Renormalization for the NLP TMD Quark-Gluon-Quark Correlators

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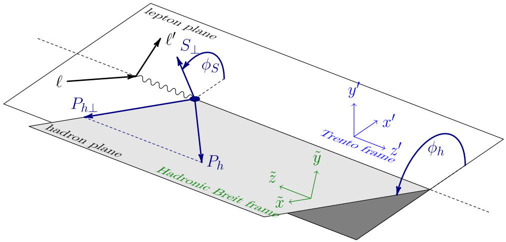
Work with Johannes Michel, Iain Stewart

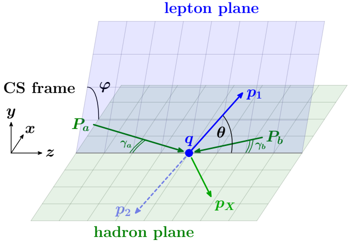
SCET 2023

Intro: Azimuthal Dependence in SIDIS and Drell-Yan

Semi-Inclusive DIS (SIDIS)



Drell-Yan



- SIDIS $\left(\epsilon = \frac{1-y}{1-y+\frac{1}{2}y^2},\ y = \frac{P_N\cdot q}{P_N\cdot p_\ell}\right)$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\vec{P}_{hT}} \sim W_{UU,T} + \sqrt{2\epsilon(1+\epsilon)}\cos(\phi_h) W_{UU}^{\cos(\phi_h)} + \epsilon\cos(2\phi_h) W_{UU}^{\cos(2\phi_h)} + \cdots$$

- Drell-Yan

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4q\,\mathrm{d}\cos\theta\,\mathrm{d}\phi} \sim (1+\cos^2\theta) W_{\text{unpol}} + \sin(2\theta)\cos\varphi\, W_1 + \cdots$$

- At TMD limit $\lambda \sim \frac{P_{hT}}{zQ},\ \frac{q_T}{Q} \ll 1$, several structure functions start at NLP $\mathcal{O}(\lambda)$

See also John Terry's talk

Review of Bare Factorization at NLP using SCET

Previously at SCET, I showed [Ebert, AG, Stewart 2112.07680]
all-order bare factorization for the NLP structure functions

- Subl. soft contributions (subl. Lagrangian and operator) vanish
- Kinematic corrections and $C^{(0)}\bar\chi_{\bar n}\not{\mathcal{P}}_\perp\chi_n$ contributions can be expressed in terms of LP TMDs
- The only nontrivial part: $C^{(1)}\bar\chi_{\bar n}\not{\mathcal{B}}_{n\perp}\chi_n$ operator contribution

$$W^{(1)\mu\nu}_{\mathcal{B}\ \mathrm{SIDIS}} \sim \hat t^\nu\int\mathrm{d}\xi\,\mathcal{H}^{(1)}(\xi)\mathrm{Tr}\Big[\tilde B^\rho_{\mathcal{B}}(x,\xi,\vec b_T)\,\gamma^\mu_\perp\,\mathcal{G}(z,\vec b_T)\,\gamma_{\perp\rho}+B(x,\vec b_T)\,\gamma^\mu_\perp\,\tilde{\mathcal{G}}^\rho_{\mathcal{B}}(z,\xi,\vec b_T)\,\gamma_{\perp\rho}\Big]$$

$$W^{(1)\mu\nu}_{\mathcal{B}\ \mathrm{DY}} \sim \hat z^\nu\int\mathrm{d}\xi\,\mathcal{H}^{(1)}(\xi)\mathrm{Tr}\Big[\tilde B^\rho_{\mathcal{B}}(x,\xi,\vec b_T)\,\gamma^\mu_\perp\,\bar B(z,\vec b_T)\,\gamma_{\perp\rho}+B(x,\vec b_T)\,\gamma^\mu_\perp\,\tilde{\bar B}^\rho_{\mathcal{B}}(z,\xi,\vec b_T)\,\gamma_{\perp\rho}\Big]$$

where the TMD quark-gluon-quark (qgq) correlators are defined as

$$\tilde B^{\rho\,\beta'\beta}_{\mathcal{B}\,f/N}(x,\xi,\vec b_T)\equiv\langle N|\,\bar\chi^\beta_n(b^\mu_\perp)\,\big[\mathcal{B}^\rho_{n\perp,-\xi Q}\,\chi^{\beta'}_{n,(1-\xi)Q}\big](0)\,|N\rangle\,\sqrt{S(b_T)}\,,$$

