Factorization at subleading power for DY and DIS

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(work done with M. Inglis-Whalen, J. Roy and A. Spourdalakis)

Outline:

- Power corrections and endpoint divergences
- Factorization without modes
- DIS and DY
- Conclusions

SCET has been around for over 20 years …

… but power corrections beyond tree level have only been calculated in the last few years … why?

- manifest decoupling of soft/ultrasoft fields fails at NLP (but can be extended to NLP using *radiative functions*)

  [I. Moult et. al. (2019)]

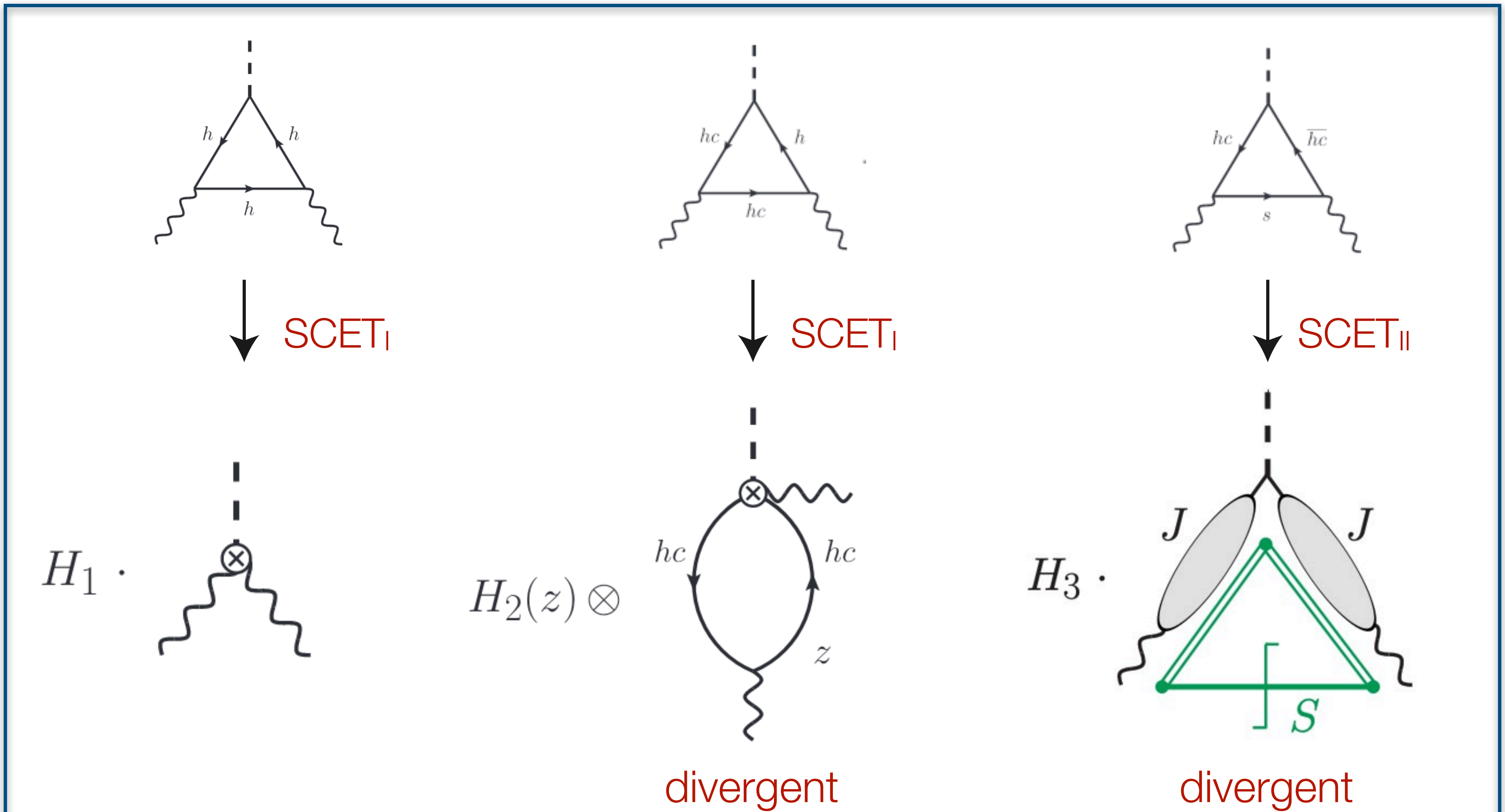
- “bare” factorization breaks down because of radiative corrections due to appearance of spurious divergences (requires *refactorization*)

  [Z.L. Liu et. al. (2020), M. Beneke et. al. (2020,2022)]

Ex: $h \to \gamma\gamma$ (through b loops)

[Z.L. Liu and M. Neubert (2020), Z.L. Liu et. al. (2021)]

- Rate naturally factorizes from SCET mode expansion, but individual terms are divergent ("bare factorization")



The SCET rate is finite, and terms can be rearranged to also make individual contributions finite (“refactorization”):

$$\begin{aligned}\mathcal{M}_b &= \left(H_1^{(0)} + \Delta H_1^{(0)}\right) \langle\gamma\gamma|\, O_1^{(0)} |h\rangle \\ &\quad + 2\lim_{\delta\to 0}\int_\delta^{1-\delta} dz \left[H_2^{(0)}(z)\, \langle\gamma\gamma|\, O_2^{(0)}(z)\, |h\rangle - \frac{[\![\bar{H}_2^{(0)}(z)]\!]}{z} [\![\langle\gamma\gamma|\, O_2^{(0)}(z)\, |h\rangle]\!] \right. \\ &\qquad \left. - \frac{[\![\bar{H}_2^{(0)}(1-z)]\!]}{1-z} [\![\langle\gamma\gamma|\, O_2^{(0)}(1-z)\, |h\rangle]\!] \right] \\ &\quad + g_\perp^{\mu\nu} \lim_{\sigma\to -1} H_3^{(0)} \int_0^{M_h} \frac{d\ell_-}{\ell_-} \int_0^{\sigma M_h} \frac{d\ell_+}{\ell_+} J^{(0)}(M_h\ell_-)\, J^{(0)}(-M_h\ell_+)\, S^{(0)}(\ell_+\ell_-)\end{aligned}$$

$$[[\langle\gamma\gamma|O_2(z)|h\rangle]] \equiv \langle\gamma\gamma|O_2(z)|h\rangle|_{z\to 0}$$

- singular terms cancel between the second two terms - gives refactorized rate with individually finite terms
- more complex refactorization conditions for other processes [G. Bell et. al. (2022)]
- no universal construction .. will be more complicated for processes with more operators

An alternative framework:

Drop the mode expansion altogether

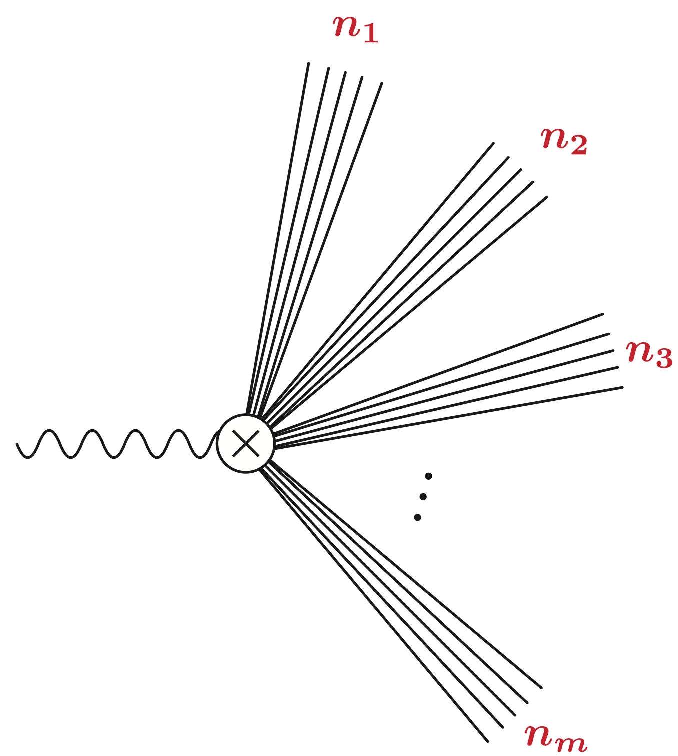
[R. Goerke and ML (2018)]

- usual EFT - integrate out physics above cutoff, leave IR unchanged
- usual SCET: integrate out physics above cutoff, *factorize IR into soft+collinear (others?) modes (~method of regions)*

Making SCET more like a "traditional" EFT (4-fermi, HQET, SMEFT, …) gives a new perspective on issues which arise from the mode expansion:

- **What you get:** factorization, resummation (RGE, rapidity) through successive matching and running (ex: integrate out hard at high scale, collinear at intermediate scale, remaining theory just has soft degrees of freedom remaining)
- **What you DON'T get:** factorization into modes (i.e. $H \otimes J \otimes J \otimes S$) where J and S arise from separate collinear/soft graphs)
- factorization formulas in terms of SCALES not MODES look different, but still gives resummed cross sections/rates. "Refactorization" doesn't arise, since these terms were never factorized in the first place.
- **greatly simplifies power corrections, issues of factorization at NLP**

The EFT:



$$q^2 \gg p_i^2$$

$$p_i \cdot p_j \sim q^2$$

- Integrating out physics at $\mu^2 \geq Q^2$ requires us to define "sectors": states in a given sector have small invariant mass; invariant mass between sectors is large. Sectors contain all degrees of freedom below the cutoff.

