Heavy Meson Light-cone Distribution Amplitude: SCET to bHQET Matching

Gael Finauri

SCET 2023 - Lawrence Berkeley National Laboratory

28 March 2023

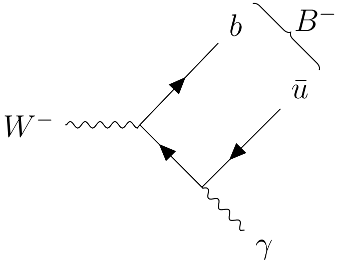
based on M. Beneke, GF, K. K. Vos, Y. Wei 2304.xxxxx

Heavy Mesons LCDAs: Why?

They arise in factorization theorems involving boosted heavy mesons

- $\bar{B}$ meson

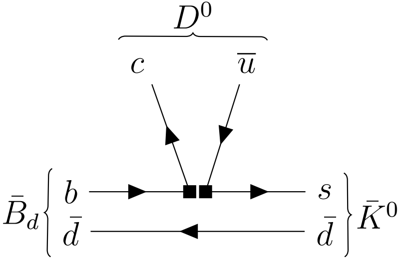
Hard processes at colliders:



boosted since $\Lambda_{\text{QCD}} \ll m_b \ll m_W$

- D meson

Exclusive two-body $\bar{B}$ decays:



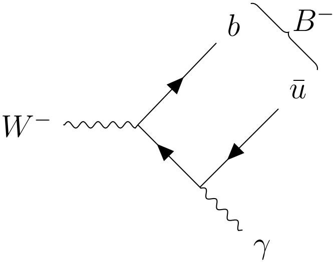
considering $\Lambda_{\text{QCD}} \ll m_c \ll m_b$

Heavy Mesons LCDAs: Why?

They arise in factorization theorems involving boosted heavy mesons

- $\bar{B}$ meson

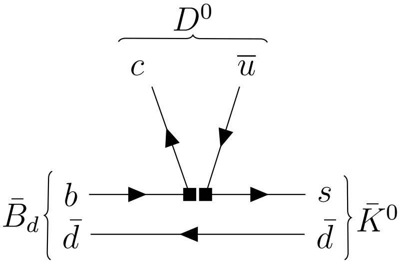
Hard processes at colliders:



boosted since $\Lambda_{\text{QCD}} \ll m_b \ll m_W$

- D meson

Exclusive two-body $\bar{B}$ decays:



considering $\Lambda_{\text{QCD}} \ll m_c \ll m_b$

Three distinct physical scales to separate with EFT machinery!

Light-cone Distribution Amplitude: Definition in QCD

We take H as a pseudoscalar **heavy meson** and tn_+^μ a light-like distance

$$\langle H(p_H)|\bar{Q}(0)\not{n}_+\gamma^5[0,tn_+]q(tn_+)|0\rangle = -if_H n_+p_H \int_0^1 du\, e^{iutn_+p_H}\phi(u;\mu)$$

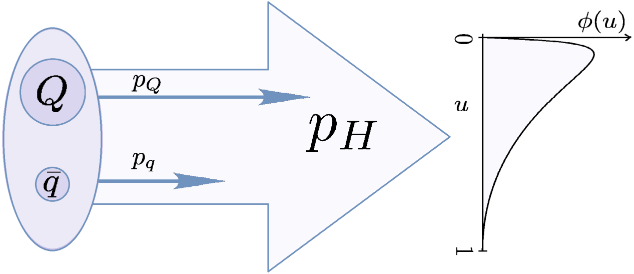
$\phi(u)$ encodes the **perturbative scale** m_H and the **non-perturbative** Λ_{QCD}

Light-cone Distribution Amplitude: Definition in QCD

We take H as a pseudoscalar **heavy meson** and tn_+^μ a light-like distance

$$\langle H(p_H)|\bar{Q}(0)\not{n}_+\gamma^5[0,tn_+]q(tn_+)|0\rangle = -i f_H n_+ p_H \int_0^1 du\, e^{iutn_+p_H}\phi(u;\mu)$$

$\phi(u)$ encodes the **perturbative scale** m_H and the **non-perturbative** Λ_{QCD}

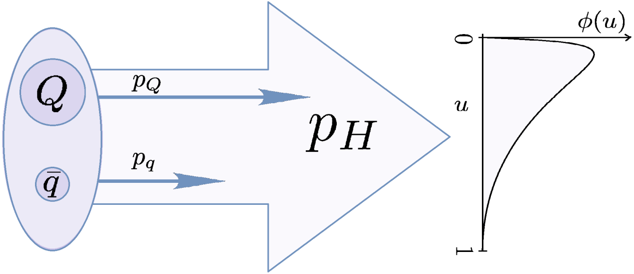


Light-cone Distribution Amplitude: Definition in QCD

We take H as a pseudoscalar **heavy meson** and tn_+^μ a light-like distance

$$\langle H(p_H)|\bar{Q}(0)\not{n}_+\gamma^5[0,tn_+]q(tn_+)|0\rangle = -if_H n_+p_H \int_0^1 du\, e^{iutn_+p_H}\phi(u;\mu)$$

$\phi(u)$ encodes the **perturbative scale** m_H and the **non-perturbative** Λ_{QCD}



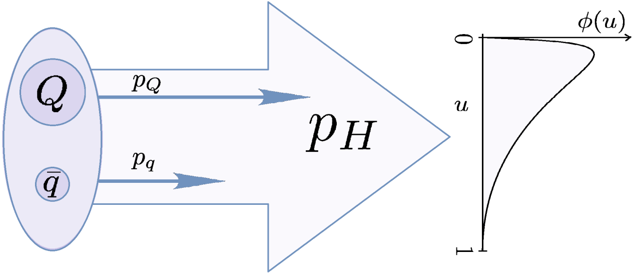
- $u = n_+p_q/n_+p_H \in [0,1]$

Light-cone Distribution Amplitude: Definition in QCD

We take H as a pseudoscalar **heavy meson** and tn_+^μ a light-like distance

$$\langle H(p_H)|\bar{Q}(0)\not{n}_+\gamma^5[0,tn_+]q(tn_+)|0\rangle = -if_H n_+p_H \int_0^1 du\, e^{iutn_+p_H}\phi(u;\mu)$$

$\phi(u)$ encodes the **perturbative scale** m_H and the **non-perturbative** Λ_{QCD}



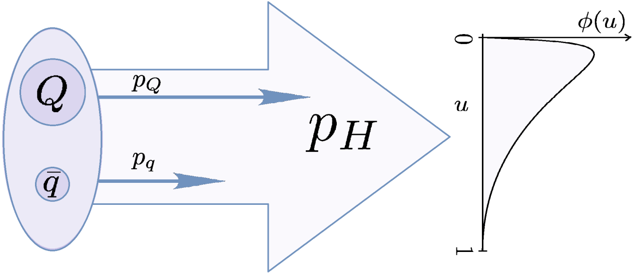
- $u = n_+p_q/n_+p_H \in [0,1]$
- normalized to 1

Light-cone Distribution Amplitude: Definition in QCD

We take H as a pseudoscalar **heavy meson** and tn_+^μ a light-like distance

$$\langle H(p_H)|\bar{Q}(0)\not{n}_+\gamma^5[0,tn_+]q(tn_+)|0\rangle = -if_H n_+p_H \int_0^1 du\, e^{iutn_+p_H}\phi(u;\mu)$$

$\phi(u)$ encodes the **perturbative scale** m_H and the **non-perturbative** Λ_{QCD}