$$\tilde{\mathcal{G}}^{\rho\,\alpha'\alpha}_{\mathcal{B}\,h/f}(z,\xi,\vec b_T)\equiv\frac{1}{2zN_c}\sum_{X_{\bar n}}\mathrm{tr}\Big\langle 0\Big|\chi^{\alpha'}_{\bar n}(b_\perp)\Big|h,X_{\bar n}\Big\rangle\Big\langle h,X_{\bar n}\Big|\bar\chi^\alpha_{\bar n,\,-(1-\xi)Q}\,\mathcal{B}^\rho_{\bar n\perp,\,\xi Q}(0)\Big|0\Big\rangle\sqrt{S(b_T)}\,,$$

and the hard function is $\mathcal{H}^{(1)}(\xi)=C^{(1)}(\xi)\,C^{(0)*}$

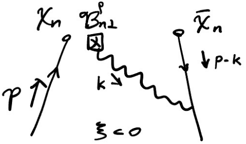
Task today: renormalize qgq

Novel Rapidity Divergence

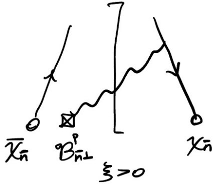
$$W^{(1)\mu\nu}_{\mathcal{B}\ \mathrm{SIDIS}} \sim \hat{t}^{\nu} \int \mathrm{d}\xi\, \mathcal{H}^{(1)}(\xi) \mathrm{Tr}\Big[\underbrace{\tilde{B}^{\rho}_{\mathcal{B}\,f}(x,\xi,\vec{b}_T)}_{+\frac{1}{\eta}} \gamma^{\mu}_{\perp}\, \mathcal{G}(z,\vec{b}_T)\, \gamma_{\perp\rho} + B(x,\vec{b}_T)\, \gamma^{\mu}_{\perp}\, \underbrace{\tilde{\mathcal{G}}^{\rho}_{\mathcal{B}}(z,\xi,\vec{b}_T)}_{-\frac{1}{\eta}}\, \gamma_{\perp\rho} \Big]$$

$$\tilde{B}^{\rho\,\beta'\beta}_{\mathcal{B}\,f/N}(x,\xi,\vec{b}_T) \equiv \langle N|\, \bar{\chi}^{\beta}_{n}(b^{\mu}_{\perp})\, \big[\mathcal{B}^{\rho}_{\perp n,-\xi Q}\, \chi^{\beta'}_{n,(1-\xi)Q}\big](0)\, |N\rangle\, \sqrt{S(b_T)}\,,$$

At leading order (LO), matching onto quark PDF $f_{q/N}(\frac{x}{z})$, FF $D_{h/q}(\frac{z_h}{z})$



$$= -\alpha_s\, 2C_F\, \underbrace{\frac{\theta(-\xi)}{(-\xi)^{1+\eta}}}_{= -\frac{\delta(\xi)}{\eta} + \mathcal{L}_0(-\xi)} \left(\frac{Q}{\nu}\right)^{-\eta} \delta\left(\frac{1-z}{z} + \xi\right) \underbrace{\mu^{2\epsilon} \int d^{2-2\epsilon} k_\perp (i\, k^{\rho}_{\perp})}_{\equiv\, \mathcal{I}^{\rho}_{NLP}(b_\perp) = \frac{b^{\rho}_{\perp}}{b^2_{\perp}} \frac{1}{2\pi} + \mathcal{O}(\epsilon)} + \mathcal{O}(\xi^0)$$



$$= \alpha_s\, 2C_F\, \underbrace{\frac{\theta(\xi)}{\xi^{1+\eta}}}_{= -\frac{\delta(\xi)}{\eta} + \mathcal{L}_0(\xi)} \left(\frac{Q}{\nu}\right)^{-\eta} \delta(1-\xi-z)\, \mathcal{I}^{\rho}_{NLP}(b_\perp) + \mathcal{O}(\xi^0)$$

$$\mathcal{L}_0(x) \equiv \left[\frac{\theta(x)}{x}\right]_+$$

- The two terms in $[\dots]$ are individually rapidity divergent, although divergence cancels in $W^{(1)\mu\nu}_{\mathcal{B}}$ [first observed at LO in Rodini, Vladimirov '22]