- Each sector is above the cutoff of all the others - decouple except for hard external current

2 sectors:

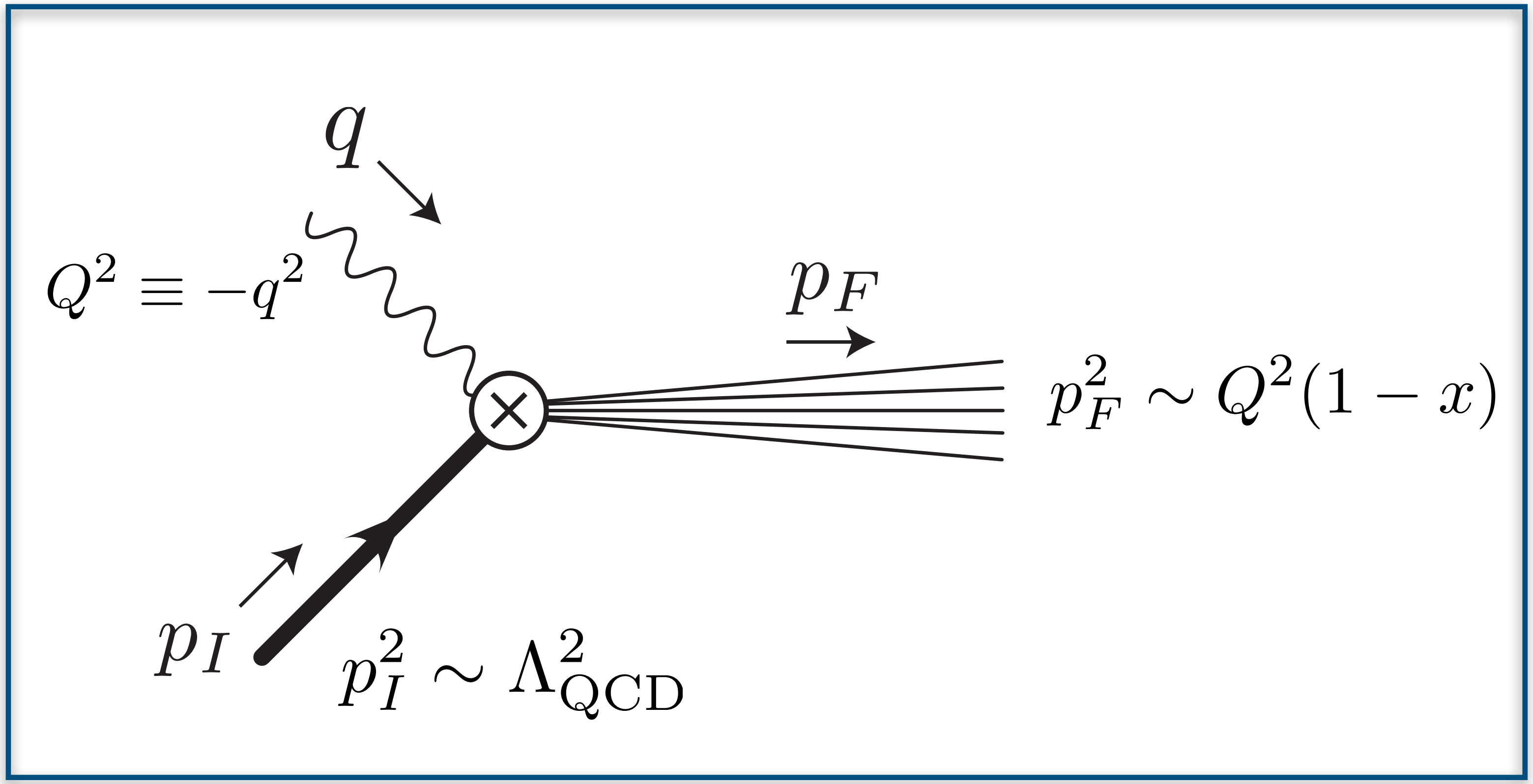
$$\mathcal{L} = \mathcal{L}^{n}_{\mathrm{QCD}} + \mathcal{L}^{\bar{n}}_{\mathrm{QCD}} + \mathcal{J}^{\mu} A_{\mu}$$

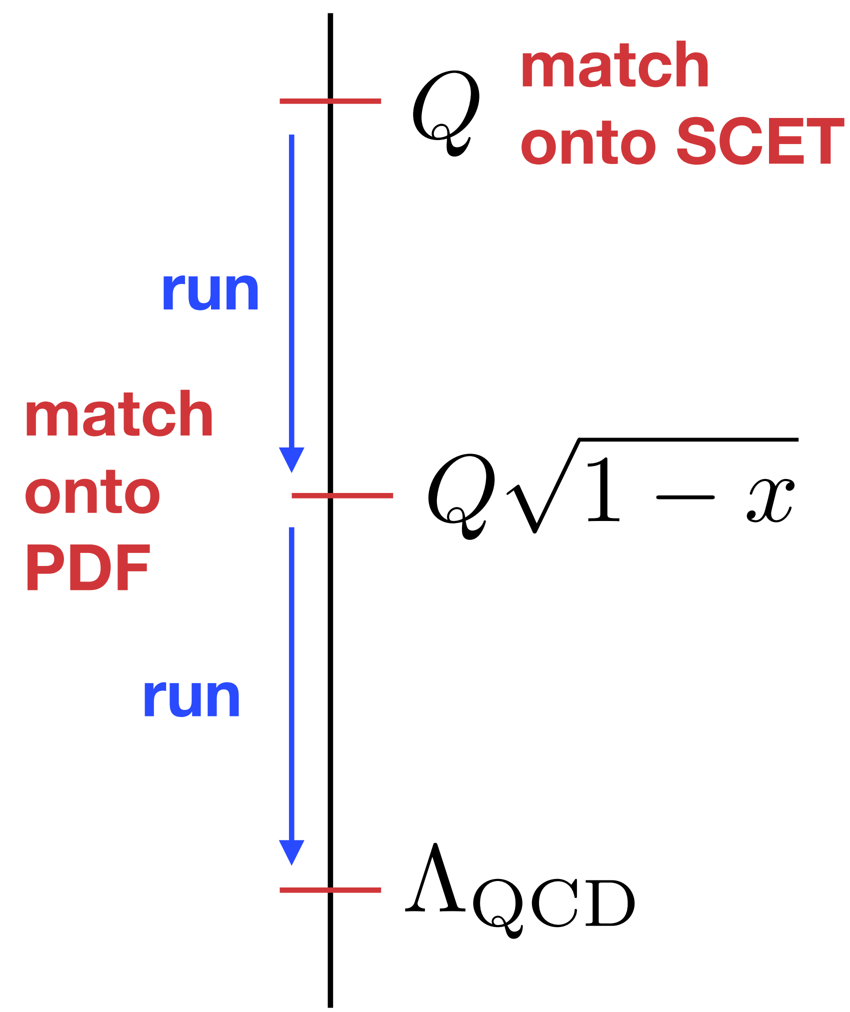
$$\mathcal{J}^{\mu}(x) = \sum_i \frac{1}{Q^{[i]}} C_2^{(i)} O_2^{(i)}$$

- no power corrections mixing modes in the Lagrangian - power corrections only arise from expansion of external current

ex: **Deep Inelastic Scattering as $x \to 1$** (trad. SCET$_\mathrm{I}$)

[R. Goerke and ML (2018)],
ML, J. Roy and A. Spourdalakis, 2022]





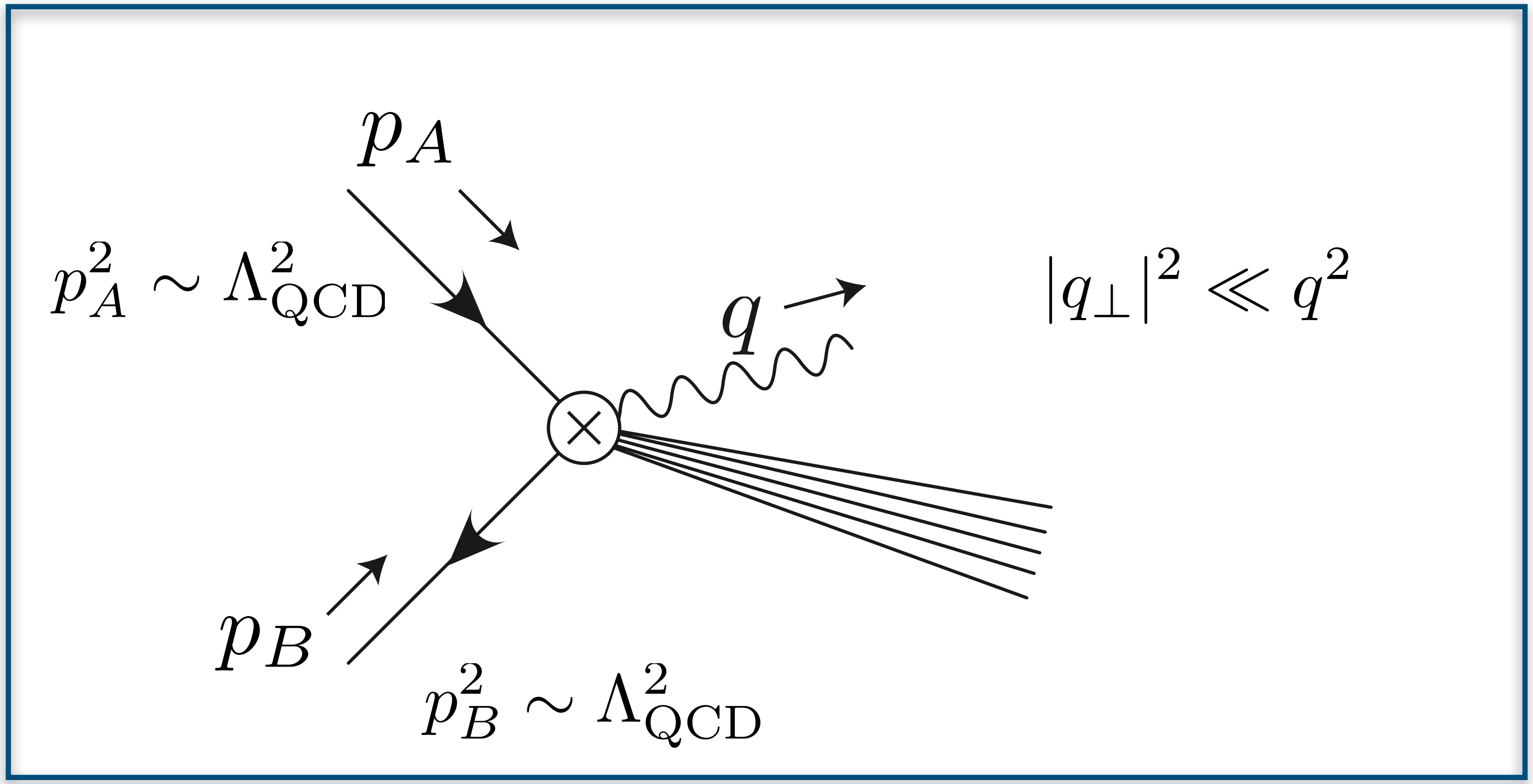
$$\left.\begin{array}{l} Q^2 \gg p_F^2 \gg p_I^2 \\ p_F \cdot p_I \sim O(Q^2) \end{array}\right\}$$