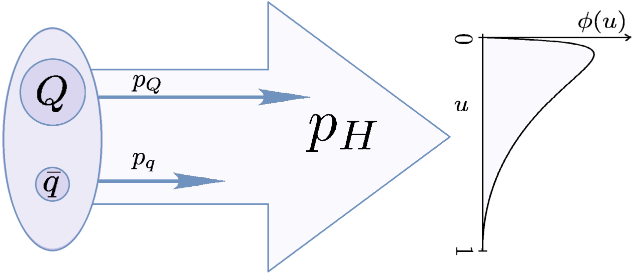
- $u = n_+p_q/n_+p_H \in [0,1]$
- normalized to 1
- highly asymmetric at ren. scales $\mu \lesssim m_H$

Light-cone Distribution Amplitude: Definition in QCD

We take H as a pseudoscalar **heavy meson** and tn_+^μ a light-like distance

$$\langle H(p_H)|\bar{Q}(0)\not{n}_+\gamma^5[0,tn_+]q(tn_+)|0\rangle = -if_H n_+p_H \int_0^1 du\, e^{iutn_+p_H}\phi(u;\mu)$$

$\phi(u)$ encodes the **perturbative scale** m_H and the **non-perturbative** Λ_{QCD}



- $u = n_+p_q/n_+p_H \in [0,1]$
- normalized to 1
- highly asymmetric at ren. scales $\mu \lesssim m_H$
- symmetric at scales $\mu \gg m_H$ (from RGE) [Efremov, Radyushkin, Brodsky, Lepage 1979,1980]

Light-cone Distribution Amplitude: Definition in HQET

In the limit $m_Q \to \infty$ we define the **universal** HQET LCDA

$$\langle H_v | \bar{h}_v(0) \not{n}_+ \gamma^5 [0, tn_+] q_s(tn_+) | 0 \rangle = -i F_{\text{stat}}(\mu) n_+ v \int_0^\infty d\omega \, e^{i\omega t n_+ v} \varphi_+(\omega; \mu)$$

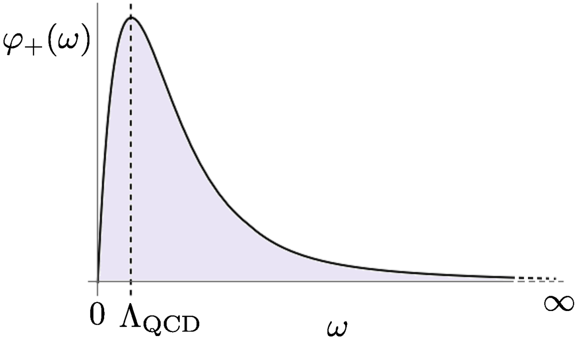
$\varphi_+(\omega) \sim 1/\Lambda_{\text{QCD}}$ encodes only the hadronic physics of order Λ_{QCD}

Light-cone Distribution Amplitude: Definition in HQET

In the limit $m_Q \to \infty$ we define the **universal** HQET LCDA

$$\langle H_v | \bar{h}_v(0) \not{n}_+ \gamma^5 [0, tn_+] q_s(tn_+) | 0 \rangle = -i F_{\text{stat}}(\mu) n_+ v \int_0^\infty d\omega \, e^{i\omega t n_+ v} \varphi_+(\omega;\mu)$$

$\varphi_+(\omega) \sim 1/\Lambda_{\text{QCD}}$ encodes only the hadronic physics of order Λ_{QCD}

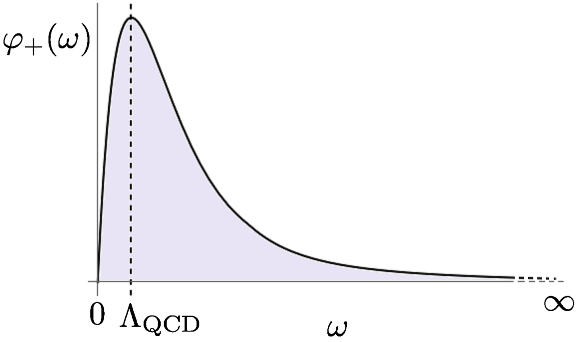


Light-cone Distribution Amplitude: Definition in HQET

In the limit $m_Q \to \infty$ we define the **universal** HQET LCDA

$$\langle H_v | \bar{h}_v(0) \not{n}_+ \gamma^5 [0, tn_+] q_s(tn_+) | 0 \rangle = -iF_{\text{stat}}(\mu) n_+ v \int_0^\infty d\omega \, e^{i\omega t n_+ v} \varphi_+(\omega; \mu)$$

$\varphi_+(\omega) \sim 1/\Lambda_{\text{QCD}}$ encodes only the hadronic physics of order Λ_{QCD}



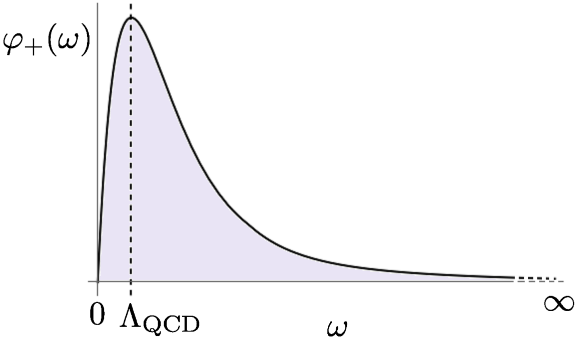
- $\omega = n_+ p_q / n_+ v \in [0, \infty]$

Light-cone Distribution Amplitude: Definition in HQET

In the limit $m_Q \to \infty$ we define the **universal** HQET LCDA

$$\langle H_v | \bar{h}_v(0) \not{n}_+ \gamma^5 [0, tn_+] q_s(tn_+) | 0 \rangle = -i F_{\text{stat}}(\mu) n_+ v \int_0^\infty d\omega \, e^{i\omega t n_+ v} \varphi_+(\omega; \mu)$$

$\varphi_+(\omega) \sim 1/\Lambda_{\text{QCD}}$ encodes only the hadronic physics of order Λ_{QCD}



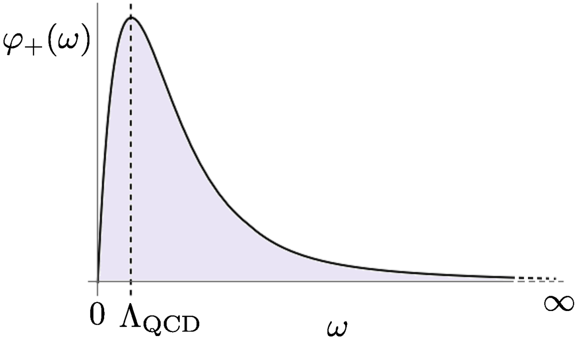
- $\omega = n_+ p_q / n_+ v \in [0, \infty]$
- peaked at $\omega \sim \Lambda_{\text{QCD}}$

Light-cone Distribution Amplitude: Definition in HQET

In the limit $m_Q \to \infty$ we define the **universal** HQET LCDA

$$\langle H_v | \bar{h}_v(0) \not{n}_+ \gamma^5 [0, tn_+] q_s(tn_+) | 0 \rangle = -i F_{\text{stat}}(\mu) n_+ v \int_0^\infty d\omega \, e^{i\omega t n_+ v} \varphi_+(\omega; \mu)$$

$\varphi_+(\omega) \sim 1/\Lambda_{\text{QCD}}$ encodes only the hadronic physics of order Λ_{QCD}



- $\omega = n_+ p_q / n_+ v \in [0, \infty]$
- peaked at $\omega \sim \Lambda_{\text{QCD}}$
- $\varphi_+(\omega) \sim \omega$ for $\omega \to 0$