Novel Rapidity Divergence

$+\frac{1}{\eta}$ $-\frac{1}{\eta}$

$$W^{(1)\mu\nu}_{\mathcal{B}\ \mathrm{SIDIS}} \sim \hat{t}^{\nu} \int \mathrm{d}\xi\, \mathcal{H}^{(1)}(\xi) \mathrm{Tr}\Big[\tilde{B}^{\rho}_{\mathcal{B}\, f}(x,\xi,\vec{b}_T)\, \gamma^{\mu}_{\perp}\, \mathcal{G}(z,\vec{b}_T)\, \gamma_{\perp\rho} + B(x,\vec{b}_T)\, \gamma^{\mu}_{\perp}\, \tilde{\mathcal{G}}^{\rho}_{\mathcal{B}}(z,\xi,\vec{b}_T)\, \gamma_{\perp\rho}\Big]$$

where the quark-gluon-quark (qgq) correlators are defined as

$$\tilde{B}^{\rho\,\beta'\beta}_{\mathcal{B}\, f/N}(x,\xi,\vec{b}_T) \equiv \langle N|\, \bar{\chi}^{\beta}_{n}(b^{\mu}_{\perp})\, \big[\mathcal{B}^{\rho}_{\perp n,-\xi Q}\, \chi^{\beta'}_{n,(1-\xi)Q}\big](0)\, |N\rangle \sqrt{S(b_T)}\,,$$

- Rapidity poles start at LO, which is $\mathcal{O}(\alpha_s)$!

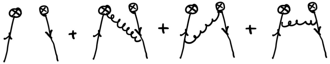
$\Rightarrow$ Can never be canceled by a multiplicative counterterm $Z = 1 + \mathcal{O}(\alpha_s)$

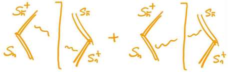
$\Rightarrow$ Need additive counterterm to get separately finite matrix elements
[LO counterterm involving $\partial^{\rho}\gamma_{\zeta}^{1-\mathrm{loop}}$ proposed in Rodini, Vladimirov '22]

- Additive counterterm also in subleading $\mathrm{SCET_I}$ applications: dijet [Moult et al '18], threshold [Beneke et al '18], ...
- Here $\mathrm{SCET_{II}}$ has $1/\eta$'s at $\mathcal{O}(\alpha_s\lambda)$
  See also $\mathrm{SCET_{II}}$ @ $\mathcal{O}(\alpha_s\lambda^2)$: q_T [Ebert et al '18], EEC [Moult et al '19], $h \to \gamma\gamma$ [Liu et al '19], ...

Rap. Div. Cancellation: Use Soft Matrix Elements (ME)

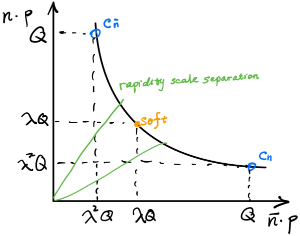
- At LP, rap. div. cancels multiplicatively (eg. η reg.)

$\hat{B}_{q/q} =$  $+\,\mathcal{O}(\alpha_s^2) = 1 - \alpha_s 4C_F \frac{1}{\eta}\mathcal{I}_{\rm LP} + \mathcal{O}(\eta^0) + \mathcal{O}(\alpha_s^2)$

$S = 1 +$  $+\,\mathcal{O}(\alpha_s^2) = 1 + \alpha_s 8C_F \frac{1}{\eta}\mathcal{I}_{\rm LP} + \mathcal{O}(\eta^0) + \mathcal{O}(\alpha_s^2)$

$\Rightarrow \hat{B}_{q/q}\sqrt{S} \sim \mathcal{O}(\eta^0)$

- From def. of qgq, rap. div. is manifest: it depends on what we mean by a “collinear” gluon field; not compensated by the LP soft function



- Idea: exploit individual pieces of vanishing NLP soft contribution

Construction of the "Counterterm"

$$W_{\mathcal{B}}^{(1)\mu\nu} \sim \hat{t}^{\nu}\int \mathrm{d}\xi\, \mathcal{H}^{(1)}(\xi)\mathrm{Tr}\Big[\tilde{B}^{\rho}_{\mathcal{B}}(x,\xi,\vec{b}_T)\,\gamma_{\perp}^{\mu}\,\mathcal{G}(z,\vec{b}_T)\,\gamma_{\perp\rho} + B(x,\vec{b}_T)\,\gamma_{\perp}^{\mu}\,\tilde{\mathcal{G}}^{\rho}_{\mathcal{B}}(z,\xi,\vec{b}_T)\,\gamma_{\perp\rho}\Big]\,.$$

- Use the fact that rap. div. cancels between the two terms in $W_{\mathcal{B}}^{(1)}$
- Can take $\mathcal{B}^{\rho}_{\bar{n}\perp}$ to soft $\mathcal{B}^{(n)\rho}_{s\perp}$ in SCET without changing the rap. div.