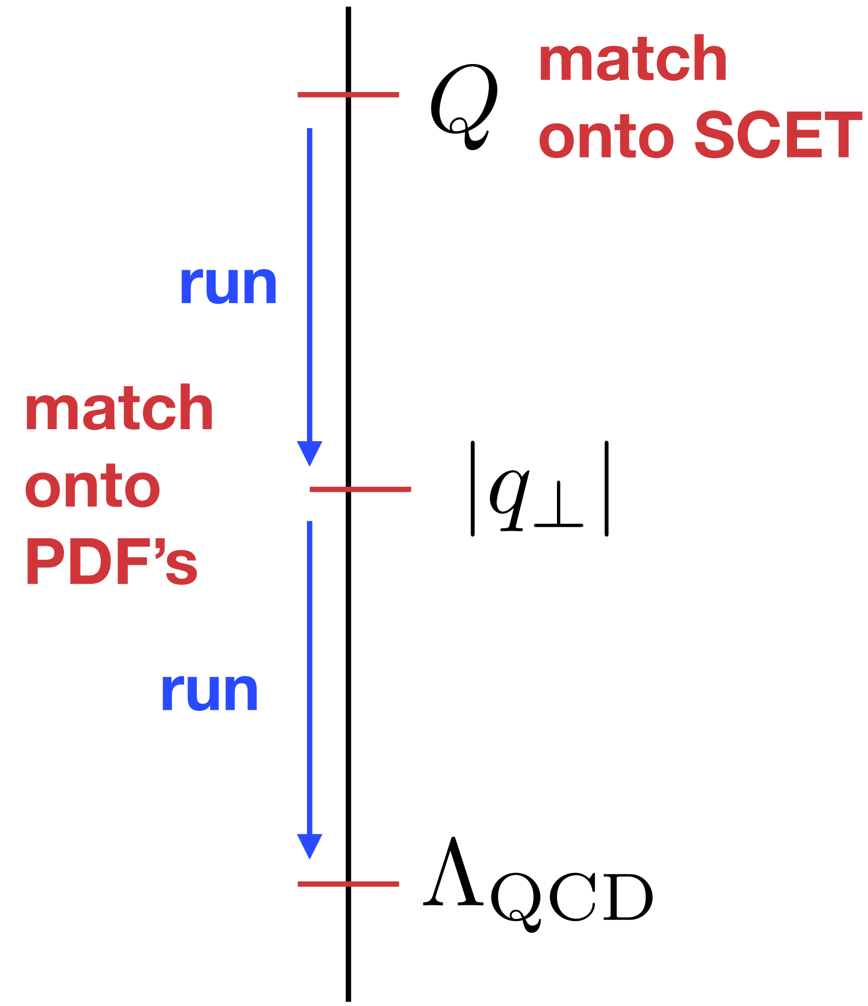
final state in n sector
initial state in $\bar{n}$ sector

(*not* n- and n̄-collinear)

ex: **Drell-Yan at small $q_\perp^2 \gg \Lambda_{\mathrm{QCD}}^2$** (trad. SCET$_{\mathrm{II}}$)

[M. Inglis-Whalen, ML, J. Roy and A. Spourdalakis, 2021]





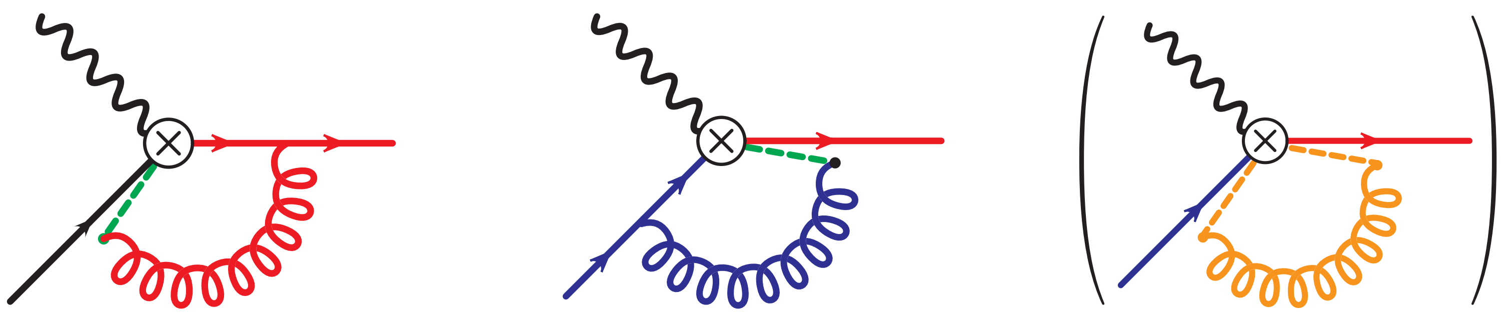
$$\left.\begin{aligned} &Q^2 \gg p_A^2, p_B^2 \\ &p_A \cdot p_B \sim O(Q^2) \end{aligned}\right\}$$

hadron A in n sector
hadron B in $\bar{n}$ sector

The complication: Double-Counting (aka 0-bin)

[A. Manohar and I. Stewart (2007), A. Idilibi and T. Mehen (2007), C. Lee and G. Sterman (2007)]

- some degrees of freedom have momenta that fall below the cutoff of more than one sector - these get double counted
- matrix elements in SCET are only well-defined if double counting between modes/sectors has been removed
- particularly acute in this framework: without subtraction, loop graphs are not well-defined (IR-dependent UV divergences)



$$I \sim -i\frac{\mu^{2\epsilon}e^{\gamma_E\epsilon}}{(4\pi)^{2-2\epsilon}}\Gamma(\epsilon)\left(\int_0^1 m_g^{-2\epsilon}(1-z)^{1-\epsilon}\frac{dz}{z} - \int_0^\infty \left(m_g^{-2\epsilon} - (Q^2 z + m_g^2)^{-\epsilon}\right)\frac{dz}{z} - \int_0^\infty m_g^{-2\epsilon}\frac{dz}{z}\right)$$

(+ wave function)

$$= \frac{C_F\alpha_s}{4\pi}\left[\frac{2}{\epsilon^2} + \frac{3 - 2\log\frac{Q^2}{\mu^2}}{\epsilon} - \log^2\frac{m_g^2}{\mu^2} + 2\log\frac{Q^2}{\mu^2}\log\frac{m_g^2}{\mu^2} - 3\log\frac{m_g^2}{\mu^2} - \frac{5\pi^2}{6} + \frac{9}{2}\right]$$

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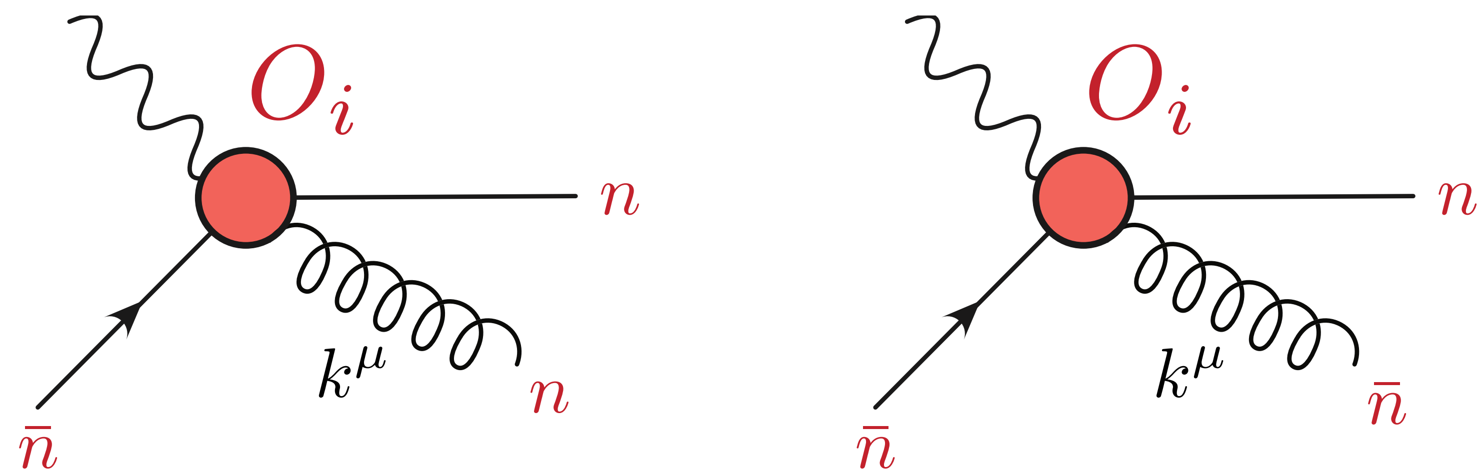
- some degrees of freedom have momenta that fall below the cutoff of more than one sector - these get double counted
- matrix elements in SCET are only well-defined if double counting between modes/sectors has been removed
- particularly acute in this framework: without subtraction, loop graphs are not well-defined (IR-dependent UV divergences)
- rapidity and endpoint divergences arise from double counting - must cancel when consistently implemented