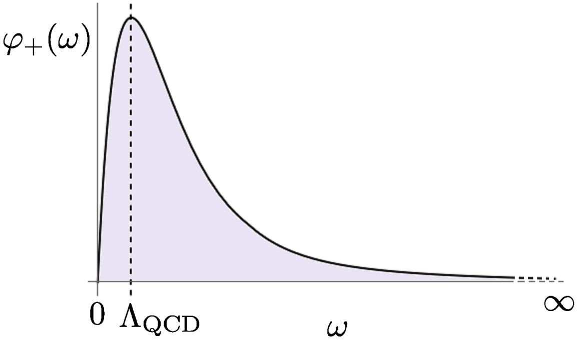
[Grozin, Neubert '96]

Light-cone Distribution Amplitude: Definition in HQET

In the limit $m_Q \to \infty$ we define the **universal** HQET LCDA

$$\langle H_v | \bar{h}_v(0) \not{n}_+ \gamma^5 [0, tn_+] q_s(tn_+) | 0 \rangle = -i F_{\text{stat}}(\mu) n_+ v \int_0^\infty d\omega\, e^{i\omega t n_+ v} \varphi_+(\omega; \mu)$$

$\varphi_+(\omega) \sim 1/\Lambda_{\text{QCD}}$ encodes only the hadronic physics of order Λ_{QCD}



- $\omega = n_+ p_q / n_+ v \in [0, \infty]$
- peaked at $\omega \sim \Lambda_{\text{QCD}}$
- $\varphi_+(\omega) \sim \omega$ for $\omega \to 0$

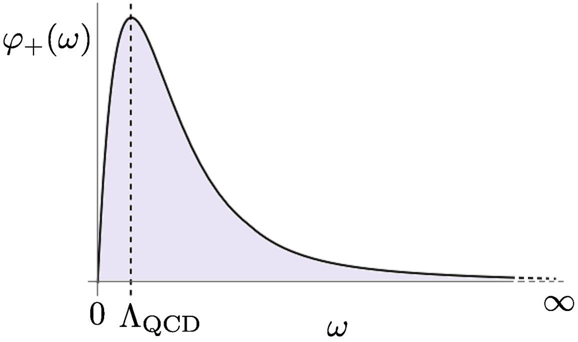
  [Grozin, Neubert '96]
- divergent normalization

Light-cone Distribution Amplitude: Definition in HQET

In the limit $m_Q \to \infty$ we define the **universal** HQET LCDA

$$\langle H_v | \bar{h}_v(0) \not{n}_+ \gamma^5 [0, tn_+] q_s(tn_+) | 0 \rangle = -iF_{\text{stat}}(\mu) n_+v \int_0^\infty d\omega\, e^{i\omega t n_+ v} \varphi_+(\omega; \mu)$$

$\varphi_+(\omega) \sim 1/\Lambda_{\text{QCD}}$ encodes only the hadronic physics of order Λ_{QCD}



- $\omega = n_+p_q / n_+v \in [0, \infty]$
- peaked at $\omega \sim \Lambda_{\text{QCD}}$
- $\varphi_+(\omega) \sim \omega$ for $\omega \to 0$

  [Grozin, Neubert '96]
- divergent normalization
- perturbative for $\omega \gg \Lambda_{\text{QCD}}$

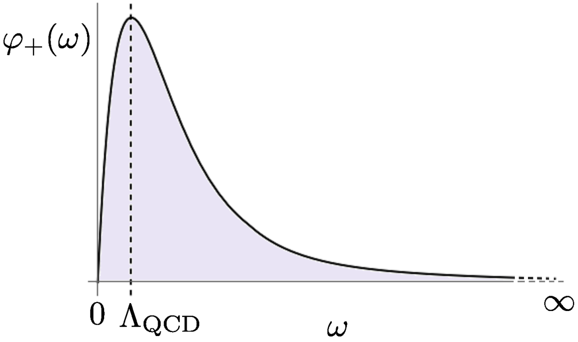
  [Lee, Neubert '05]

Light-cone Distribution Amplitude: Definition in HQET

In the limit $m_Q \to \infty$ we define the **universal** HQET LCDA

$$\langle H_v | \bar{h}_v(0) \not{n}_+ \gamma^5 [0, tn_+] q_s(tn_+) | 0 \rangle = -i F_{\text{stat}}(\mu) n_+ v \int_0^\infty d\omega \, e^{i\omega t n_+ v} \varphi_+(\omega; \mu)$$

$\varphi_+(\omega) \sim 1/\Lambda_{\text{QCD}}$ encodes only the hadronic physics of order Λ_{QCD}



- $\omega = n_+ p_q / n_+ v \in [0, \infty]$
- peaked at $\omega \sim \Lambda_{\text{QCD}}$
- $\varphi_+(\omega) \sim \omega$ for $\omega \to 0$

  [Grozin, Neubert '96]
- divergent normalization
- perturbative for $\omega \gg \Lambda_{\text{QCD}}$

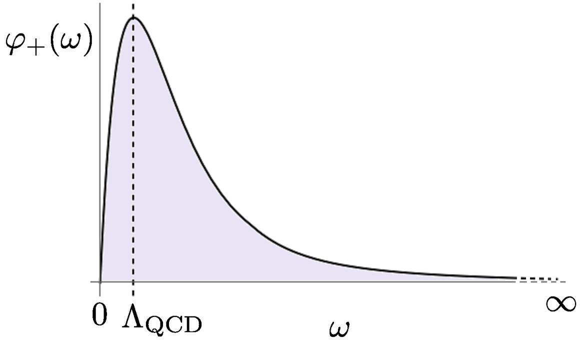
  [Lee, Neubert '05]
- valid at scales $\mu \lesssim m_H$

Light-cone Distribution Amplitude: Definition in HQET

In the limit $m_Q \to \infty$ we define the **universal** HQET LCDA

$$\langle H_v | \bar{h}_v(0) \not{n}_+ \gamma^5 [0, tn_+] q_s(tn_+) | 0 \rangle = -i F_{\text{stat}}(\mu) n_+ v \int_0^\infty d\omega \, e^{i\omega t n_+ v} \varphi_+(\omega; \mu)$$

$\varphi_+(\omega) \sim 1/\Lambda_{\text{QCD}}$ encodes only the hadronic physics of order Λ_{QCD}



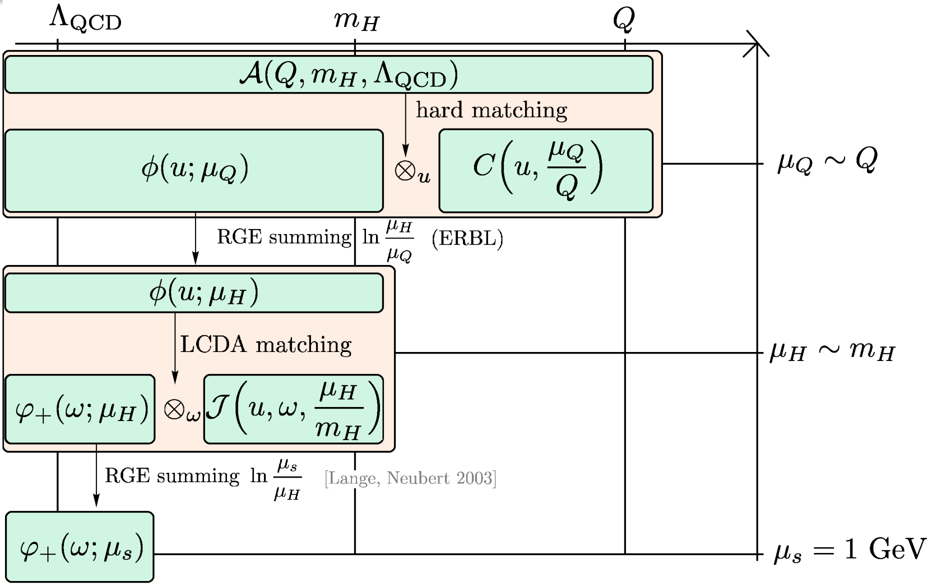
- $\omega = n_+ p_q / n_+ v \in [0, \infty]$
- peaked at $\omega \sim \Lambda_{\text{QCD}}$
- $\varphi_+(\omega) \sim \omega$ for $\omega \to 0$

  [Grozin, Neubert '96]
- divergent normalization
- perturbative for $\omega \gg \Lambda_{\text{QCD}}$

  [Lee, Neubert '05]
- valid at scales $\mu \lesssim m_H$

We are looking for a factorization formula that connects both LCDAs!

