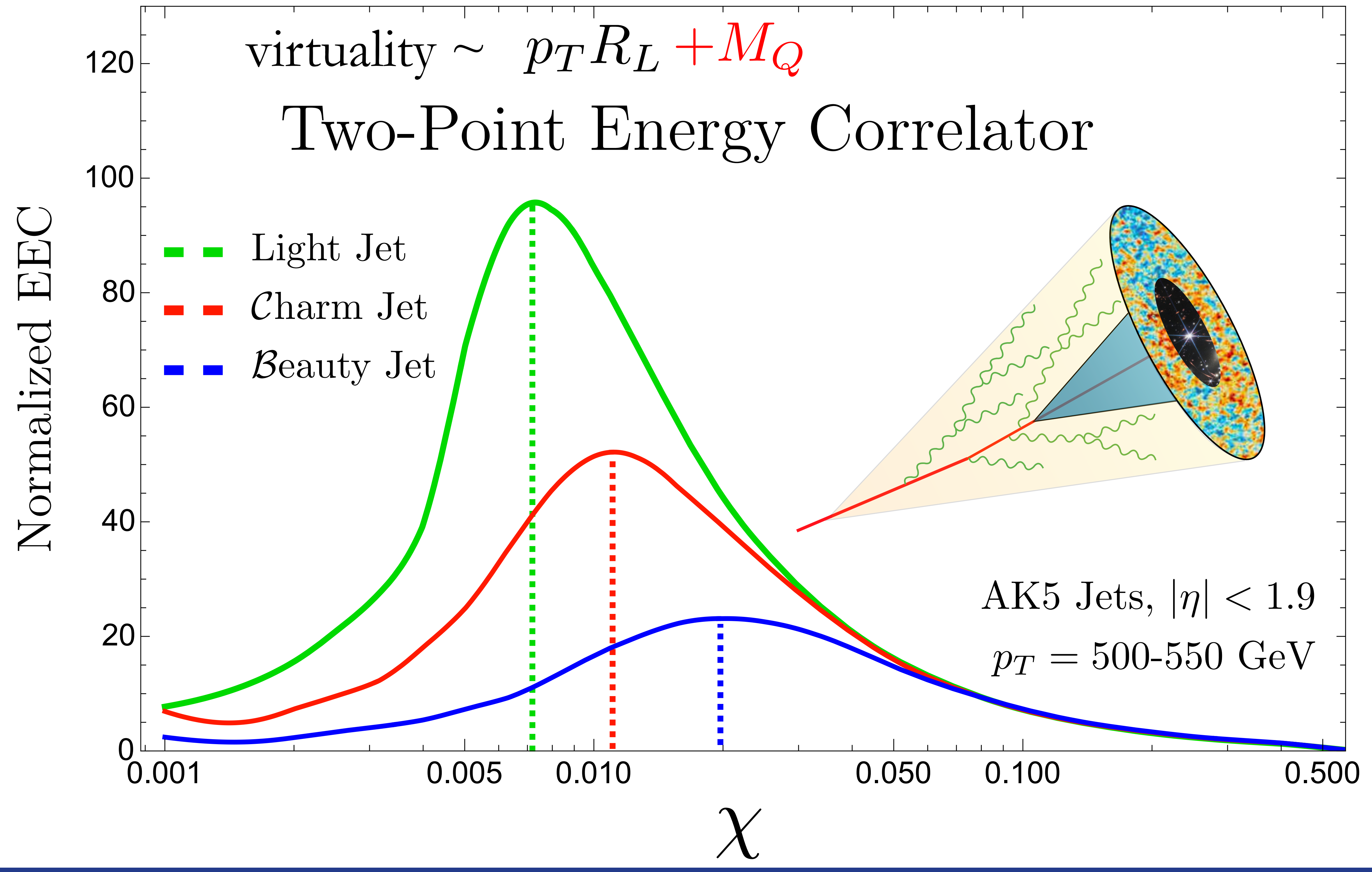
Collinear Energy-Energy Correlators with mass