$$B(x,\vec{b}_T)\tilde{\mathcal{G}}^{\rho}_{\mathcal{B}}(z,\xi,\vec{b}_T) \to \delta(\xi)B\mathcal{G}\,\frac{S_{\mathcal{B}}^{(n)\rho}}{S}$$

$$S_{\mathcal{B}}^{(n)\rho}(b_\perp) \equiv \frac{1}{N_c}\,\mathrm{tr}\left\langle 0\middle|\left[S_n^{\dagger}(b_\perp)\,S_{\bar{n}}(b_\perp)\right]\left[S_{\bar{n}}^{\dagger}(0)\,S_n(0)g\mathcal{B}_{s\perp}^{(n)\rho}(0)\right]\middle|0\right\rangle,$$

- By construction, $\boxed{\tilde{B}'^{\rho[\Gamma]}_{\mathcal{B}}(\xi) \equiv \tilde{B}^{\rho[\Gamma]}_{\mathcal{B}}(\xi) + \delta(\xi)\,\frac{S_{\mathcal{B}}^{(n)\rho}}{S}\,B^{[\Gamma]}} \sim \mathcal{O}(\eta^0)$,

- Similarly, $S_{\mathcal{B}}^{(\bar{n})\rho} \equiv \frac{1}{N_c}\,\mathrm{tr}\left\langle 0\middle|\left[S_n^{\dagger}(b_\perp)\,S_{\bar{n}}(b_\perp)\right]\left[g\mathcal{B}_{s\perp}^{(\bar{n})\rho}(0)S_{\bar{n}}^{\dagger}(0)\,S_n(0)\right]\middle|0\right\rangle$ absorbs rap. div. of the second term: $\tilde{\mathcal{G}}'^{\rho[\Gamma]}_{\mathcal{B}}(\xi) \equiv \tilde{\mathcal{G}}^{\rho[\Gamma]}_{\mathcal{B}}(\xi) + \delta(\xi)\,\frac{S_{\mathcal{B}}^{(\bar{n})\rho}}{S}\,\mathcal{G}^{[\Gamma]} \sim \mathcal{O}(\eta^0)$

Construction of the "Counterterm"

- Note that $S_{\mathcal{B}}^{(n)\rho} + S_{\mathcal{B}}^{(\bar{n})\rho}$

$$= \frac{1}{N_c} \operatorname{tr} \Big\langle 0 \Big| \big[S_n^\dagger S_{\bar{n}}\big](b_\perp) \Big[S_{\bar{n}}^\dagger S_n g\mathcal{B}_{s\perp}^{(n)\rho} + g\mathcal{B}_{s\perp}^{(\bar{n})\rho} S_{\bar{n}}^\dagger S_n\Big](0) \Big| 0 \Big\rangle = 0$$

  due to C & P & Poincaré invariance of the vacuum!

  [Ebert, AG, Stewart 19']

- $$\tilde{B}_{\mathcal{B}}^{\rho}\mathcal{G} + B\tilde{\mathcal{G}}_{\mathcal{B}}^{\rho} = \underbrace{\left[\tilde{B}_{\mathcal{B}}^{\rho} + \delta(\xi)\, B\, \frac{S_{\mathcal{B}}^{(n)\rho}}{S}\right]}\mathcal{G} + B \underbrace{\left[\delta(\xi)\mathcal{G}\, \frac{S_{\mathcal{B}}^{(\bar{n})\rho}}{S} + \tilde{\mathcal{G}}_{\mathcal{B}}^{\rho}\right]}$$

  free of $\frac{1}{\eta}$ divergences to all orders (additive + LP multiplicative div.)

  $$= \tilde{B}_{\mathcal{B}}^{\prime\rho}\mathcal{G} + B\tilde{\mathcal{G}}_{\mathcal{B}}^{\prime\rho}$$

- So we can simply replace the old by the new qgq correlators

A Neat Property of the "Counterterm"

$$S_{\mathcal{B}}^{(n)\rho}(b_\perp) \equiv \frac{1}{N_c}\,\mathrm{tr}\,\Big\langle 0\Big|\big[S_n^\dagger(b_\perp)\,S_{\bar n}(b_\perp)\big]\big[S_{\bar n}^\dagger(0)\,S_n(0) g\mathcal{B}_{s\perp}^{(n)\rho}(0)\big]\Big|0\Big\rangle$$

$$S_{\mathcal{B}}^{(\bar n)\rho}(b_\perp) \equiv \frac{1}{N_c}\,\mathrm{tr}\,\Big\langle 0\Big|\big[S_n^\dagger(b_\perp)\,S_{\bar n}(b_\perp)\big]\big[g\mathcal{B}_{s\perp}^{(\bar n)\rho}(0) S_{\bar n}^\dagger(0)\,S_n(0)\big]\Big|0\Big\rangle$$