Note: this has to happen in a consistent EFT, otherwise IR degrees of freedom aren't correctly described

"Overlap Subtraction" Prescription:

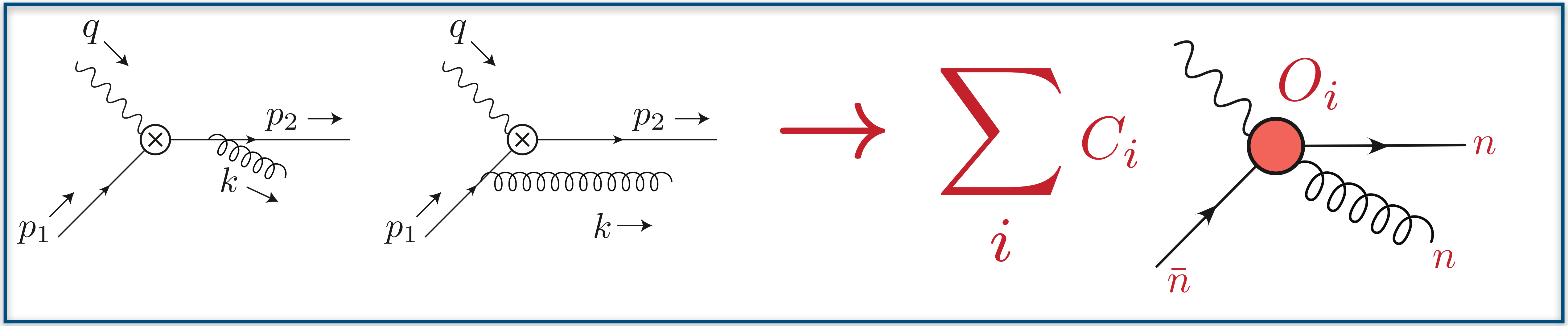


- in the regime $k \cdot n, k \cdot \bar{n} \ll Q$, the same gluon in QCD is double counted in the EFT
- as in 0-bin, expand "wrong sector" graph in $\bar{n} \to n$ limit and subtract
- to work consistently to NLP, must expand overlap to NLP - crucial for cancellations of endpoint divergences at NLP
- scheme dependence of subtraction gives rapidity renormalization group equations

[M. Inglis-Whalen, A. Spourdalakis and ML, 2020]

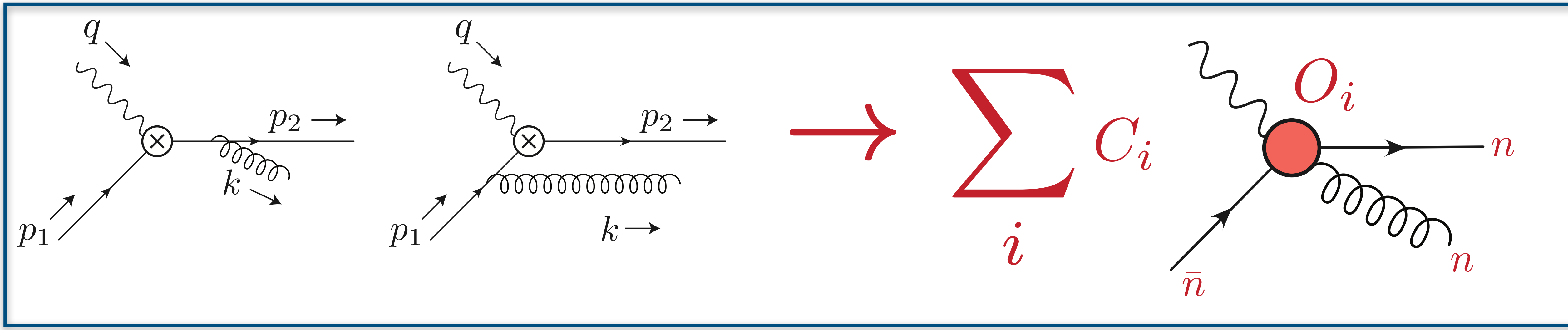
Example: DIS as $x \to 1$

$\mu = Q$: Match QCD to SCET



Expand QCD amplitude in powers of $\frac{p_1 \cdot \bar{n}}{Q}, \frac{p_2 \cdot \bar{n}}{Q}, \frac{k \cdot n}{Q}, \frac{p_{i\perp}}{Q}, \frac{k_\perp}{Q}$

$\mu = Q$: Match QCD to SCET



$$\begin{aligned} i\mathcal{A}^\mu = -\, g_s T^a \bar{u}(p_2)\Bigg[& \frac{2p_2^\alpha + \gamma^\alpha \not{k}}{2p_2\cdot k} P_{\bar{n}}\gamma^\mu P_{\bar{n}} - P_{\bar{n}}\gamma^\mu P_{\bar{n}}\frac{\bar{n}^\alpha}{\bar{n}\cdot k} \\ & + \frac{1}{Q}\left(\overline{\Delta}^{\alpha\beta}(k)\gamma_\beta^\perp \frac{\bar{\not{n}}}{2}\gamma^\mu P_{\bar{n}} + P_{\bar{n}}\gamma^\mu \frac{\not{n}}{2}\gamma_\beta^\perp \overline{\Delta}^{\alpha\beta}(k)\right) \\ & + \frac{1}{Q^2} P_{\bar{n}}\gamma^\mu P_{\bar{n}} \gamma_\beta^\perp \gamma_\gamma^\perp k^\beta \overline{\Delta}^{\alpha\gamma}(k)\left(1 + \frac{\bar{n}\cdot p_2}{\bar{n}\cdot k}\right) \\ & - \frac{1}{Q^2}\gamma_\beta^\perp \frac{\bar{\not{n}}}{2}\gamma^\mu \frac{\not{n}}{2}\gamma_\gamma^\perp k^\beta \overline{\Delta}^{\alpha\gamma}(k)\left(1 + \frac{\bar{n}\cdot k}{\bar{n}\cdot p_2}\right)\Bigg] \\ & \times u(p_1)\varepsilon_\alpha^*(k) + \dots \end{aligned}$$

$$O_2^{(0)\mu}(x) = \bar{\chi}_n(x)\gamma^\mu \chi_{\bar{n}}(x)$$

$$\begin{aligned} O_2^{(1A)\mu}(x,\hat{t}) = & -\bar{\chi}_n(x)\mathcal{B}_n^\alpha(x+\bar{n}t) \\ & \times \left(\gamma_\alpha^\perp \frac{\bar{\not{n}}}{2}\gamma^\mu + \gamma^\mu \frac{\not{n}}{2}\gamma_\alpha^\perp\right)\chi_{\bar{n}}(x) \end{aligned}$$

$$\begin{aligned} O_2^{(2A_1)\mu}(x,\hat{t}) = & 2\pi i\theta(\hat{t}) \\ & \otimes \bar{\chi}_n(x)\mathcal{B}_n^{\alpha\beta}(x+\bar{n}t)\gamma^\mu \gamma_\alpha^\perp \gamma_\beta^\perp \chi_{\bar{n}}(x) \end{aligned}$$

$$\begin{aligned} O_2^{(2A_2)\mu}(x,\hat{t}) = & -2\pi i\theta(\hat{t}) \\ & \otimes \bar{\chi}_n(x+\bar{n}t)\mathcal{B}_n^{\alpha\beta}(x)\gamma_\alpha^\perp \frac{\bar{\not{n}}}{2}\gamma^\mu \frac{\not{n}}{2}\gamma_\beta^\perp \chi_{\bar{n}}(x) \end{aligned}$$

$Q > \mu > Q\sqrt{1-x}$: RG Evolution

[M. Inglis-Whalen, and R. Goerke, 2018]

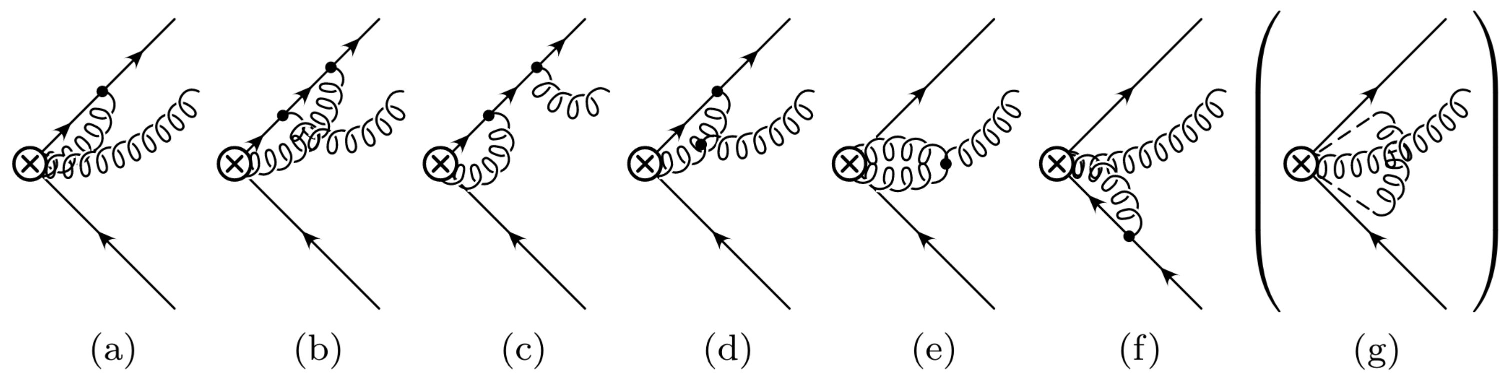


Figure 2. The Feynman diagrams for any operator with the $q^n \bar{q}^{\bar{n}} g^n$ configuration. The Feynman rules for the effective vertex are determined by the structure of each operator. Diagram (g) is the overlap amplitude, and must be subtracted.