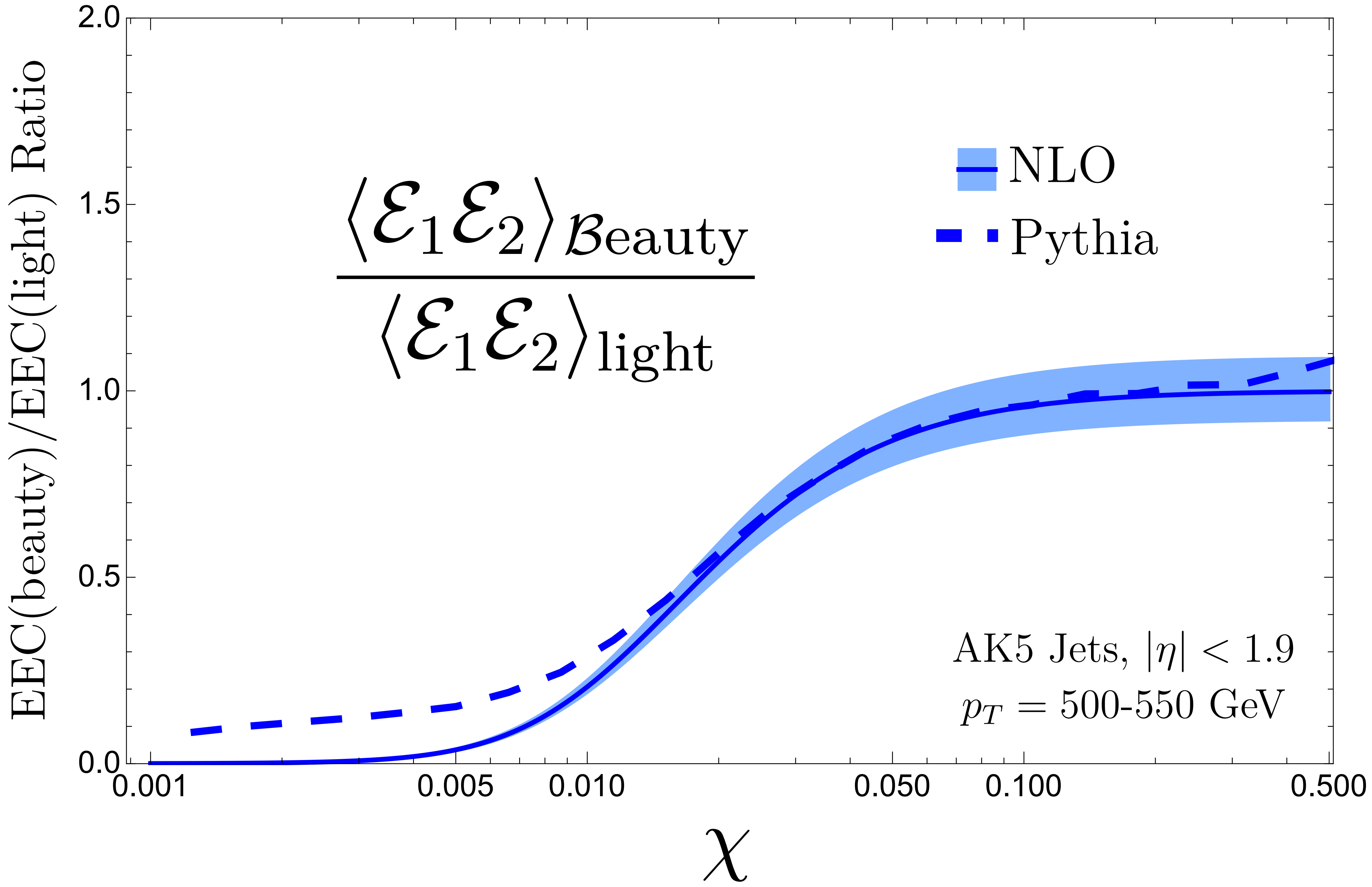




$$\Sigma\left(\chi, p_T^2, m_Q, \mu\right) = \int_0^1 dx\, x^2 \vec{J}\left(\chi, x, m_Q, \mu\right) \cdot \vec{H}\left(x, p_T^2, \mu\right)$$

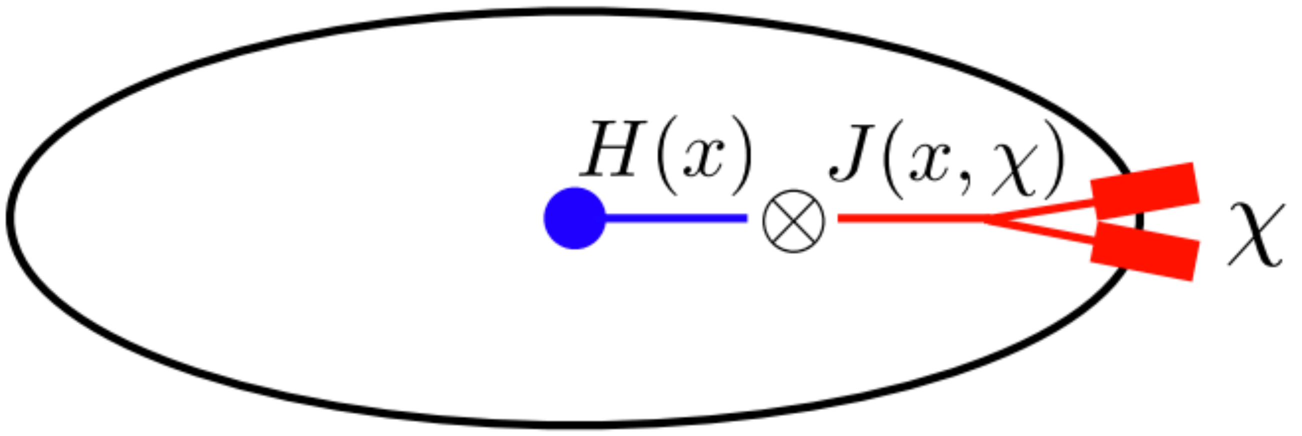
Collinear dynamics factorize identically for different collider environment

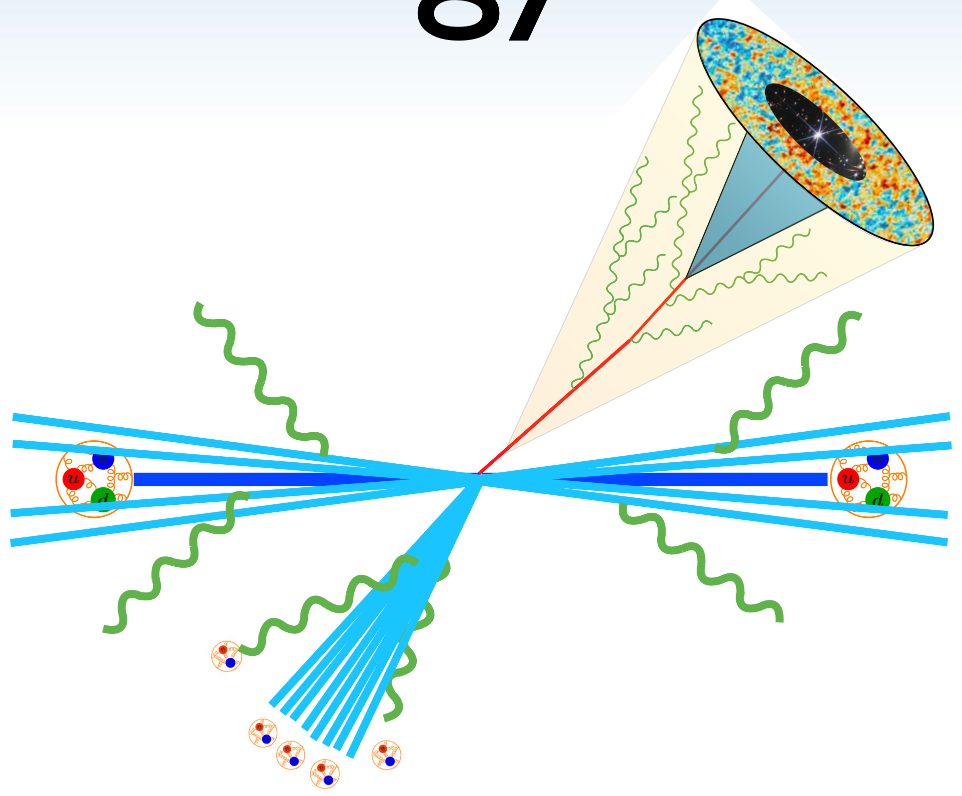




- Presence of the mass forces heavy flavor hadrons to be formed at an earlier time. This is clearly probed via energy correlators!