Collinear Factorization Picture



Peak & Tail Separation

Working setup:

- leading power (LP) in $\lambda \equiv \Lambda_{\text{QCD}}/m_Q \ll 1$ and $b \equiv m_H/Q \ll 1$

Peak & Tail Separation

Working setup:

- leading power (LP) in $\lambda \equiv \Lambda_{\text{QCD}}/m_Q \ll 1$ and $b \equiv m_H/Q \ll 1$
- factorization scale $\mu \sim \mathcal{O}(m_Q) \Rightarrow$ highly asymmetric LCDA!

Peak & Tail Separation

Working setup:

- leading power (LP) in $\lambda \equiv \Lambda_{\mathrm{QCD}}/m_Q \ll 1$ and $b \equiv m_H/Q \ll 1$
- factorization scale $\mu \sim \mathcal{O}(m_Q) \Rightarrow$ highly asymmetric LCDA!

Goal: Integrating out m_Q at one loop

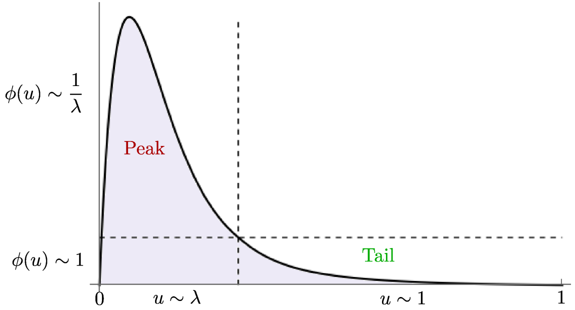
Peak & Tail Separation

Working setup:

- leading power (LP) in $\lambda \equiv \Lambda_{\text{QCD}}/m_Q \ll 1$ and $b \equiv m_H/Q \ll 1$
- factorization scale $\mu \sim \mathcal{O}(m_Q) \Rightarrow$ highly asymmetric LCDA!

Goal: Integrating out m_Q at one loop

before matching:



- We have to match separately peak and tail to have a consistent power counting

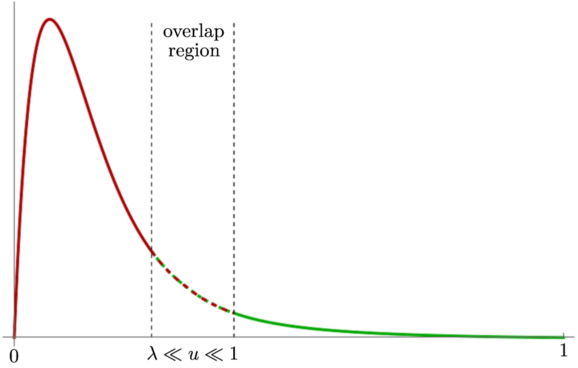
Peak & Tail Separation

Working setup:

- leading power (LP) in $\lambda \equiv \Lambda_{\mathrm{QCD}}/m_Q \ll 1$ and $b \equiv m_H/Q \ll 1$
- factorization scale $\mu \sim \mathcal{O}(m_Q) \Rightarrow$ highly asymmetric LCDA!

Goal: Integrating out m_Q at one loop

after matching:



- We have to match separately peak and tail to have a consistent power counting
- The two resulting functions have to overlap in the region $\lambda \ll u \ll 1$

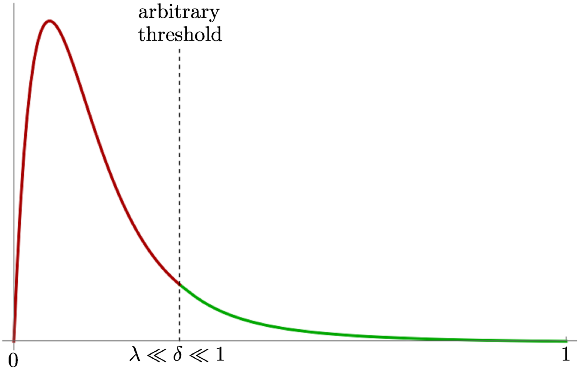
Peak & Tail Separation

Working setup:

- leading power (LP) in $\lambda \equiv \Lambda_{\text{QCD}}/m_Q \ll 1$ and $b \equiv m_H/Q \ll 1$
- factorization scale $\mu \sim \mathcal{O}(m_Q) \Rightarrow$ highly asymmetric LCDA!

Goal: Integrating out m_Q at one loop

merging the two results:



- We have to match separately peak and tail to have a consistent power counting
- The two resulting functions have to overlap in the region $\lambda \ll u \ll 1$
- We will merge them by choosing a threshold parameter δ

SCET for Heavy Quarks (bHQET)

HQET field h_v with highly boosted velocity $v = (n_+ v, v_\perp, n_- v) \sim (\frac{1}{b}, 1, b)$ interacts with soft-collinear modes $k \sim (\frac{1}{b}, 1, b)\Lambda_{\text{QCD}}$ [Fleming, Hoang, Mantry, Stewart '07]

SCET for Heavy Quarks (bHQET)

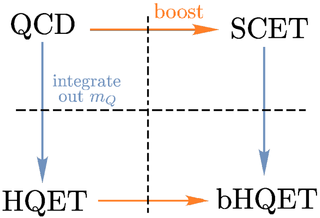
HQET field h_v with highly boosted velocity $v = (n_+v, v_\perp, n_-v) \sim (\frac{1}{b}, 1, b)$
interacts with soft-collinear modes $k \sim (\frac{1}{b}, 1, b)\Lambda_{\text{QCD}}$ [Fleming, Hoang, Mantry, Stewart '07]

Goal: write down an EFT homogeneous in b

SCET for Heavy Quarks (bHQET)

HQET field h_v with highly boosted velocity $v = (n_+v, v_\perp, n_-v) \sim (\frac{1}{b}, 1, b)$ interacts with soft-collinear modes $k \sim (\frac{1}{b}, 1, b)\Lambda_{\mathrm{QCD}}$ [Fleming, Hoang, Mantry, Stewart '07]

Goal: write down an EFT homogeneous in b
$\Rightarrow$ same as SCET from QCD! (here we have 2 ways)



SCET for Heavy Quarks (bHQET)

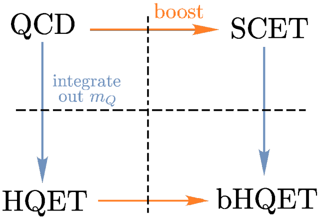
HQET field h_v with highly boosted velocity $v = (n_+v, v_\perp, n_-v) \sim (\frac{1}{b}, 1, b)$ interacts with soft-collinear modes $k \sim (\frac{1}{b}, 1, b)\Lambda_{\text{QCD}}$ [Fleming, Hoang, Mantry, Stewart '07]