$$\frac{d}{d\log\mu} O_2^{(j)}(x,u) = -\int dv\, \gamma_2^{(j)}(u,v) O_2^{(j)}(x,v)$$

$$\begin{aligned}
\gamma_{(1a)}(u,v) = {} & \frac{\alpha_s \delta(u-v)}{\pi}\left[C_F\left(\log\frac{-Q^2}{\mu^2} - \frac{3}{2} + \log\bar{v}\right) + \frac{C_A}{2}\left(1 + \log\frac{v}{\bar{v}}\right)\right] \\
& + \frac{\alpha_s}{\pi}\left(C_F - \frac{C_A}{2}\right)\bar{u}\left(\frac{uv}{\bar{u}\bar{v}}\theta(1-u-v) + \frac{uv+u+v-1}{uv}\theta(u+v-1)\right) \\
& + \frac{\alpha_s}{\pi}\frac{C_A}{2}\bar{u}\left(\frac{\bar{v}-uv}{u\bar{v}}\theta(u-v) + \frac{\bar{u}-uv}{v\bar{u}}\theta(v-u)\right. \\
& \left. - \frac{1}{\bar{u}\bar{v}}\left[\bar{u}\frac{\theta(u-v)}{u-v} + \bar{v}\frac{\theta(v-u)}{v-u}\right]_+\right)
\end{aligned}$$

$$\begin{aligned}
\gamma_{(1c)}(u,v) = {} & \frac{\alpha_s \delta(u-v)}{\pi}\left[\frac{1}{2}C_F + C_A\left(\log\frac{-Q^2}{\mu^2} - 1 + \frac{1}{2}\log v\bar{v}\right)\right] \\
& - \frac{\alpha_s}{\pi}\left(C_F - \frac{C_A}{2}\right)\frac{1}{v\bar{v}}\left(v\bar{u}\theta(u-v) + u\bar{v}\theta(v-u)\right. \\
& \left. + \left[\bar{u}v\frac{\theta(u-v)}{u-v} + \bar{v}u\frac{\theta(v-u)}{v-u}\right]_+\right).
\end{aligned}$$

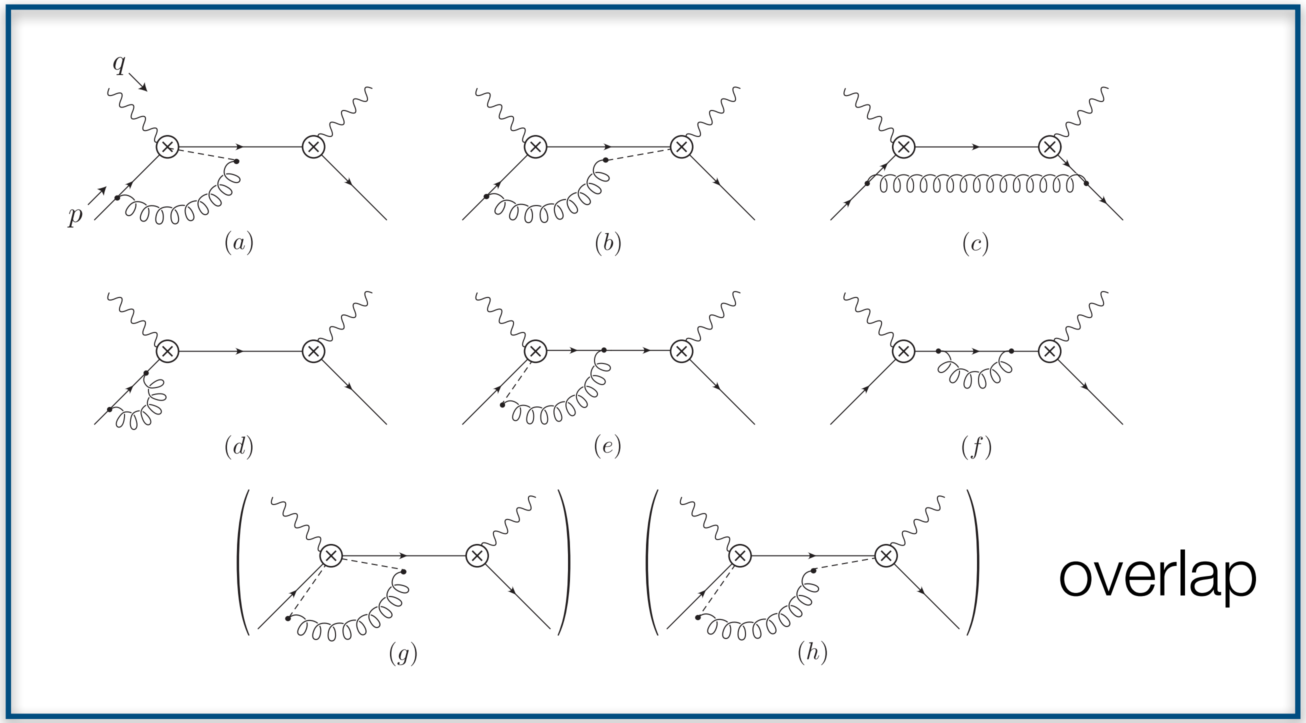
$$\begin{aligned}
\gamma_2^{(2a_1)}(u,v) = {} & \frac{\alpha_s}{\pi}\delta(u-v)\left[C_F\left(\log\frac{-Q^2}{\mu^2} + \log(\bar{v}) - \frac{3}{2}\right) + \frac{C_A}{2}\left(\log\frac{v}{\bar{v}} + \frac{5}{2}\right)\right] \\
& + \frac{\alpha_s}{\pi}\left(C_F - \frac{C_A}{2}\right)\frac{1}{v\bar{v}^2}\left(\bar{u}^2\bar{v}^2\,\theta(u+v-1) + uv(\bar{u}\bar{v} + \bar{u} + \bar{v} - 1)\theta(1-u-v)\right) \\
& - \frac{\alpha_s}{\pi}\frac{C_A}{2}\frac{1}{v\bar{v}^2}\left(v\bar{u}^2(1+\bar{v})\theta(u-v) + u\bar{v}^2(1+\bar{u})\theta(v-u)\right. \\
& \left. + \left[v\bar{u}^2\frac{\theta(u-v)}{u-v} + u\bar{v}^2\frac{\theta(v-u)}{v-u}\right]_+\right),
\end{aligned}$$

$$\begin{aligned}
\gamma_2^{(2a_2)}(u,v) = {} & \frac{\alpha_s}{\pi}\delta(u-v)\left[C_F\left(\log\frac{-Q^2}{\mu^2} + \log(v) - \frac{3}{2}\right) + \frac{C_A}{2}\left(\log\frac{\bar{v}}{v} + \frac{5}{2}\right)\right] \\
& + \frac{\alpha_s}{\pi}\left(C_F - \frac{C_A}{2}\right)\frac{1}{\bar{v}v^2}\left(\frac{uv}{\bar{u}\bar{v}}(\bar{u}-v)(\bar{v}-u)\theta(1-u-v)\right) \\
& - \frac{\alpha_s}{\pi}\frac{C_A}{2}\frac{1}{\bar{v}v^2}\left(\frac{v\bar{u}(\bar{v}-u)}{\bar{v}}\theta(u-v) + \frac{u\bar{v}(\bar{u}-v)}{\bar{u}}\theta(v-u)\right. \\
& \left. + \left[\bar{u}v^2\frac{\theta(u-v)}{u-v} + \bar{v}u^2\frac{\theta(v-u)}{v-u}\right]_+\right).
\end{aligned}$$

$\mu = Q\sqrt{1-x}$: Match SCET to PDF

$$T^{\mu\nu} = \text{Disc}\,\frac{1}{2\pi}\int d^dx\; e^{-iq\cdot x}T\left[\mathcal{J}^{\mu\dagger}(x)\mathcal{J}^{\nu}(0)\right] \to \int \frac{dw}{w}C^{\mu\nu}(w)\phi(-q^+/w) + \ldots$$

$$\phi(r^+) = \frac{1}{4\pi}\int_{-\infty}^{\infty} dt\, e^{-ir^+t}\,\bar{\chi}_{\bar{n}}(nt)\not{n}\chi_{\bar{n}}(0)$$



$$T^{\mu\nu} = \text{Disc}\,\frac{1}{2\pi}\int d^dx\; e^{-iq\cdot x}T\left[\mathcal{J}^{\mu\dagger}(x)\mathcal{J}^{\nu}(0)\right]$$

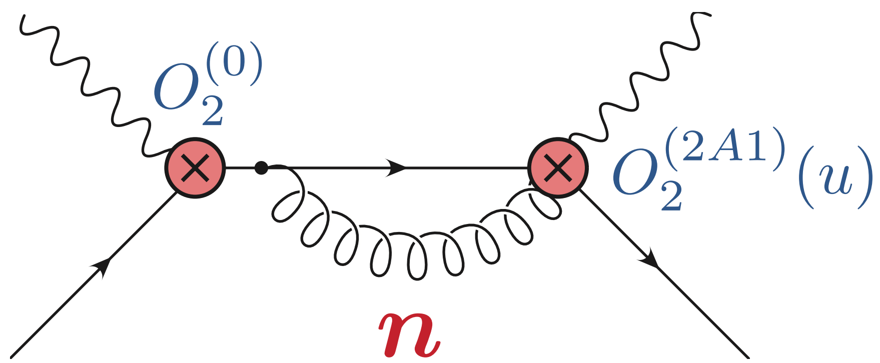
$$\to \int \frac{dw}{w}C^{\mu\nu}(w)\phi(-q^+/w) + \ldots$$