Collinear Energy-Energy Correlators with mass



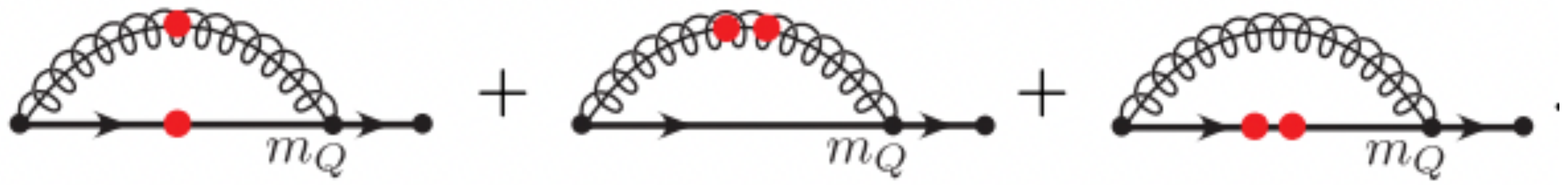


$$\Sigma\left(\chi, p_T^2, m_Q, \mu\right) = \int_0^1 dx\, x^2 \vec{J}\left(\chi, x, m_Q, \mu\right) \cdot \vec{H}\left(x, p_T^2, \mu\right)$$

Collinear dynamics factorize identically for different collider environment

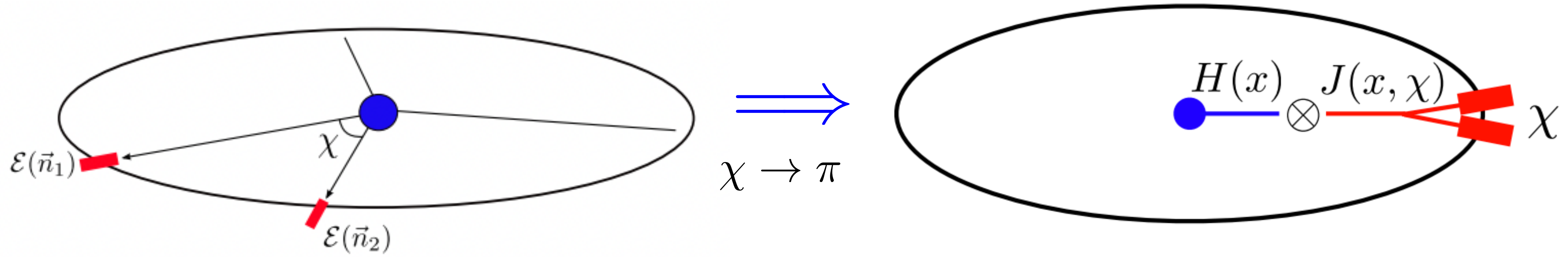
SCET definition $$J_Q\left(\chi, m_Q\right) = \sum_X \sum_{i,j \in X} \langle 0 \left|\bar{\chi}_n\right| X \rangle \frac{E_i E_j}{p_T^2} \Theta\left(\theta_{ij} < \chi\right) \langle X \left|\chi_n\right| 0 \rangle$$

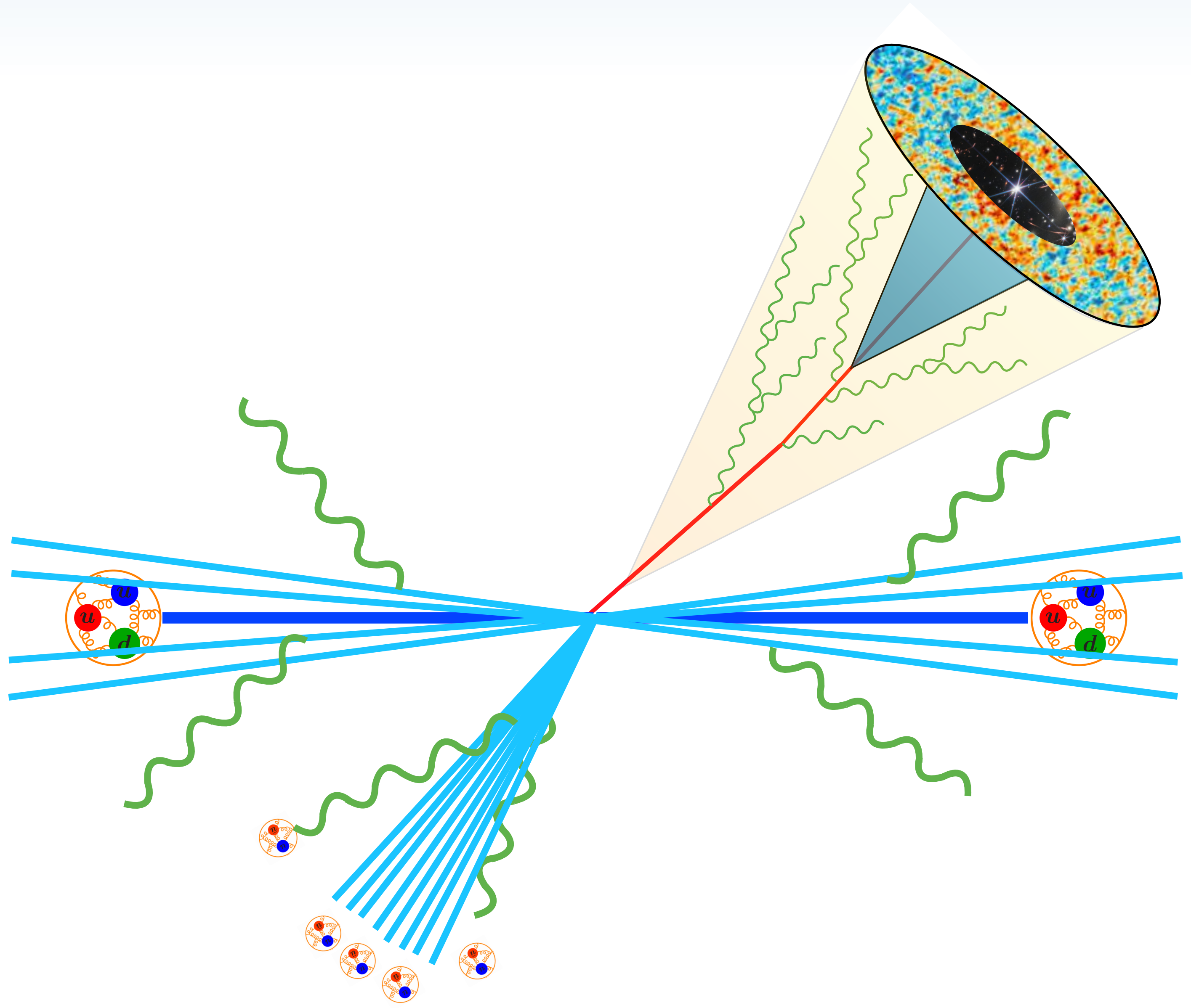
At NLO,



$$J_Q\big|_{\chi \neq 0}\left(\chi, m\right) = \frac{\alpha_s C_F}{4\pi} \left\{ \left[\delta^4 + 4i\delta^3 - 2\delta^2 - 3\right] \ln\left(\frac{i\delta}{1 + i\delta}\right) + \frac{1}{2}\left(9\delta^2 - \frac{31}{6}\right) \right\} + c.c$$ where $$\delta = \frac{m}{p_T \chi}$$