Goal: write down an EFT homogeneous in b

$\Rightarrow$ same as SCET from QCD! (here we have 2 ways)

one instructive realization is given by the definition:

$$h_n \equiv \sqrt{\frac{2}{n_+v}}\, \frac{\not{n}_-\not{n}_+}{4} h_v$$

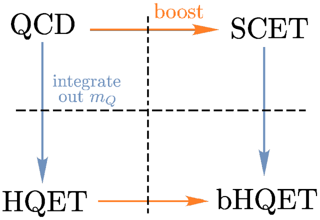


SCET for Heavy Quarks (bHQET)

HQET field h_v with highly boosted velocity $v = (n_+v, v_\perp, n_-v) \sim (\frac{1}{b}, 1, b)$
interacts with soft-collinear modes $k \sim (\frac{1}{b}, 1, b)\Lambda_{\text{QCD}}$ [Fleming, Hoang, Mantry, Stewart '07]

Goal: write down an EFT homogeneous in b
$\Rightarrow$ same as SCET from QCD! (here we have 2 ways)
one instructive realization is given by the definition:

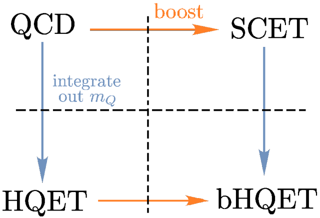
$$h_n \equiv \sqrt{\frac{2}{n_+v}}\,\frac{\not{n}_-\not{n}_+}{4} h_v \quad \Rightarrow \quad \frac{\not{n}_+\not{n}_-}{4} h_v = \sqrt{\frac{n_+v}{2}}\,\frac{1+\not{v}_\perp}{n_+v}\,\frac{\not{n}_+}{2} h_n$$



SCET for Heavy Quarks (bHQET)

HQET field h_v with highly boosted velocity $v = (n_+ v, v_\perp, n_- v) \sim (\frac{1}{b}, 1, b)$
interacts with soft-collinear modes $k \sim (\frac{1}{b}, 1, b)\Lambda_{\text{QCD}}$ [Fleming, Hoang, Mantry, Stewart '07]

Goal: write down an EFT homogeneous in b
$\Rightarrow$ same as SCET from QCD! (here we have 2 ways)
one instructive realization is given by the definition:



$$h_n \equiv \sqrt{\frac{2}{n_+ v}} \frac{\not{n}_- \not{n}_+}{4} h_v \quad \Rightarrow \quad \frac{\not{n}_+ \not{n}_-}{4} h_v = \sqrt{\frac{n_+ v}{2}} \frac{1 + \not{v}_\perp}{n_+ v} \frac{\not{n}_+}{2} h_n$$

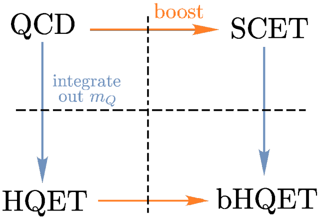
the (**exact!**) relation between HQET and bHQET field is

$$h_v = \left(\frac{\not{n}_- \not{n}_+}{4} + \frac{\not{n}_+ \not{n}_-}{4} \right) h_v$$

SCET for Heavy Quarks (bHQET)

HQET field h_v with highly boosted velocity $v = (n_+v, v_\perp, n_-v) \sim (\frac{1}{b}, 1, b)$
interacts with soft-collinear modes $k \sim (\frac{1}{b}, 1, b)\Lambda_{\text{QCD}}$ [Fleming, Hoang, Mantry, Stewart '07]

Goal: write down an EFT homogeneous in b
$\Rightarrow$ same as SCET from QCD! (here we have 2 ways)
one instructive realization is given by the definition:



$$h_n \equiv \sqrt{\frac{2}{n_+v}}\frac{\not{n}_-\not{n}_+}{4}h_v \quad \Rightarrow \quad \frac{\not{n}_+\not{n}_-}{4}h_v = \sqrt{\frac{n_+v}{2}}\frac{1+\not{v}_\perp}{n_+v}\frac{\not{n}_+}{2}h_n$$

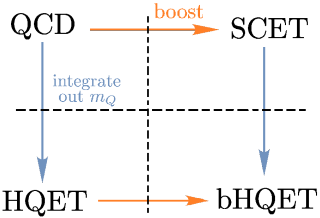
the (**exact!**) relation between HQET and bHQET field is

$$h_v = \left(\frac{\not{n}_-\not{n}_+}{4}+\frac{\not{n}_+\not{n}_-}{4}\right)h_v = \sqrt{\frac{n_+v}{2}}\left(1+\frac{1+\not{v}_\perp}{n_+v}\frac{\not{n}_+}{2}\right)h_n$$

SCET for Heavy Quarks (bHQET)

HQET field h_v with highly boosted velocity $v = (n_+v, v_\perp, n_-v) \sim (\frac{1}{b}, 1, b)$
interacts with soft-collinear modes $k \sim (\frac{1}{b}, 1, b)\Lambda_{\text{QCD}}$ [Fleming, Hoang, Mantry, Stewart '07]

Goal: write down an EFT homogeneous in b
$\Rightarrow$ same as SCET from QCD! (here we have 2 ways)
one instructive realization is given by the definition:



$$h_n \equiv \sqrt{\frac{2}{n_+v}}\frac{\not{n}_-\not{n}_+}{4}h_v \quad \Rightarrow \quad \frac{\not{n}_+\not{n}_-}{4}h_v = \sqrt{\frac{n_+v}{2}}\frac{1+\not{v}_\perp}{n_+v}\frac{\not{n}_+}{2}h_n$$

the (**exact!**) relation between HQET and bHQET field is

$$h_v = \left(\frac{\not{n}_-\not{n}_+}{4}+\frac{\not{n}_+\not{n}_-}{4}\right)h_v = \sqrt{\frac{n_+v}{2}}\left(1+\frac{1+\not{v}_\perp}{n_+v}\frac{\not{n}_+}{2}\right)h_n$$

Lagrangian derivation [Dai, Kim, Leibovich 2021]

$$\mathcal{L}_{\text{HQET}} = \bar{h}_v iv\cdot D h_v = \ \dots \ = \bar{h}_n iv\cdot D\frac{\not{n}_+}{2}h_n \equiv \mathcal{L}_{\text{bHQET}}$$

Operator Definition

Operator in SCET (momentum space)

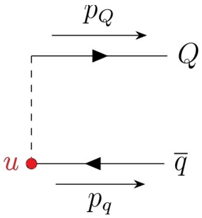
$$\mathcal{O}_C(u) = \int \frac{dt}{2\pi} e^{-iutn_+p_H} \bar{\chi}_C^{(Q)}(0) \, \not{n}_+ \gamma^5 \, \chi_C(tn_+) \quad \text{with} \quad \chi_C(x) = W_C^\dagger(x)\xi_C(x)$$

Operator Definition

Operator in SCET (momentum space)

$$\mathcal{O}_C(u) = \int \frac{dt}{2\pi} e^{-iutn_+p_H} \bar{\chi}_C^{(Q)}(0) \, \not{n}_+ \gamma^5 \, \chi_C(tn_+) \qquad \text{with} \qquad \chi_C(x) = W_C^\dagger(x)\xi_C(x)$$

Feynman rules: Tree level $\longrightarrow$



$$= \not{n}_+ \gamma^5 \delta(n_+p_q - un_+p_H)$$

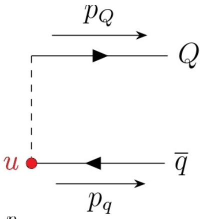
Operator Definition

Operator in SCET (momentum space)