LP factorization: $\boldsymbol{C^{\mu\nu}(w) = \left|C_2^{(0)}\right|^2 C_J^{(0,T)}(w,\mu)g_\perp^{\mu\nu}}$

(the same result everyone else gets)

$$C_J^{(0,T)}(w) = -\,\delta(1-w) \;\; - \frac{\alpha_s C_F}{2\pi}\left\{\left(\log^2\frac{Q^2}{\mu^2} - \frac{3}{2}\log\frac{Q^2}{\mu^2} - \frac{\pi^2}{2} + \frac{7}{2}\right)\delta(1-w)\right.$$
$$\left. + (1+w^2)\left[\frac{\log(1-w)}{1-w}\right]_+ + \left((1+w^2)\log\frac{Q^2}{\mu^2 w} - \frac{3}{2}\right)\frac{1}{[1-w]_+} + \frac{1}{2}(3+w)\theta(1-w)\right\} + O(\alpha_s^2)$$

Endpoint divergence at NLP:



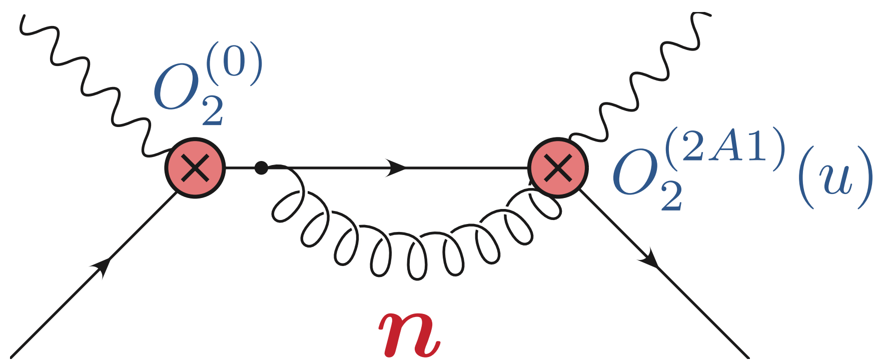
$$\sim \frac{1}{u^{1+\epsilon}}$$

$$u = k \cdot \bar{n}$$

$$\int du\, F_{T,n}^{(0,2A_1)} = \frac{\alpha_s C_F}{2\pi} \frac{\theta(1-y)}{y} \left(-\frac{1}{\epsilon} + \log \frac{Q^2(1-y)}{\mu^2 y} - \frac{3}{2} \right)$$

- spurious divergence - neither UV (no counterterm) or IR (not in matrix element of distribution function)
- arises from region of loop integration where both sectors contribute - should be fixed by overlap subtraction

Endpoint divergence at NLP:

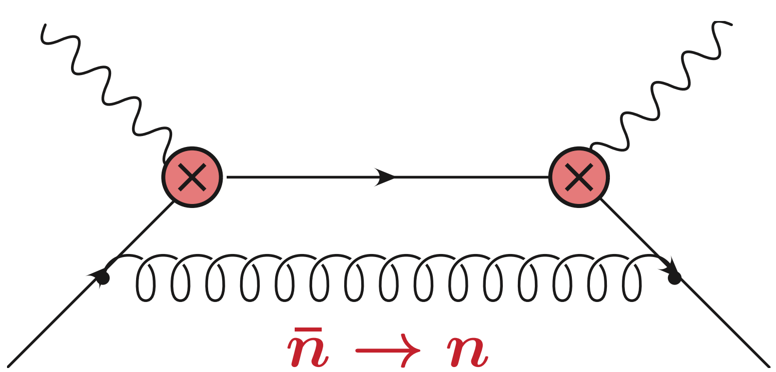


$$\sim \frac{1}{u^{1+\epsilon}}$$

$$u = k \cdot \bar{n}$$

Overlap:

$$\int du\, F_{T,n}^{(0,2A_1)} = \frac{\alpha_s C_F}{2\pi} \frac{\theta(1-y)}{y} \left(-\frac{1}{\epsilon} + \log \frac{Q^2(1-y)}{\mu^2 y} - \frac{3}{2} \right)$$



$O(1)$: LP overlap

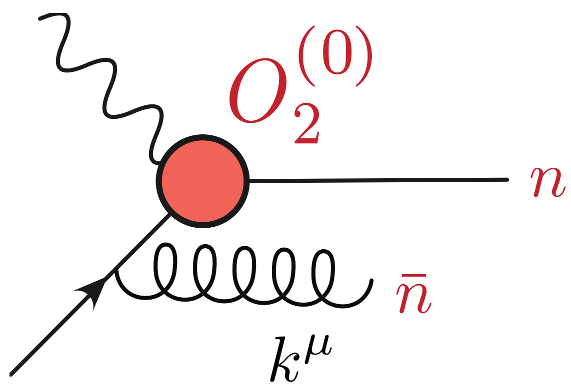
$O(1/Q^2)$: NLP overlap

$$F_{T,\bar{n}\to n}^{(0,0),\mathrm{NLP}} = \frac{\alpha_s C_F}{\pi} \frac{\theta(1-y)}{y} \left(-\frac{1}{\epsilon} + \log \frac{Q^2(1-y)}{\mu^2 y} - 1 \right)$$

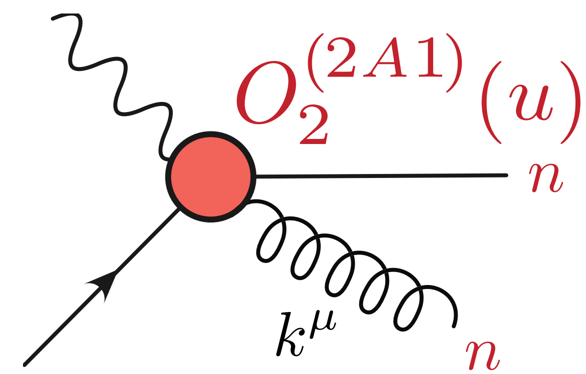
cancels the spurious divergence

Consistency:

Cancellation must occur at all renormalization scales, although terms arise from different operators -> nontrivial constraint on anomalous dimensions



$$\gamma_2^{(0)} = \frac{\alpha_s C_F}{\pi}\left(\log\frac{Q^2}{\mu^2} - \frac{3}{2}\right)$$



$$\begin{aligned}
\gamma_2^{(2A_1)}(u,v) = \frac{\alpha_s}{\pi}\Bigg(&\delta(u-v)\left\{C_F\left(\log\frac{Q^2}{\mu^2} - \frac{3}{2} + \log\bar{v}\right)\right. \\
&\left. + \frac{C_A}{2}\left(\frac{5}{2} + \log\frac{v}{\bar{v}}\right)\right\} \\
&+ \left(C_F - \frac{C_A}{2}\right)\left\{\frac{\bar{u}^2}{u}\theta(u+v-1)\right. \\
&\left. + \frac{v}{\bar{v}^2}(\bar{u}\bar{v} + \bar{u} + \bar{v} - 1)\theta(1-u-v)\right\} \\
&- \frac{C_A}{2u\bar{v}^2}\left\{v\bar{u}^2(1+\bar{v})\theta(u-v) + u\bar{v}^2(1+\bar{u})\theta(v-u)\right. \\
&\left. + \left[v\bar{u}^2\frac{\theta(u-v)}{u-v} + u\bar{v}^2\frac{\theta(v-u)}{v-u}\right]_+\right\}\Bigg),
\end{aligned}$$

BUT rate only depends on the integrated operator

$$\overline{O}_2^{(2A_1)}(\mu) \equiv \int_0^1 du\; O_2^{(2A_1)}(u,\mu)$$

and

$$\int_0^1 du\; \gamma_2^{(2A_1)}(u,v) = \gamma_2^{(0)} = \frac{\alpha_s C_F}{\pi}\left(\log\frac{Q^2}{\mu^2} - \frac{3}{2}\right)$$

so the subleading operator runs the same as the leading order current: cancellation occurs independent of scale (RPI? presumably, but not obviously …)

Final factorization:

Matching onto PDF

$$C^{\mu\nu}(w) = \left|C_2^{(0)}(\mu)\right|^2 \left[C_J^{(0,T)}(w) + C_J^{(2,T)}(w)\right] g_\perp^{\mu\nu}$$

$$+ \int du\, dv\, C_2^{(1A)\dagger}(u,\mu) C_2^{(1A)}(v,\mu) C_J^{(2,L)}(u,v,w) L^{\mu\nu} + \ldots$$