Collinear Energy-Energy Correlators with mass





Making contact with the general angle result:

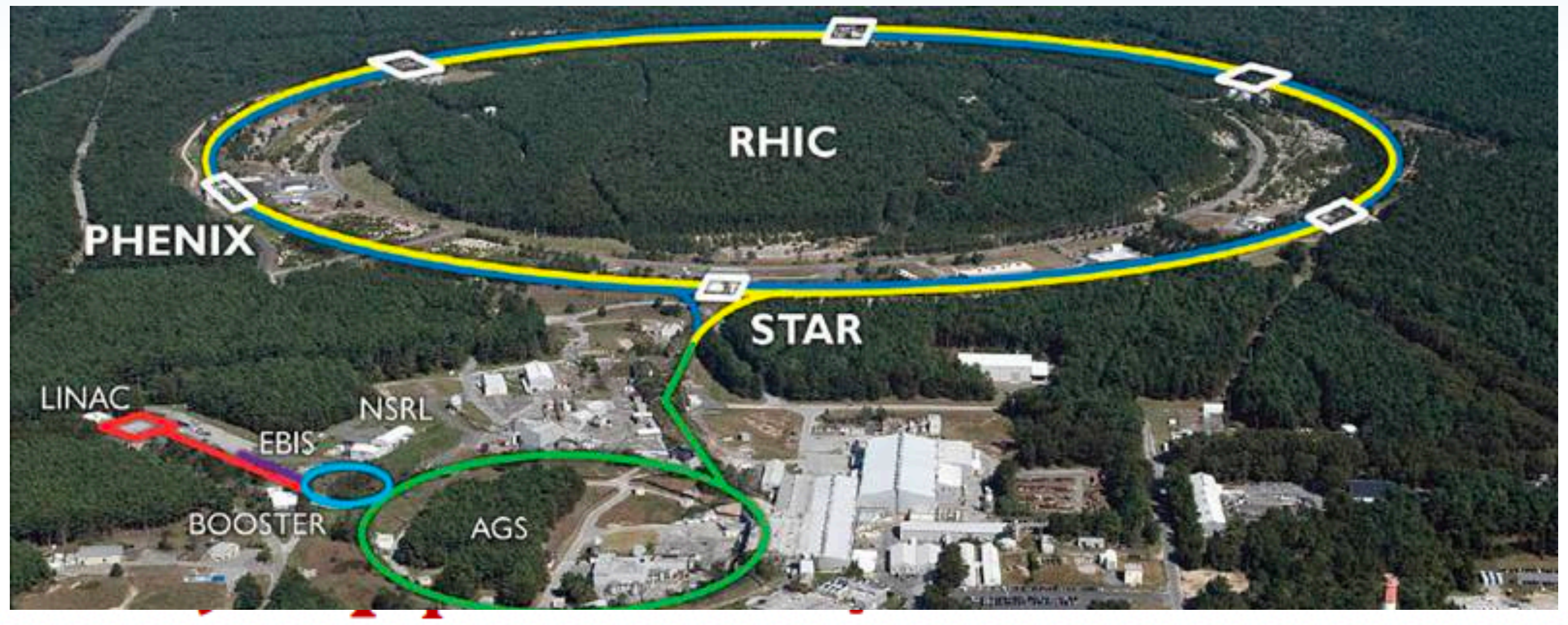
$$\left.\frac{d\sigma}{dz}\right|_{Qg+\bar{Q}g}^{z\to 0} = \text{LP} + \text{NLP} + \text{NNLP} + \cdots$$

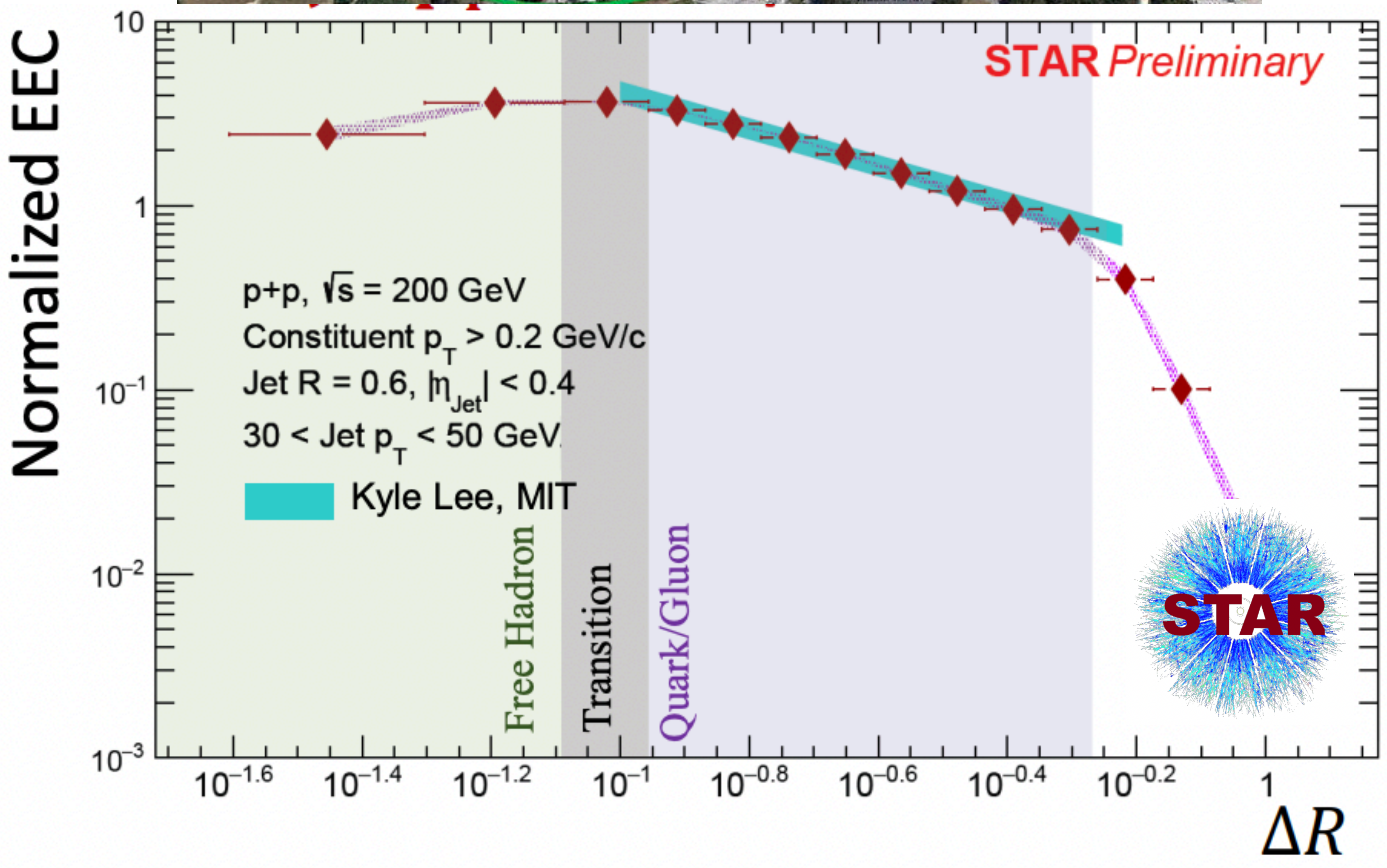
$$\text{LP} \propto \frac{1}{2z}\left\{\left[-4(i\delta)^4 + 12(i\delta)^3 - 4(i\delta)^2\right]\ln\left(\frac{i\delta}{1+i\delta}\right) + \left[-(i\delta)^4 + 4(i\delta)^3 - 2(i\delta)^2 + 3\right]\frac{1}{(1+i\delta)} + 9(i\delta)^2\right\} + c.c,$$