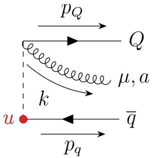
$$\mathcal{O}_C(u) = \int \frac{dt}{2\pi} e^{-iutn_+p_H}\, \bar{\chi}_C^{(Q)}(0)\, \not{n}_+ \gamma^5\, \chi_C(tn_+) \quad \text{with} \quad \chi_C(x) = W_C^\dagger(x)\xi_C(x)$$

Feynman rules: Tree level $\longrightarrow$

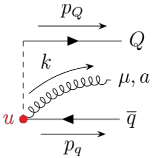
$\downarrow$ One gluon from the Wilson lines



$$= \not{n}_+ \gamma^5 \delta(n_+p_q - un_+p_H)$$



$$= g_s T^a \not{n}_+ \gamma^5 \frac{n_+^\mu}{n_+k + i\eta} \delta(n_+p_q - un_+p_H)$$



$$= -g_s T^a \not{n}_+ \gamma^5 \frac{n_+^\mu}{n_+k + i\eta} \delta(n_+p_q + n_+k - un_+p_H)$$

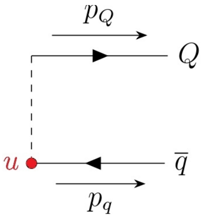
Operator Definition

Operator in SCET (momentum space)

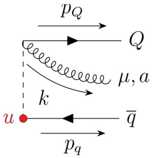
$$\mathcal{O}_C(u) = \int \frac{dt}{2\pi} e^{-iutn_+p_H}\, \bar{\chi}_C^{(Q)}(0)\, \not{n}_+ \gamma^5\, \chi_C(tn_+) \quad \text{with} \quad \chi_C(x) = W_C^\dagger(x)\xi_C(x)$$

Feynman rules: Tree level $\longrightarrow$

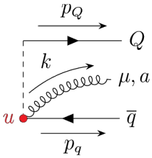
$\downarrow$ One gluon from the Wilson lines



$$= \not{n}_+ \gamma^5 \boxed{\delta(n_+p_q - un_+p_H)}$$



$$= g_s T^a \not{n}_+ \gamma^5 \frac{n_+^\mu}{n_+k + i\eta} \boxed{\delta(n_+p_q - un_+p_H)}$$



$$= -g_s T^a \not{n}_+ \gamma^5 \frac{n_+^\mu}{n_+k + i\eta} \boxed{\delta(n_+p_q + n_+k - un_+p_H)}$$

Crucial: the delta functions force the momentum fraction coming out from the dot • to assume the value u

Peak & Tail Matching

Final state scalings: $p_Q \sim \overset{\text{hard}-\text{collinear}}{(1,b,b^2)Q}$ (hc), $p_q \sim \overset{\text{soft}-\text{collinear}}{\lambda(1,b,b^2)Q}$ (sc)

Jet functions are given by hc loops ($k \sim (1,b,b^2)Q$) $\Rightarrow$ to all orders in α_s:

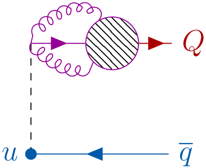
Peak & Tail Matching

Final state scalings: $p_Q \sim \overset{\text{hard—collinear}}{(1, b, b^2)Q}$ (hc), $p_q \sim \overset{\text{soft—collinear}}{\lambda(1, b, b^2)Q}$ (sc)

Jet functions are given by hc loops ($k \sim (1, b, b^2)Q$) $\Rightarrow$ to all orders in α_s:

- **Peak** $u \sim \lambda$

if a hc gluon is emitted from $\bar{q} \Rightarrow u \sim 1$, outside of the peak region! ($L \equiv \ln \frac{\mu}{m_H}$)



$$\phi_p(u) = \frac{\tilde{f}_H}{f_H} \int_0^\infty d\omega \mathcal{J}_p(u, \omega) \varphi_+(\omega)$$

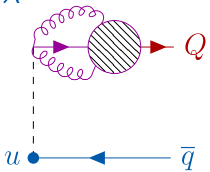
Peak & Tail Matching

Final state scalings: $p_Q \sim \overset{\text{hard—collinear}}{(1, b, b^2)Q}$ (hc), $\quad p_q \sim \overset{\text{soft—collinear}}{\lambda(1, b, b^2)Q}$ (sc)

Jet functions are given by hc loops ($k \sim (1, b, b^2)Q$) $\Rightarrow$ to all orders in α_s:

- **Peak** $u \sim \lambda$

if a hc gluon is emitted from $\bar{q} \Rightarrow u \sim 1$, outside of the peak region! ($L \equiv \ln \frac{\mu}{m_H}$)



$$\phi_p(u) = \frac{\tilde{f}_H}{f_H} \int_0^\infty d\omega \mathcal{J}_p(u, \omega) \varphi_+(\omega)$$

$$\mathcal{J}_p(u, \omega) = \theta(m_H - \omega) \delta\left(u - \frac{\omega}{m_H}\right) \mathcal{J}_{\text{peak}}$$

$$\mathcal{J}_{\text{peak}} = 1 + \frac{\alpha_s C_F}{4\pi} \left(\frac{L^2}{2} + \frac{L}{2} + \frac{\pi^2}{12} + 2\right)$$

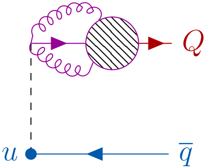
Peak & Tail Matching

Final state scalings: $p_Q \sim \overset{\text{hard—collinear}}{(1, b, b^2)Q}$ (hc), $p_q \sim \overset{\text{soft—collinear}}{\lambda(1, b, b^2)Q}$ (sc)

Jet functions are given by hc loops ($k \sim (1, b, b^2)Q$) $\Rightarrow$ to all orders in α_s:

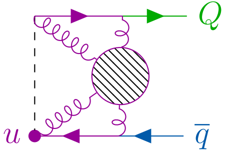
- **Peak** $u \sim \lambda$

if a hc gluon is emitted from $\bar{q} \Rightarrow u \sim 1$, outside of the peak region! ($L \equiv \ln \frac{\mu}{m_H}$)



- **Tail** $u \sim 1$

hc loop insensitive to sc $\bar{q}$ momentum $\Rightarrow \omega$ independent jet function: matching to local operators!



$$\phi_p(u) = \frac{\tilde{f}_H}{f_H} \int_0^\infty d\omega \mathcal{J}_p(u, \omega) \varphi_+(\omega)$$

$$\mathcal{J}_p(u, \omega) = \theta(m_H - \omega)\delta\left(u - \frac{\omega}{m_H}\right)\mathcal{J}_{\text{peak}}$$

$$\mathcal{J}_{\text{peak}} = 1 + \frac{\alpha_s C_F}{4\pi}\left(\frac{L^2}{2} + \frac{L}{2} + \frac{\pi^2}{12} + 2\right)$$

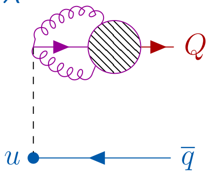
Peak & Tail Matching

Final state scalings: $p_Q \sim \overset{\text{hard—collinear}}{(1,b,b^2)Q}$ (hc), $p_q \sim \overset{\text{soft—collinear}}{\lambda(1,b,b^2)Q}$ (sc)

Jet functions are given by hc loops ($k \sim (1,b,b^2)Q$) $\Rightarrow$ to all orders in α_s:

- **Peak** $u \sim \lambda$

if a hc gluon is emitted from $\bar{q} \Rightarrow u \sim 1$, outside of the peak region! ($L \equiv \ln \frac{\mu}{m_H}$)