Matching onto SCET

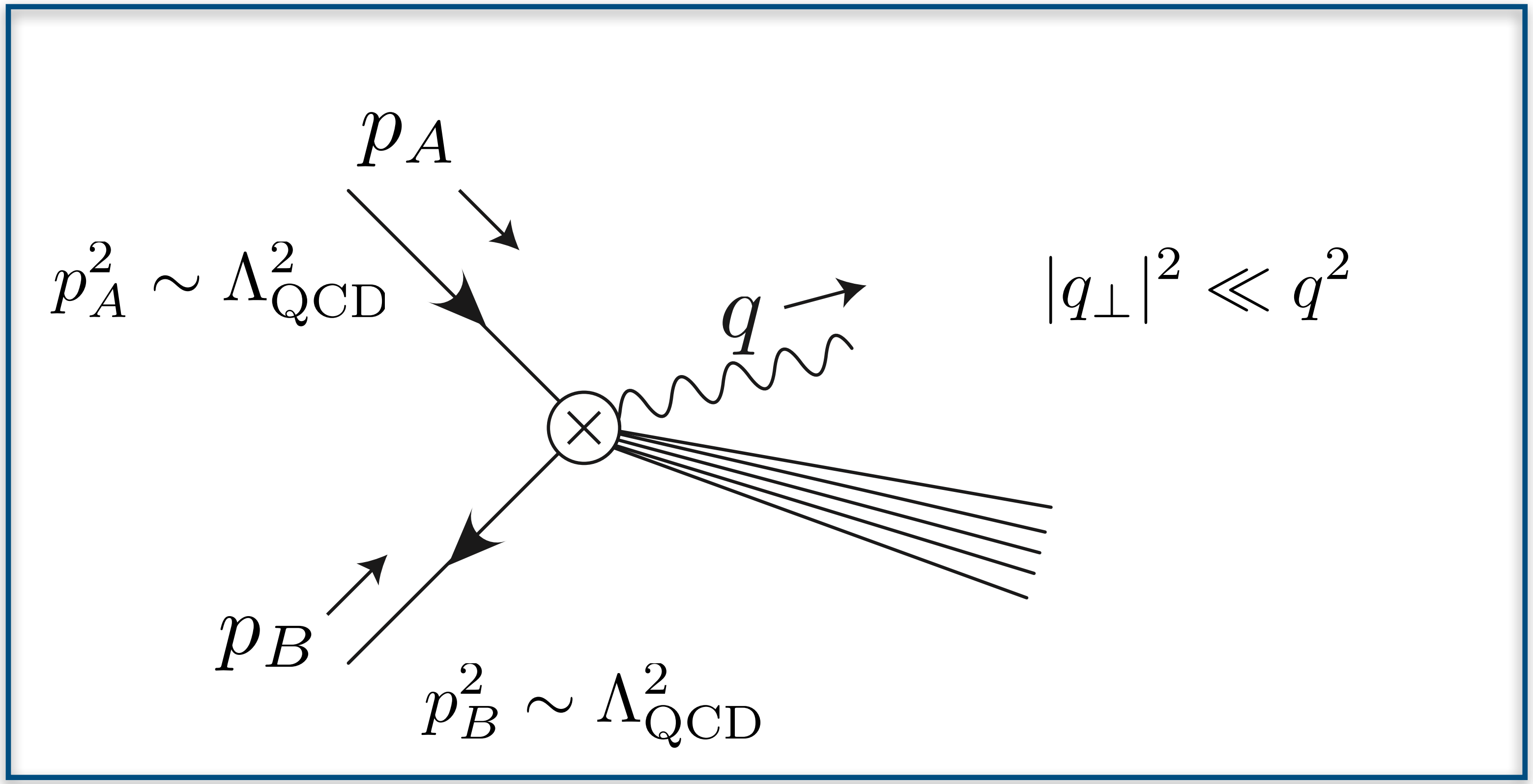
$$C_J^{(0,T)}(w) = -\,\delta(1-w) \;\; - \frac{\alpha_s C_F}{2\pi}\left\{\left(\log^2\frac{Q^2}{\mu^2} - \frac{3}{2}\log\frac{Q^2}{\mu^2} - \frac{\pi^2}{2} + \frac{7}{2}\right)\delta(1-w)\right.$$
$$\left. + (1+w^2)\left[\frac{\log(1-w)}{1-w}\right]_+ + \left((1+w^2)\log\frac{Q^2}{\mu^2 w} - \frac{3}{2}\right)\frac{1}{[1-w]_+} + \frac{1}{2}(3+w)\theta(1-w)\right\} + O(\alpha_s^2)$$

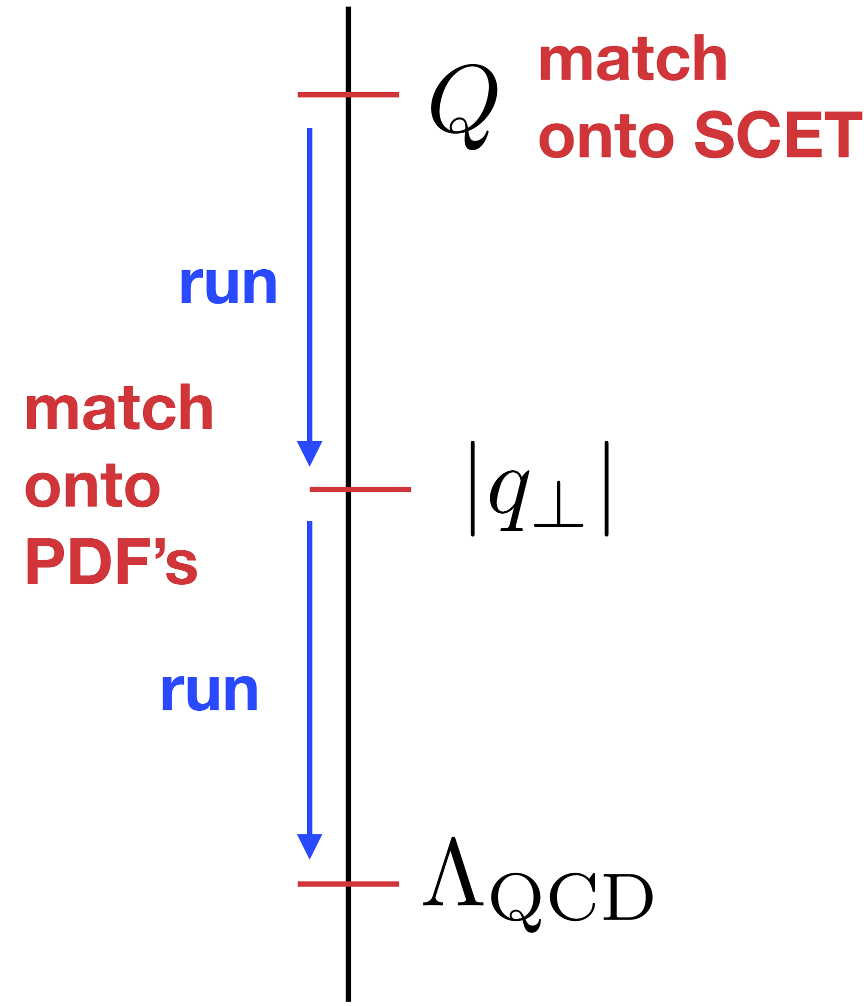
$$C_J^{(2,T)}(w) = -\,\frac{\alpha_s C_F}{2\pi}\frac{\theta(1-w)}{w} + O(\alpha_s^2)$$

$$C_J^{(2,L)}(u,v,w) = \frac{2\alpha_s C_F}{\pi}\frac{\theta(1-w)}{w}(1-u)\,\theta(u)\theta(1-u)\delta(u-v) + O(\alpha_s^2)$$

ex: Drell-Yan at small $q_\perp^2 \gg \Lambda_{\rm QCD}^2$

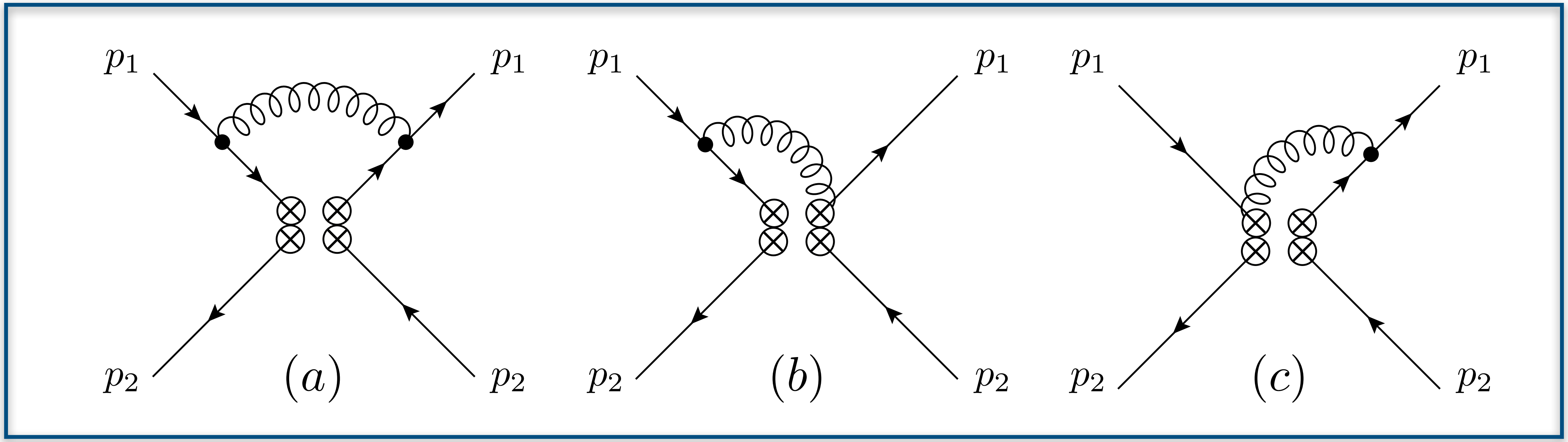
[M. Inglis-Whalen, ML, J. Roy and A. Spourdalakis, 2021]





- Essentially the same calculation until $\mu = |q_\perp|$

- Inclusive rate given by matrix elements of operator products - match onto PDF's at $\mu = q_\perp$