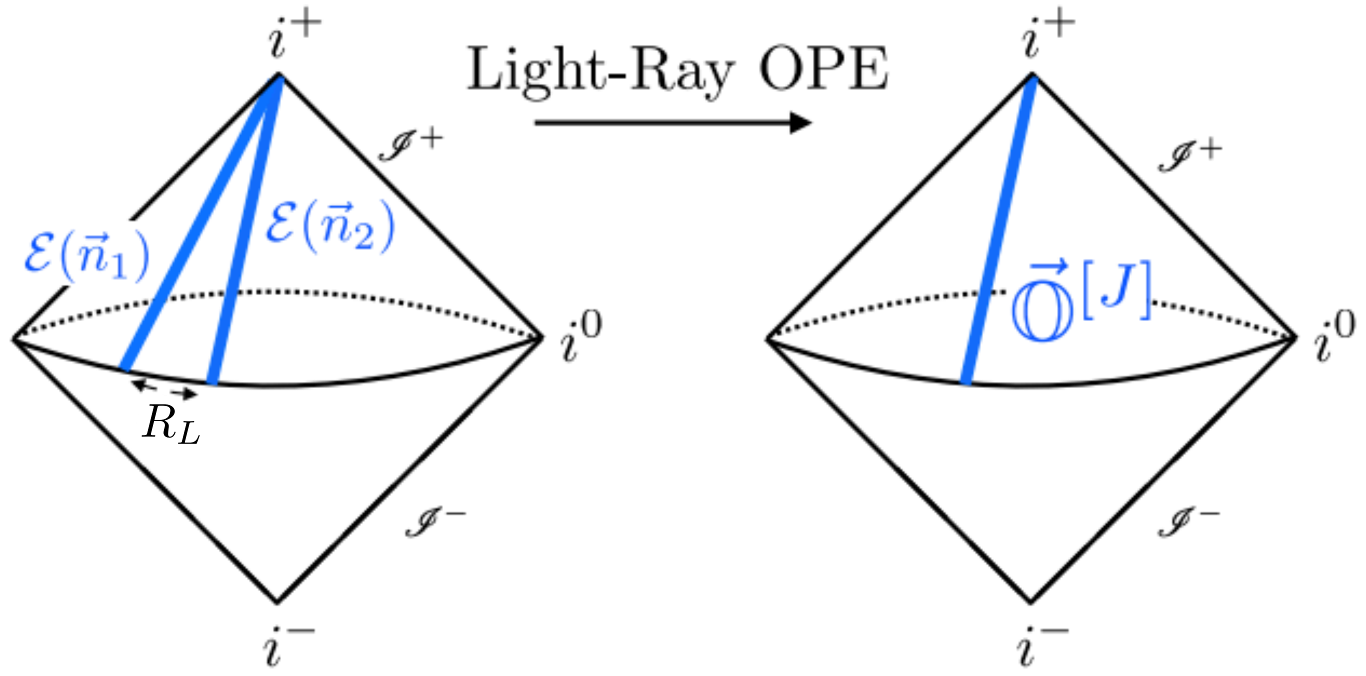
$$\text{NLP} \propto \frac{-2\left(300\delta^8 + 530\delta^6 + 208\delta^4 + 41\delta^2 - 27\right)}{15\left(\delta^2+1\right)^2} + \frac{15\delta^3\left(\delta^2+1\right)^2\left(6\delta\log\left(\frac{\delta^2+1}{\delta^2}\right) + i\left(1-20\delta^2\right)\log\left(\frac{\delta+i}{\delta-i}\right)\right)}{15\left(\delta^2+1\right)^2}$$

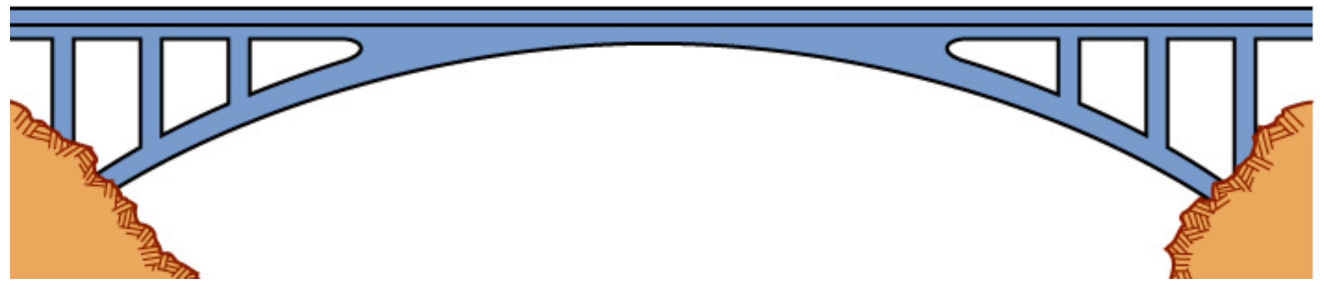
Leading power agrees and one can obtain higher power results from expanding the general angle result!