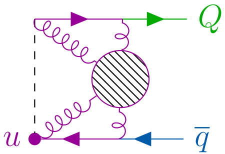
$$\phi_p(u) = \frac{\tilde{f}_H}{f_H} \int_0^\infty d\omega \mathcal{J}_p(u,\omega)\varphi_+(\omega)$$

$$\mathcal{J}_p(u,\omega) = \theta(m_H - \omega)\delta\left(u - \frac{\omega}{m_H}\right)\mathcal{J}_{\text{peak}}$$

$$\mathcal{J}_{\text{peak}} = 1 + \frac{\alpha_s C_F}{4\pi}\left(\frac{L^2}{2} + \frac{L}{2} + \frac{\pi^2}{12} + 2\right)$$

- **Tail** $u \sim 1$

hc loop insensitive to sc $\bar{q}$ momentum $\Rightarrow \omega$ independent jet function: matching to local operators!



$$\mathcal{J}_{\text{tail}}(u) = \frac{\alpha_s C_F}{4\pi}\frac{2\bar{u}}{u}\left[2(1+u)\ln\frac{\mu}{u m_H} - u + 1\right]$$

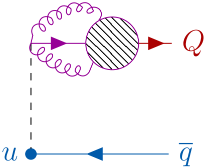
Peak & Tail Matching

Final state scalings: $p_Q \sim \overset{\text{hard—collinear}}{(1,b,b^2)Q}$ (hc), $p_q \sim \overset{\text{soft—collinear}}{\lambda(1,b,b^2)Q}$ (sc)

Jet functions are given by hc loops ($k \sim (1,b,b^2)Q$) $\Rightarrow$ to all orders in α_s:

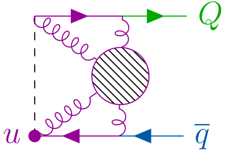
- **Peak** $u \sim \lambda$

if a hc gluon is emitted from $\bar{q} \Rightarrow u \sim 1$, outside of the peak region! ($L \equiv \ln \frac{\mu}{m_H}$)



- **Tail** $u \sim 1$

hc loop insensitive to sc $\bar{q}$ momentum $\Rightarrow \omega$ independent jet function: matching to local operators!



$$\phi_p(u) = \frac{\tilde{f}_H}{f_H}\int_0^\infty d\omega \mathcal{J}_p(u,\omega)\varphi_+(\omega)$$

$$\mathcal{J}_p(u,\omega) = \theta(m_H - \omega)\delta\left(u - \frac{\omega}{m_H}\right)\mathcal{J}_{\text{peak}}$$

$$\mathcal{J}_{\text{peak}} = 1 + \frac{\alpha_s C_F}{4\pi}\left(\frac{L^2}{2} + \frac{L}{2} + \frac{\pi^2}{12} + 2\right)$$

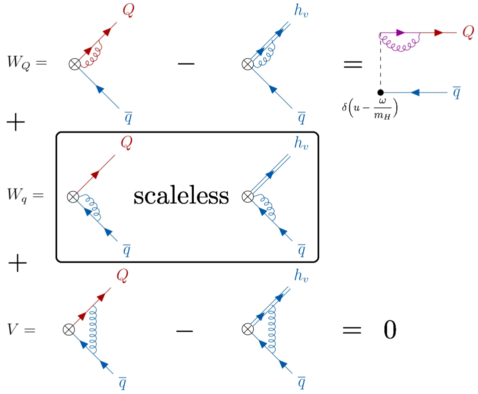
$$\mathcal{J}_{\text{tail}}(u) = \frac{\alpha_s C_F}{4\pi}\frac{2\bar{u}}{u}\left[2(1+u)\ln\frac{\mu}{u m_H} - u + 1\right]$$

$$\phi_t(u) = \frac{\tilde{f}_H}{f_H}\mathcal{J}_{\text{tail}}(u)$$

More Details...

- **Peak** $u \sim \lambda$

$$M^{(1)}\left(u, \frac{\omega}{m_H}\right) - m_H N^{(1)}(u m_H, \omega) = \mathcal{J}_p^{(1)}(u, \omega)$$



actually matching the massive hc field

$$\chi_C^{(Q)} \to \mathcal{J}_{\text{peak}} \sqrt{\frac{n_+ v}{2}} W_{sc}^\dagger h_n = \mathcal{J}_{\text{peak}} \sqrt{\frac{n_+ v}{2}} W_{sc}^\dagger Y_v h_n^{(0)}$$

- **Tail** $u \sim 1$

We match into **local** bHQET operators (OPE)

$$\mathcal{O}_C(u) = \mathcal{J}_+(u)\mathcal{O}_+ + \mathcal{J}_-(u)\mathcal{O}_-$$

two operators

(= decay constant matching)

$$\mathcal{O}_\pm \xrightarrow{\text{boost to RF}} \frac{1}{m_H n_\pm v} \bar{h}_v \not{n}_\pm \gamma^5 q_s$$

Merging of the Regions and LCDA Properties

$$\phi(u) = \frac{\tilde{f}_H}{f_H} \begin{cases} \mathcal{J}_{\text{peak}} m_H \varphi_+(u m_H)\,, & \text{for } u \sim \lambda \\ \mathcal{J}_{\text{tail}}(u)\,, & \text{for } u \sim 1 \end{cases}$$

we have to check that

$$\phi_p(u)\big|_{u \gg \lambda} \stackrel{!}{=} \phi_t(u)\big|_{u \ll 1}$$

Merging of the Regions and LCDA Properties

$$\phi(u) = \frac{\tilde{f}_H}{f_H} \begin{cases} \mathcal{J}_{\text{peak}} m_H \varphi_+(u m_H)\,, & \text{for } u \sim \lambda \\ \mathcal{J}_{\text{tail}}(u)\,, & \text{for } u \sim 1 \end{cases}$$

we have to check that

$$\phi_p(u)\big|_{u \gg \lambda} \stackrel{!}{=} \phi_t(u)\big|_{u \ll 1}$$

$$\mathcal{J}_{\text{peak}} m_H \varphi_+(u m_H)\big|_{u \gg \lambda} = m_H \varphi_+^{\text{asy}}(u m_H)$$

$$= \frac{\alpha_s C_F}{2\pi u}\left(2 \ln \frac{\mu}{u m_H} + 1\right)$$

[Lee, Neubert 2005]

Merging of the Regions and LCDA Properties

$$\phi(u) = \frac{\tilde{f}_H}{f_H} \begin{cases} \mathcal{J}_{\text{peak}} m_H \varphi_+(u m_H)\,, & \text{for } u \sim \lambda \\ \mathcal{J}_{\text{tail}}(u)\,, & \text{for } u \sim 1 \end{cases}$$

we have to check that

$$\phi_p(u)\big|_{u \gg \lambda} \stackrel{!}{=} \phi_t(u)\big|_{u \ll 1}$$

$$\mathcal{J}_{\text{peak}} m_H \varphi_+(u m_H)\big|_{u \gg \lambda} = m_H \varphi_+^{\text{asy}}(u m_H)$$

$$= \frac{\alpha_s C_F}{2\pi u}\left(2 \ln \frac{\mu}{u m_H} + 1\right)$$

[Lee, Neubert 2005]

$$\mathcal{J}_{\text{tail}}(u)\big|_{u \ll 1} = \frac{\alpha_s C_F}{2\pi u}\left(2 \ln \frac{\mu}{u m_H} + 1\right)$$