- can Fierz operator products into convolutions of subleading TMD's

$$T_{(k,\ell)}(q,\{u\}) = \int \frac{d^d x}{2(2\pi)^d} e^{-iq\cdot x} \Phi_n^{(k)}(x_n,\{u\}) \Phi_{\bar{n}}^{(\ell)}(x_{\bar{n}},\{u\})$$

$$\Phi_n^{(0)}(x_n) \equiv \bar{\chi}_n(x_n) \frac{\bar{\not{n}}}{2} \chi_n(0)$$

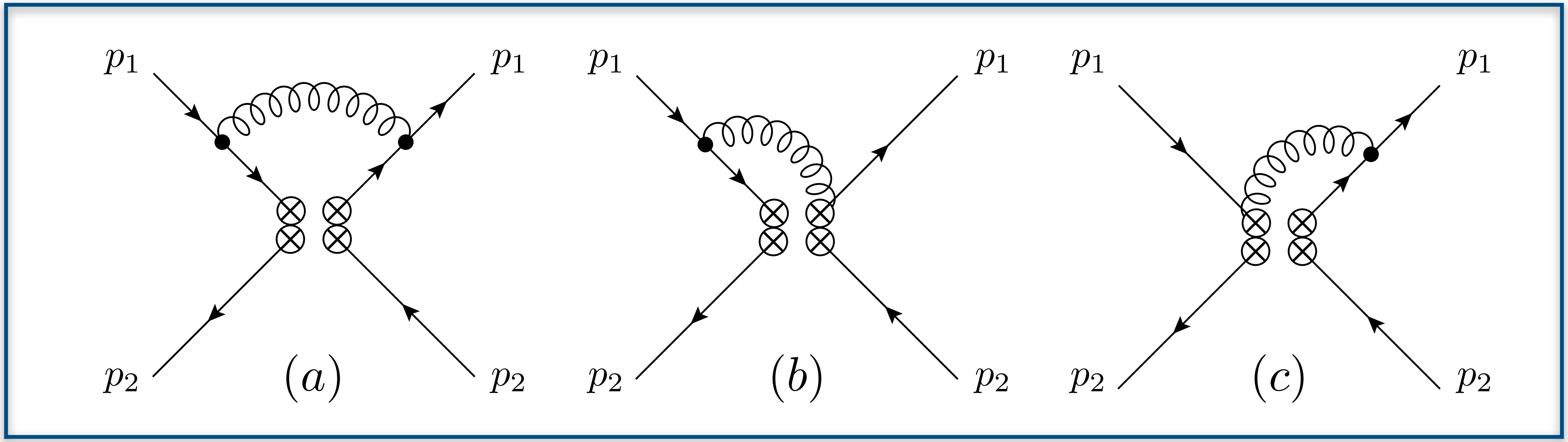
$$\Phi_n^{(2_1)}(x_n,\hat{t}) \equiv (i\partial^\mu \bar{\chi}_n(x_n)) \frac{\bar{\not{n}}}{2} \gamma_\mu^\perp \gamma_\nu^\perp \mathcal{B}_n^{\dagger\nu}(-\bar{n}t) \chi_n(0)$$

$$\Phi_n^{(2_2)}(x_n,\hat{t}_1,\hat{t}_2) \equiv -\bar{\chi}_n(x_n) \mathcal{B}_n^\mu(x_n - \bar{n}t_1) \frac{\bar{\not{n}}}{2} \gamma_\mu^\perp \gamma_\nu^\perp$$

$$\Phi_n^{(2_3)}(x_n,\hat{t}) \equiv 2\pi i \theta(\hat{t}) \otimes \bar{\chi}_n(x_n) \mathcal{B}_n^{\mu\nu}(x_n - \bar{n}t) \frac{\bar{\not{n}}}{2} \gamma_\nu^\perp \gamma_\mu^\perp \chi_n(0)$$

$$\Phi_n^{(2_4)}(x_n) \equiv q^+ q^- \frac{x^-}{2} (n\cdot\partial\, \bar{\chi}_n(x_n)) \frac{\bar{\not{n}}}{2} \chi_n(0)$$

- Inclusive rate given by matrix elements of operator products - match onto PDF's at $\mu = q_\perp$



- individual sector and overlap graphs are divergent in *d* dimensions - need to introduce rapidity regulator ("pure rapidity regulator") [Ebert et. al., 2019]

- scheme dependence in overlap subtraction gives rapidity renormalization group - sum rapidity logs (for future work)

$$\frac{d}{d\log\nu_{n,\bar{n}}} T_{(i,j)}\left(q^-, q^+, \mathbf{q}_T; \nu_{n,\bar{n}}\right) = \sum_{k,\ell} \left(\gamma^{n,\bar{n}}_{(i,j),(k,\ell)} * T_{(k,\ell)}\right)\left(q^-, q^+, \mathbf{q}_T; \nu_{n,\bar{n}}\right)$$

Features at NLP:

- At leading power, only one operator product contributes -> rapidity divergence cancellation is simple (overlap graph). Reproduces known results. [Becher and Neubert, 2011]
- at NLP multiple individually rapidity divergent operator products contribute: cancellation of rapidity divergences occurs between different subleading operators and overlaps (leading and subleading) - gives off-diagonal rapidity mixing.

Final Factorization:

$$\frac{1}{\sigma_0}\frac{d\sigma}{dq^2 dy dq_T^2} = \int \frac{dz_1}{z_1}\frac{dz_2}{z_2} C_{f\bar{f}}(z_1, z_2, q^2, q_T^2)\; f_{q/N_1}\left(\frac{\xi_1}{z_1}\right) f_{\bar{q}/N_2}\left(\frac{\xi_2}{z_2}\right) + O\left(\frac{\Lambda_{\rm QCD}^2}{q^2}\right)$$

$$C_{f\bar{f}}(z_1, z_2, q_L^2, q_T^2) =$$

$$\int \frac{d\Omega_T}{2} \int \frac{d\omega_1}{\omega_1}\frac{d\omega_2}{\omega_2} \sum_{ijkk'\ell\ell'} \frac{H_{(i,j)}\,(\mu_S)\, K_{(k,\ell)}^{(i,j)}}{q_L^{[i]+[j]}}$$

Hard Matching/running $\mu = Q \to \mu = q_\perp$

$$\times\, d^2\mathbf{p}_T\; V_{(k,\ell),(k',\ell')}\left(\omega_1, \omega_2, \mathbf{p}_T; \mu_S; \nu^H_{n,\bar{n}}, \nu^S_{n,\bar{n}}\right)$$

Rapidity renormalization $\nu = Q \to \nu = q_\perp$

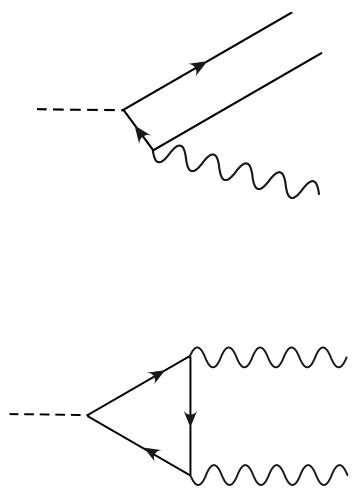
$$\times\, C_{S,(k',\ell')}\left(\frac{z_1}{\omega_1}, \frac{z_2}{\omega_2}, \mathbf{q}_T - \mathbf{p}_T; \mu_S, \nu^S_{n,\bar{n}}\right).$$

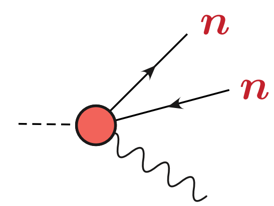
Soft matching $\mu = Q$

What about $h \to \gamma\gamma$?

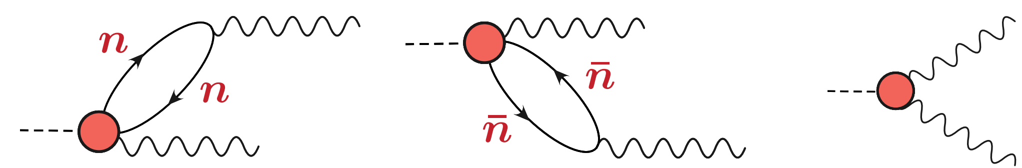
$\mu = m_h$ QCD $\longrightarrow$ SCET

RGE



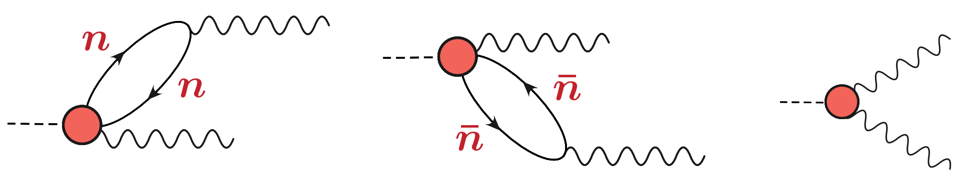


$(+\, n \to \bar{n})$

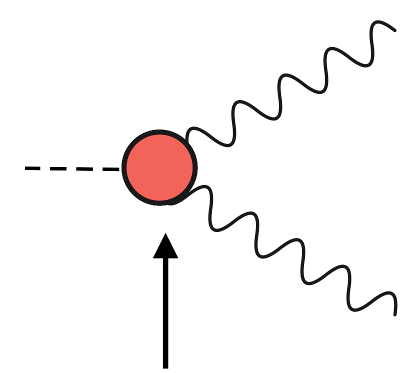


(- overlap)

$\mu = m_b$ SCET $\xrightarrow{\text{Rapidity Renormalization Group}}$ Free Field Theory



(- overlap)



Resummed coefficient

Conclusions:

- It is useful to write SCET as a theory of decoupled QCD sectors, with no mode expansion. Factorization arises through usual matching/running procedure. Scheme dependence of overlap subtraction gives rapidity renormalization group equations.
- Using this, we presented the first EFT calculation of power corrections in endpoint DIS and small-q_T Drell-Yan
- endpoint divergences naturally cancel between graphs - no refactorization prescription required. Cancellation arises due to consistent application of overlap subtraction procedure already required at LP.
- at NLP, complicated pattern of divergence cancellation between different operators, including NLP expansion of overlap
- lots of future work: application to more processes, consistency conditions, Glaubers, …