- Manipulation of Wilson lines gives

$$\begin{aligned}\big[\mathcal{P}_\perp^\rho S_{\bar n}^\dagger S_n\big] &= \big[S_{\bar n}^\dagger \mathrm{i} D_{s\perp}^\rho S_n\big] + \big[S_{\bar n}^\dagger \mathrm{i}\overleftarrow{D}_{s\perp}^\rho S_n\big] \\ &= S_{\bar n}^\dagger S_n \,[S_n^\dagger \mathrm{i} D_{s\perp}^\rho S_n] + [S_{\bar n}^\dagger \mathrm{i}\overleftarrow{D}_{s\perp}^\rho S_{\bar n}]\, S_{\bar n}^\dagger S_n \\ &= S_{\bar n}^\dagger S_n\, g\mathcal{B}_{s\perp}^{(n)\rho} - g\mathcal{B}_{s\perp}^{(\bar n)\rho}\, S_{\bar n}^\dagger S_n\end{aligned}$$

- Since $S_{\mathcal{B}}^{(n)\rho} + S_{\mathcal{B}}^{(\bar n)\rho} = 0$, we have $S_{\mathcal{B}}^{(n)} = -S_{\mathcal{B}}^{(\bar n)} = \frac{\mathrm{i}}{2}\partial_\perp^\rho S$ so that

$$S_{\mathcal{B}}^{(n)\rho}/S = \frac{\mathrm{i}}{2}\partial_\perp^\rho \ln S$$

$\Rightarrow$ Due to non-Abelian exponentiation for S, $S_{\mathcal{B}}^{(n)\rho}/S \propto 1/\eta + \mathcal{O}(\eta^0)$ (no ϵ poles, no double poles in η)

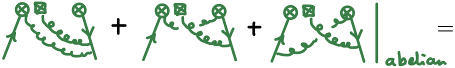
Checking Cancellation of Rapidity Divergence

Check deepest $(1/\eta^2)$ pole in $\alpha_s^2 C_F^2$ channel:

$$\mathcal{O}(C_F\eta^0)$$

$$\left[\hat{\tilde{B}}^{\rho}_{\mathcal{B}}(\xi) + \delta(\xi)\frac{S^{(n)\rho}_{\mathcal{B}}}{S}\hat{B}\right]\sqrt{S} = \mathcal{O}(\eta^0)$$

$$\mathcal{O}(C_F/\eta)$$

$\hat{\tilde{B}}^{(2)}_{\mathcal{B}}$ 

$= -\,\alpha_s^2\, 8C_F^2\,\frac{1}{\eta^2}\delta(1-z)\delta(\xi)\mathcal{I}^{\rho}_{\rm NLP}\mathcal{I}_{\rm LP} + \mathcal{O}(\frac{1}{\eta})$

$\hat{\tilde{B}}^{(1)}_{\mathcal{B}}$ 
$= +\,\alpha_s\, 2C_F\,\frac{1}{\eta}\delta(1-z)\delta(\xi)\,\mathcal{I}^{\rho}_{\rm NLP} + \mathcal{O}(\eta^0)$

$\sqrt{S}$ $\frac{1}{2}$ 
$= +\,\alpha_s\, 4C_F\,\frac{1}{\eta}\delta(1-z)\delta(\xi)\,\mathcal{I}_{\rm LP} + \mathcal{O}(\eta^0)$ ✓

Checking Cancellation of Rapidity Divergence

Check deepest $(1/\eta^2)$ pole in $\alpha_s^2 C_F C_A$ channel:

$$\Bigg[\underbrace{\hat{\tilde{B}}^{\rho}_{\mathcal{B}}(\xi)}_{\text{have to cancel within}} + \delta(\xi)\underbrace{\frac{S^{(n)\rho}_{\mathcal{B}}}{S}}_{\mathcal{O}(C_F/\eta)}\overbrace{\hat{B}\Bigg]\underbrace{\sqrt{S}}_{1+\alpha_s C_F/\eta}}^{\mathcal{O}(\eta^0)} = \mathcal{O}(\eta^0)$$

$A =$  +  $= \mathcal{O}(1/\eta^2)$

$B =$  +  +  +  + ... $= \mathcal{O}(1/\eta^2)$

$A + B = \mathcal{O}(1/\eta)$ ✓

Renormalization and Evolution

- Renormalization

$$W^{(1)\mu\nu}_{\mathcal{B}\ \mathrm{DY}} \sim \int \mathrm{d}\xi'\, \mathcal{H}^{(1)\mathrm{ren}}(\xi')\, Z^{(0)} Z^{(1)}(\xi',\xi) \otimes \Big[\tilde{B}'^{\rho}_{\mathcal{B}}(\xi)\ \bar{B} + B\, \tilde{\bar{B}}'^{\rho}_{\mathcal{B}}(\xi)\Big]$$