Energy-Energy Correlators meet the high-energy colliders





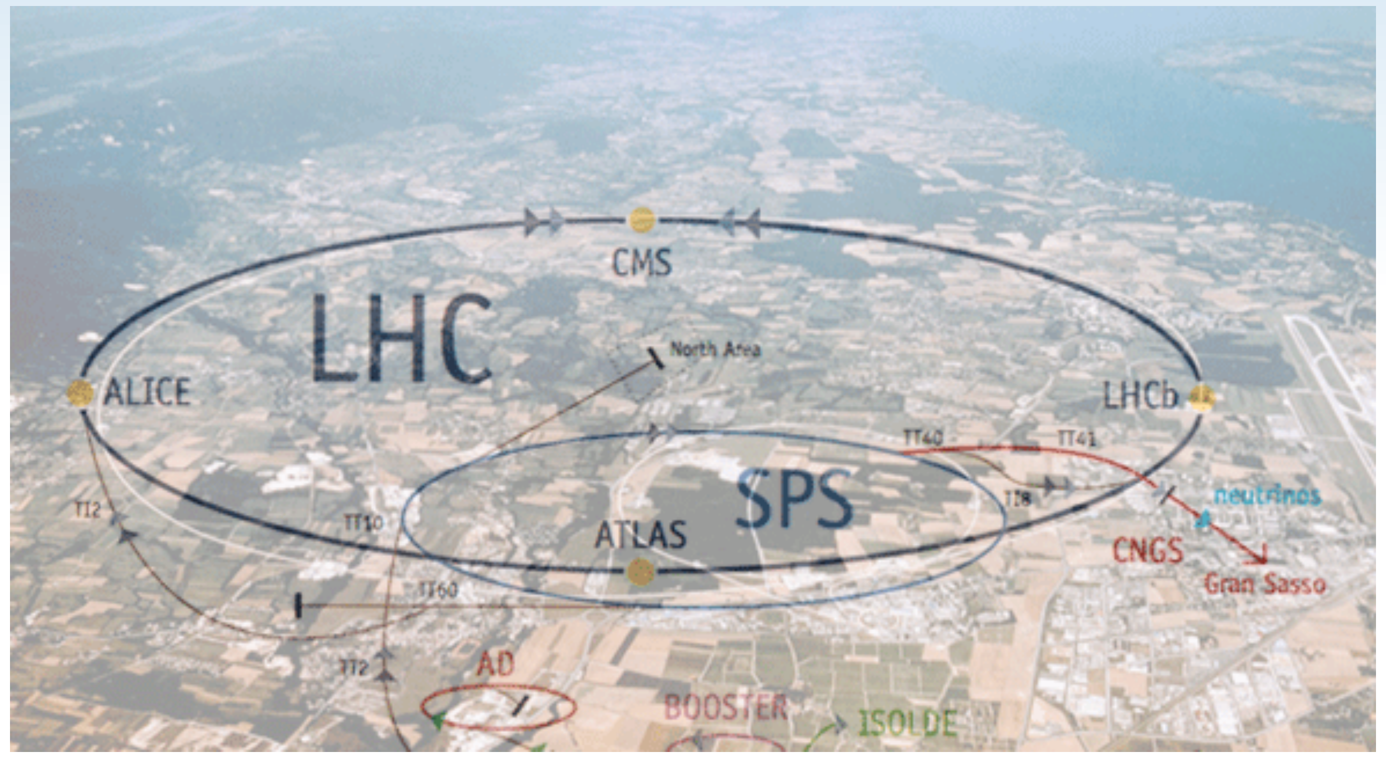


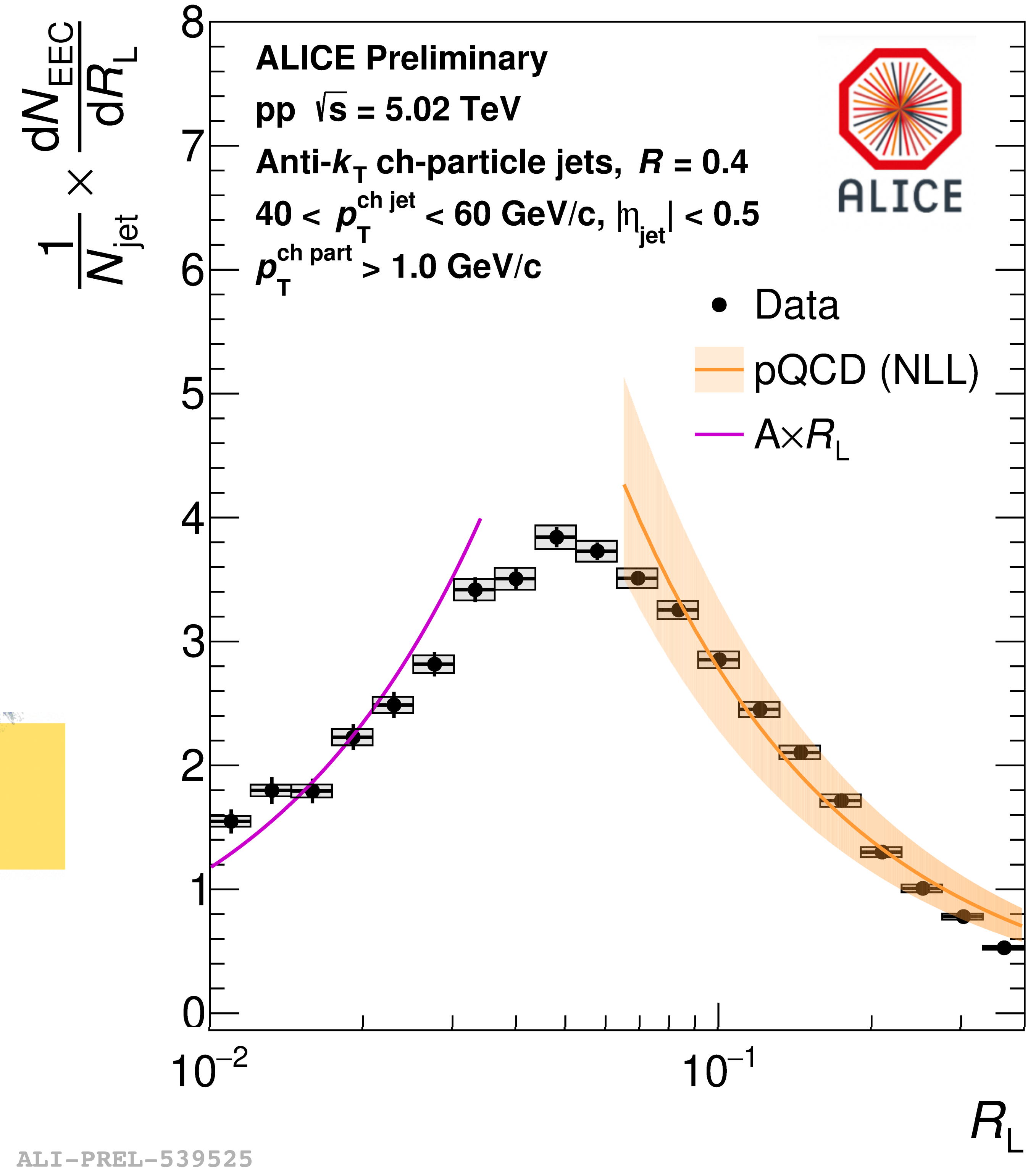


"field theory faces reality"