$Z^{(1)}(\xi',\xi)$: counterterm for $C^{(1)}$, calculated in [Freedman, Goerke '14, Goerke, Inglis-Whalen '17; Beneke et al '17; Vladimirov, Moos, Scimemi '21]

- Due to charge conjugation $\tilde{B}'^{\rho}_{\mathcal{B}}(\xi) \leftrightarrow \tilde{\bar{B}}'^{\rho}_{\mathcal{B}}(\xi)$, we know that $Z(\xi',\xi)$ renormalizes $\tilde{B}'^{\rho}_{\mathcal{B}}(\xi) = \tilde{B}^{\rho}_{\mathcal{B}}(\xi) + \delta(\xi) B \frac{\mathrm{i}}{2}\partial^{\rho}_{\perp} \ln S$,

$$\tilde{B}^{\mathrm{ren}\,\rho}_{\mathcal{B}}(\xi) \equiv Z^{(1)}(\xi',\xi) \otimes \tilde{B}'^{\rho}_{\mathcal{B}}(\xi)$$

- This implies μ anomalous dimension

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\tilde{B}^{\mathrm{ren}\,\rho[\Gamma]}_{\mathcal{B}}(\xi) = \gamma_{\mu}(\xi,\xi') \otimes \tilde{B}^{\mathrm{ren}\,\rho[\Gamma]}_{\mathcal{B}}(\xi')$$

- Also get all order ζ RGE (equivalently: ν RGE) for renormalized objects

$$2\zeta\frac{\mathrm{d}}{\mathrm{d}\zeta}\tilde{B}^{\mathrm{ren}\,\rho[\Gamma]}_{\mathcal{B}} = -\delta(\xi)\big(\mathrm{i}\partial^{\rho}_{\perp}\gamma_{\zeta}\big) B^{\mathrm{ren}[\Gamma]} + \gamma_{\zeta}\tilde{B}^{\mathrm{ren}\,\rho[\Gamma]}_{\mathcal{B}}$$

Are we done?

Are we done? NO!

Endpoint Divergence

$C^{(1)}$ calculated in [Vladimirov, Moos, Scimemi '21] where $L_Q = \ln \frac{-q^2}{\mu^2}$

$$C^{(1)}(\xi) = 1 + \frac{\alpha_s}{4\pi}\left[C_F\left(-L_Q^2 + L_Q - 3 + \frac{\pi^2}{6}\right) - C_A \frac{\ln\xi}{1-\xi} - \left(C_F - \frac{C_A}{2}\right)\frac{\ln(1-\xi)}{\xi}\left(2L_Q + \ln(1-\xi) - 4\right)\right] + \mathcal{O}(\alpha_s^2),$$

- SCET$_{\text{I}}$ ME $\sim \mathcal{O}(\alpha_s \xi^{-1/2}) \Rightarrow$ convergent,
  SCET$_{\text{II}}$ ME $\sim \mathcal{O}(\alpha_s \xi^{-1}) \Rightarrow$ endpoint divergence

$\Rightarrow$ Divergent integral as $\xi \to 0$ at $\mathcal{O}(\alpha_s^2)$

$$\alpha_s^2 \int \mathrm{d}\xi \ln\xi\, \mathcal{L}_0(\pm\xi) = ???$$

- Special: hard coefficient and rapidity divergence conspire to give an endpoint divergence in a soft gluon limit with back-to-back $n\,\&\,\bar{n}$
- Need refactorization for endpoint divergences [Liu et al, Beneke et al, ...]

  See Robert Szafron's talk for a nice overview

Solution: Hard-Collinear Matching to $\mathcal{B}_{s\perp}$

Q^2 λQ^2

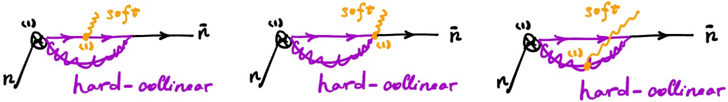
- The full $\mathcal{B}_s^\perp$ current receives hard and hard-collinear contributions ($\hat{p}_s = p_s^\pm$) [Ebert, AG, Stewart 21', (see also my SCET 2022 slides)]

$$J^{(1)\mu}_{\mathcal{B}_s^\perp}(0) = J^{(1)\mu}_{\mathrm{h}\,\mathcal{B}_s^\perp}(0) + J^{(1)\mu}_{\mathrm{hc}\,\mathcal{B}_s^\perp}(0)$$

$$J^{(1)\mu}_{\mathrm{hc}\,\mathcal{B}_s^\perp}(0) \sim \int \mathrm{d}\hat{p}_s \int \mathrm{d}\tilde{\xi}\, C^{(1)}(q^2,\tilde{\xi})\, \tilde{J}_{\mathcal{B}_s^\perp}(\hat{p}_s,\tilde{\xi})$$

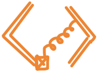
$$\times\, \bar{\chi}_{\bar{n},-\omega_2}\Big\{\big[S_{\bar{n}}^\dagger S_n g\mathcal{B}^{(n)}_{s\perp}\big](p_s^+) + \big[g\mathcal{B}^{(\bar{n})}_{s\perp} S_{\bar{n}}^\dagger S_n\big](p_s^-)\Big\}\,\chi_{n,-\omega_1}\,,$$

- $T\big[J_{\mathrm{I}}^{(1)\mu}\mathcal{L}_{\mathrm{I}}^{(1)}\big]$ in $\mathrm{SCET_I}$ $\to$ hard scattering operators in $\mathrm{SCET_{II}}$



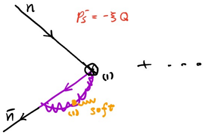
- These $\tilde{J}_{\mathcal{B}_s^\perp}$ graphs reproduce $\xi^{-\epsilon}$ behavior of $C^{(1)}$ $(p_s^- = -\xi Q)$!

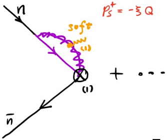
$$\int \mathrm{d}\tilde{\xi}\, \tilde{J}_{\mathcal{B}_s^\perp}(p_s^-,\tilde{\xi}) \sim \sum \text{graphs} = \frac{\alpha_s}{4\pi}\frac{C_A}{\epsilon}\left(\frac{p_s^- Q}{\mu^2}\right)^{-\epsilon} + \mathcal{O}(\epsilon^0) = \frac{\alpha_s}{4\pi}\frac{C_A}{\epsilon}\left(\frac{Q^2}{\mu^2}\right)^{-\epsilon}(1-\epsilon\ln(-\xi)) + \mathcal{O}(\epsilon^0\xi^0$$

Soft ME $\equiv \tilde{\mathcal{S}}^{(\bar{n})}_{\mathcal{B}}(p_s^-) \sim$  $(p_s^-) \sim \frac{\alpha_s}{\pi} C_F \frac{\theta(p_s^-)}{p_s^-} = \frac{\alpha_s}{\pi} C_F \frac{\theta(-\xi)}{-\xi Q} \longleftrightarrow \tilde{B}^{\mathrm{ren}}_{\mathcal{B}}(\xi \to 0)$

Solution

- Exploit the vanishing hc soft contribution for Drell-Yan





$$0 = B\bar{B}\left[\int_0^\infty \mathrm{d}p_s^- \, \frac{1}{\epsilon}\left(\frac{-p_s^- Q}{\mu^2}\right)^{-\epsilon}\frac{1}{p_s^-} - \int_0^\infty \mathrm{d}p_s^+ \, \frac{1}{\epsilon}\left(\frac{-p_s^+ Q}{\mu^2}\right)^{-\epsilon}\frac{1}{p_s^+}\right]$$

$$\propto B\bar{B}\int \mathrm{d}\xi \frac{1}{\epsilon}\left(\frac{-\xi q^2}{\mu^2}\right)^{-\epsilon}\left[\theta(|\xi| - \xi_{\mathrm{cut}}) + \theta(\xi_{\mathrm{cut}} - |\xi|)\right]\left[\frac{\theta(-\xi)}{-\xi} - \frac{\theta(-\xi)}{-\xi}\right]$$

$$= B\bar{B}\left(\frac{-q^2}{\mu^2}\right)^{-\epsilon}\int \mathrm{d}\xi \, \frac{\xi^{-\epsilon}}{\epsilon}\theta(\xi_{\mathrm{cut}} - |\xi|)\Big(\mathcal{L}_0(-\xi) - \mathcal{L}_0(-\xi)\Big)$$

- $\theta(\xi_{\mathrm{cut}} - |\xi|) \ln \xi \, \mathcal{L}_0(-\xi)$ cancels the divergence in qgq as $\xi \to 0$
- The following convolution is finite at $\xi \to 0$!

$$\int \mathrm{d}\xi \left[C^{(1)}_{\mathrm{ren}}(\xi)\tilde{B}^{\mathrm{ren}\,\rho}_{\mathcal{B}}(\xi) + \theta(\xi_{\mathrm{cut}} - |\xi|)\, C^{(1)}_{\mathrm{ren}} \otimes \tilde{J}^{\mathrm{ren}}_{\mathcal{B}_s^\perp}(-\xi Q)\frac{\tilde{S}^{(n)\mathrm{ren}\,\rho}_{\mathcal{B}}(-\xi Q)}{S^{\mathrm{ren}}} B^{\mathrm{ren}}\right]$$