arXiv:2205.03414 , 2209.11236

Andrew Tamis
29 Mar, 11.30

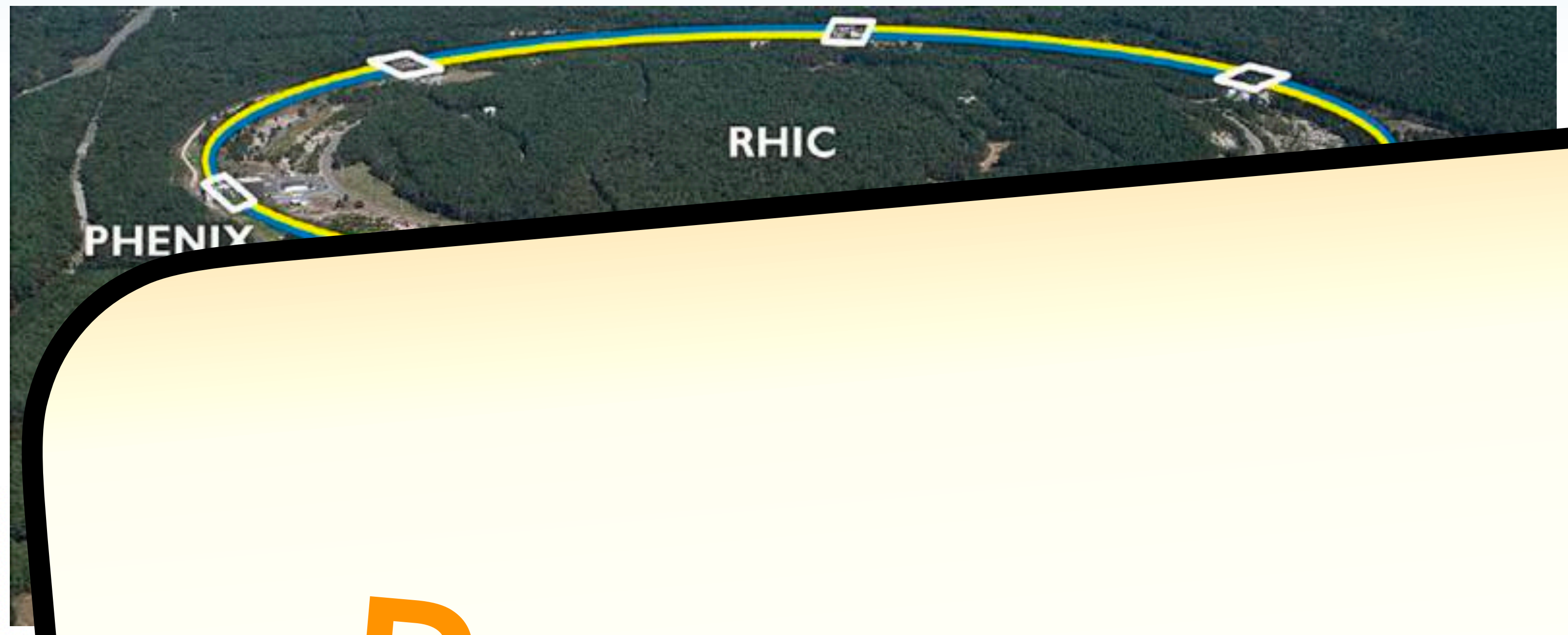


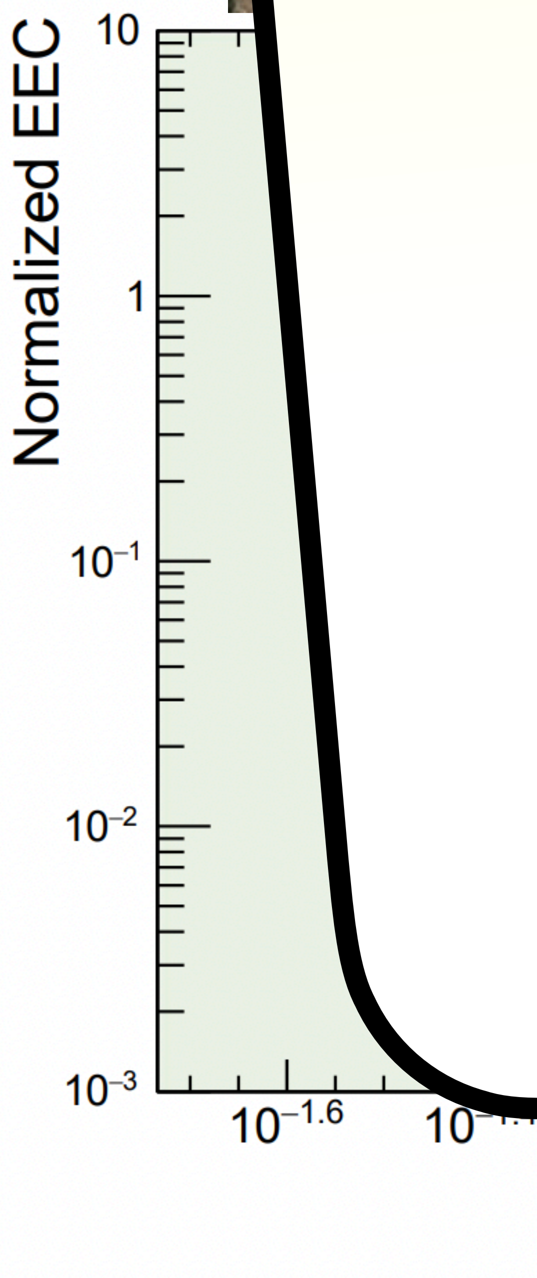


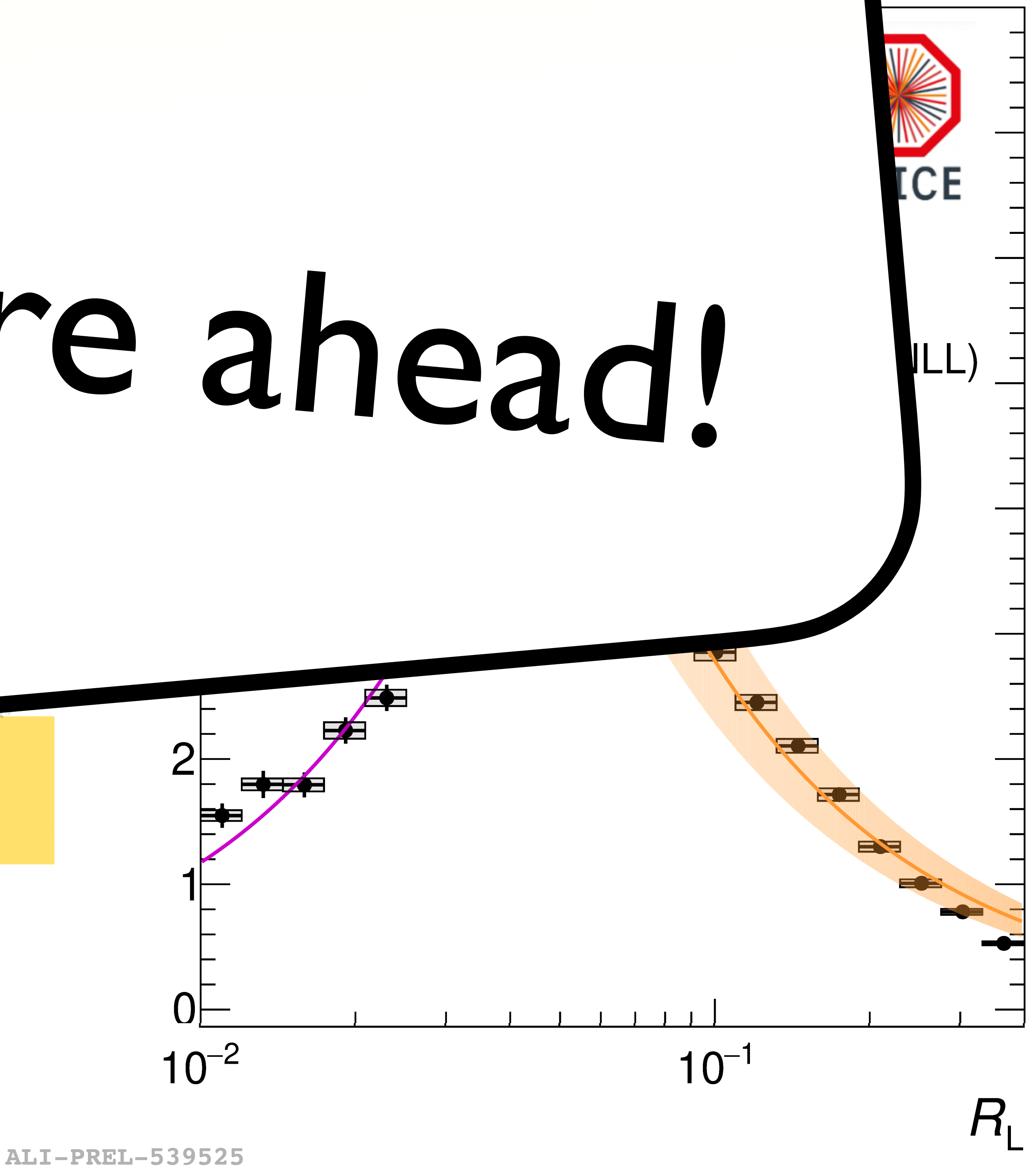
HARD PROBES 2023 Aschaffenburg