Solution

- Exploit the vanishing hc soft contribution for SIDIS

$$0 = B\mathcal{G}\left[\int_0^\infty \mathrm{d}p_s^- \,\frac{1}{\epsilon}\left(\frac{p_s^- Q}{\mu^2}\right)^{-\epsilon}\frac{1}{p_s^-} - \int_0^\infty \mathrm{d}p_s^+ \left(\frac{-p_s^+ Q}{\mu^2}\right)^{-\epsilon}\frac{1}{p_s^+}\right]$$

$$\propto B\mathcal{G}\int \mathrm{d}\xi\, \frac{1}{\epsilon}\left(\frac{-\xi q^2}{\mu^2}\right)^{-\epsilon}\left[\theta(|\xi| - \xi_{\rm cut}) + \theta(\xi_{\rm cut} - |\xi|)\right]\left[\frac{\theta(-\xi)}{-\xi} - \frac{\theta(\xi)}{\xi}\right]$$

$$= B\mathcal{G}\left(\frac{-q^2}{\mu^2}\right)^{-\epsilon}\left[-\mathrm{i}\pi\left(\frac{1}{\epsilon} - \ln\xi_{\rm cut}\right) - \frac{1}{2}\pi^2 + \int \mathrm{d}\xi\,\frac{\xi^{-\epsilon}}{\epsilon}\theta(\xi_{\rm cut} - |\xi|)\Big(\mathcal{L}_0(-\xi) - \mathcal{L}_0(\xi)\Big)\right]$$

after including h.c.

- $\theta(\xi_{\rm cut} - |\xi|)\ln\xi\, \mathcal{L}_0(\mp\xi)$ cancels the divergence in $\int \mathrm{d}\xi C^{(1)}_{\rm ren}\tilde{B}^{{\rm ren}\,\rho}_{\mathcal{B}}$, $\tilde{\mathcal{G}}^{{\rm ren}\,\rho}_{\mathcal{B}}$

$$\int \mathrm{d}\xi\left[C^{(1)}_{\rm ren}(\xi)\tilde{B}^{{\rm ren}\,\rho}_{\mathcal{B}}(\xi) + \theta(\xi_{\rm cut} - |\xi|)\, C^{(1)}_{\rm ren} \otimes \tilde{J}^{\rm ren}_{\mathcal{B}_s^\perp}(-\xi Q)\tilde{S}^{(n){\rm ren}\,\rho}_{\mathcal{B}}(-\xi Q)/S^{\rm ren}\, B^{\rm ren}\right]$$

$$\int \mathrm{d}\xi\left[C^{(1)}_{\rm ren}(\xi)\tilde{\mathcal{G}}^{{\rm ren}\,\rho}_{\mathcal{B}}(\xi) + \theta(\xi_{\rm cut} - |\xi|)\, C^{(1)}_{\rm ren} \otimes \tilde{J}^{\rm ren}_{\mathcal{B}_s^\perp}(\xi Q)\tilde{S}^{(\bar{n}){\rm ren}\,\rho}_{\mathcal{B}}(\xi Q)/S^{\rm ren}\, \mathcal{G}^{\rm ren}\right]$$

- We are left with a remainder term $R_{\rm SIDIS} \propto \alpha_s C_A \alpha_s \pi^2 C_F B\mathcal{G} + \mathcal{O}(\alpha_s^3)$
- For $q_T \sim \Lambda_{\rm QCD}$, $\alpha_s\pi^2 C_F \longrightarrow$ NP function defined by $\tilde{S}^\rho_{\mathcal{B}}(\hat{p}_s, b_\perp)$

Summary

- Constructed all-order definitions of the renormalized TMD qgq correlators which involves both additive and multiplicative soft factors
  - ▷ soft subtraction for $\frac{\delta(\xi)}{\eta}$ divergences
  - ▷ soft subtraction for removal of endpoint $\ln\xi/\xi$ divergences
- Perturbative cross checks at $\mathcal{O}(\alpha_s)$ and C_F^2/η^2, $C_F C_A/\eta^2$ for $\mathcal{O}(\alpha_s^2)$
- Obtained an all order ζ evolution equation for renormalized TMD qgq correlators
- Renormalization procedure for SIDIS leaves behind remainder R_{SIDIS}
- Yields renormalized factorization theorems for $\mathcal{O}(\lambda)$ SIDIS and Drell-Yan

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Thanks for your attention!