DIS23 Michigan State University

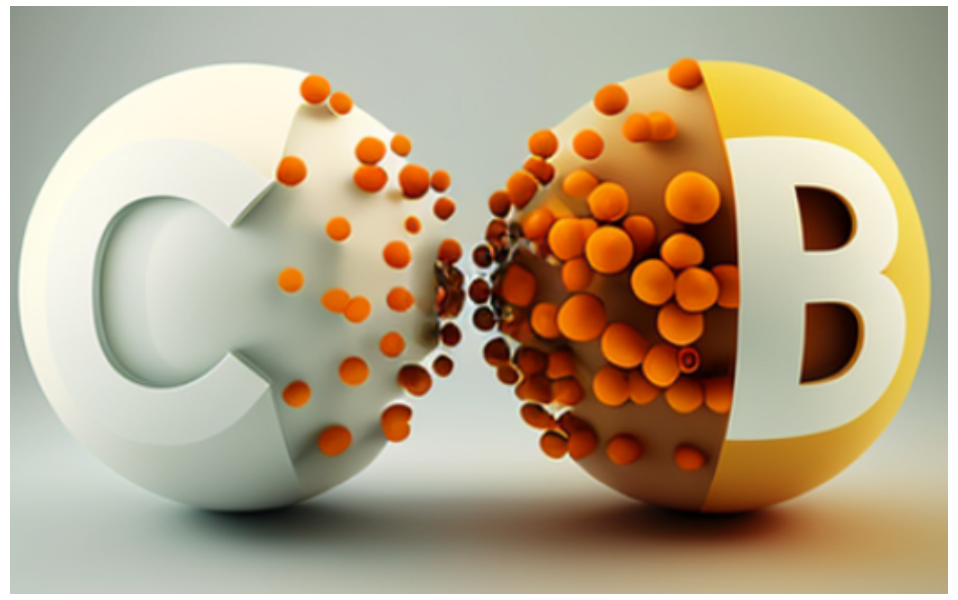
Energy-Energy Correlators meet the high-energy colliders







Beautiful and **Charming** future ahead!



Andrew Tamis
29 Mar, 11.30

arXiv:2205.03414 , 2209.11236