Merging of the Regions and LCDA Properties

$$\phi(u) = \frac{\tilde{f}_H}{f_H} \begin{cases} \mathcal{J}_{\text{peak}} m_H \varphi_+(u m_H)\,, & \text{for } u \sim \lambda \\ \mathcal{J}_{\text{tail}}(u)\,, & \text{for } u \sim 1 \end{cases}$$

we have to check that

$$\phi_p(u)\big|_{u \gg \lambda} \overset{!}{=} \phi_t(u)\big|_{u \ll 1}$$

$$\mathcal{J}_{\text{peak}} m_H \varphi_+(u m_H)\big|_{u \gg \lambda} = m_H \varphi_+^{\text{asy}}(u m_H)$$

$$= \frac{\alpha_s C_F}{2\pi u}\left(2 \ln \frac{\mu}{u m_H} + 1\right)$$

[Lee, Neubert 2005]

$$\mathcal{J}_{\text{tail}}(u)\big|_{u \ll 1} = \frac{\alpha_s C_F}{2\pi u}\left(2 \ln \frac{\mu}{u m_H} + 1\right)$$

$\Rightarrow$ merge the functions with a "**merging function**" $\vartheta(u; \delta, \sigma)$

$$\phi(u) = \vartheta(u; \delta, \sigma)\phi_p(u) + (1 - \vartheta(u; \delta, \sigma))\phi_t(u)$$

Merging of the Regions and LCDA Properties

$$\phi(u) = \frac{\tilde{f}_H}{f_H} \begin{cases} \mathcal{J}_{\text{peak}} m_H \varphi_+(u m_H)\,, & \text{for } u \sim \lambda \\ \mathcal{J}_{\text{tail}}(u)\,, & \text{for } u \sim 1 \end{cases}$$

we have to check that

$$\phi_p(u)|_{u \gg \lambda} \stackrel{!}{=} \phi_t(u)|_{u \ll 1}$$

$$\mathcal{J}_{\text{peak}} m_H \varphi_+(u m_H)|_{u \gg \lambda} = m_H \varphi_+^{\text{asy}}(u m_H)$$

$$= \frac{\alpha_s C_F}{2\pi u}\left(2 \ln \frac{\mu}{u m_H} + 1\right)$$

[Lee, Neubert 2005]

$$\mathcal{J}_{\text{tail}}(u)|_{u \ll 1} = \frac{\alpha_s C_F}{2\pi u}\left(2 \ln \frac{\mu}{u m_H} + 1\right)$$

$\Rightarrow$ merge the functions with a "**merging function**" $\vartheta(u; \delta, \sigma)$

$$\phi(u) = \vartheta(u; \delta, \sigma)\phi_p(u) + (1 - \vartheta(u; \delta, \sigma))\phi_t(u)$$

δ sets the threshold in the overlap region, $\sigma \sim 10^{-2}$ the smoothness

- $\vartheta(u; \delta, \sigma)|_{u \ll \delta} = 1,$
- $\vartheta(u; \delta, \sigma)|_{u \gg \delta} = 0,$
- $\vartheta(u; \delta, 0) = \theta(\delta - u)$

Merging of the Regions and LCDA Properties

$$\phi(u) = \frac{\tilde{f}_H}{f_H} \begin{cases} \mathcal{J}_{\text{peak}} m_H \varphi_+(u m_H)\,, & \text{for } u \sim \lambda \\ \mathcal{J}_{\text{tail}}(u)\,, & \text{for } u \sim 1 \end{cases}$$

we have to check that

$$\phi_p(u)|_{u \gg \lambda} \stackrel{!}{=} \phi_t(u)|_{u \ll 1}$$

Properties of $\phi(u)$

- endpoint behaviour ✓

$$\mathcal{J}_{\text{peak}} m_H \varphi_+(u m_H)|_{u \gg \lambda} = m_H \varphi_+^{\text{asy}}(u m_H)$$

$$= \frac{\alpha_s C_F}{2\pi u}\left(2 \ln \frac{\mu}{u m_H} + 1\right)$$

[Lee, Neubert 2005]

$$\mathcal{J}_{\text{tail}}(u)|_{u \ll 1} = \frac{\alpha_s C_F}{2\pi u}\left(2 \ln \frac{\mu}{u m_H} + 1\right)$$

$\Rightarrow$ merge the functions with a "**merging function**" $\vartheta(u; \delta, \sigma)$

$$\phi(u) = \vartheta(u; \delta, \sigma)\phi_p(u) + (1 - \vartheta(u; \delta, \sigma))\phi_t(u)$$

δ sets the threshold in the overlap region, $\sigma \sim 10^{-2}$ the smoothness

- $\vartheta(u; \delta, \sigma)|_{u \ll \delta} = 1$,
- $\vartheta(u; \delta, \sigma)|_{u \gg \delta} = 0$,
- $\vartheta(u; \delta, 0) = \theta(\delta - u)$

Merging of the Regions and LCDA Properties

$$\phi(u) = \frac{\tilde{f}_H}{f_H} \begin{cases} \mathcal{J}_{\text{peak}} m_H \varphi_+(u m_H)\,, & \text{for } u \sim \lambda \\ \mathcal{J}_{\text{tail}}(u)\,, & \text{for } u \sim 1 \end{cases}$$

we have to check that
$\phi_p(u)|_{u \gg \lambda} \overset{!}{=} \phi_t(u)|_{u \ll 1}$

Properties of $\phi(u)$

- endpoint behaviour ✓
- normalization = 1, using $M_0(\delta m_H)$ [Lee, Neubert 2005] ✓

$$\mathcal{J}_{\text{peak}} m_H \varphi_+(u m_H)|_{u \gg \lambda} = m_H \varphi_+^{\text{asy}}(u m_H)$$
$$= \frac{\alpha_s C_F}{2\pi u}\left(2 \ln \frac{\mu}{u m_H} + 1\right)$$ [Lee, Neubert 2005]

$$\mathcal{J}_{\text{tail}}(u)|_{u \ll 1} = \frac{\alpha_s C_F}{2\pi u}\left(2 \ln \frac{\mu}{u m_H} + 1\right)$$

$\Rightarrow$ merge the functions with a "**merging function**" $\vartheta(u;\delta,\sigma)$

$$\phi(u) = \vartheta(u;\delta,\sigma)\phi_p(u) + (1 - \vartheta(u;\delta,\sigma))\phi_t(u)$$

δ sets the threshold in the overlap region, $\sigma \sim 10^{-2}$ the smoothness

- $\vartheta(u;\delta,\sigma)|_{u \ll \delta} = 1$,
- $\vartheta(u;\delta,\sigma)|_{u \gg \delta} = 0$,
- $\vartheta(u;\delta,0) = \theta(\delta - u)$

Merging of the Regions and LCDA Properties

$$\phi(u) = \frac{\tilde{f}_H}{f_H} \begin{cases} \mathcal{J}_{\text{peak}} m_H \varphi_+(u m_H)\,, & \text{for } u \sim \lambda \\ \mathcal{J}_{\text{tail}}(u)\,, & \text{for } u \sim 1 \end{cases}$$

we have to check that

$$\phi_p(u)|_{u \gg \lambda} \stackrel{!}{=} \phi_t(u)|_{u \ll 1}$$

Properties of $\phi(u)$

- endpoint behaviour ✓
- normalization = 1, using $M_0(\delta m_H)$ [Lee, Neubert 2005] ✓
- RG evolution (ERBL) ✓

$$\mathcal{J}_{\text{peak}} m_H \varphi_+(u m_H)|_{u \gg \lambda} = m_H \varphi_+^{\text{asy}}(u m_H)$$

$$= \frac{\alpha_s C_F}{2\pi u}\left(2 \ln \frac{\mu}{u m_H} + 1\right)$$ [Lee, Neubert 2005]

$$\mathcal{J}_{\text{tail}}(u)|_{u \ll 1} = \frac{\alpha_s C_F}{2\pi u}\left(2 \ln \frac{\mu}{u m_H} + 1\right)$$

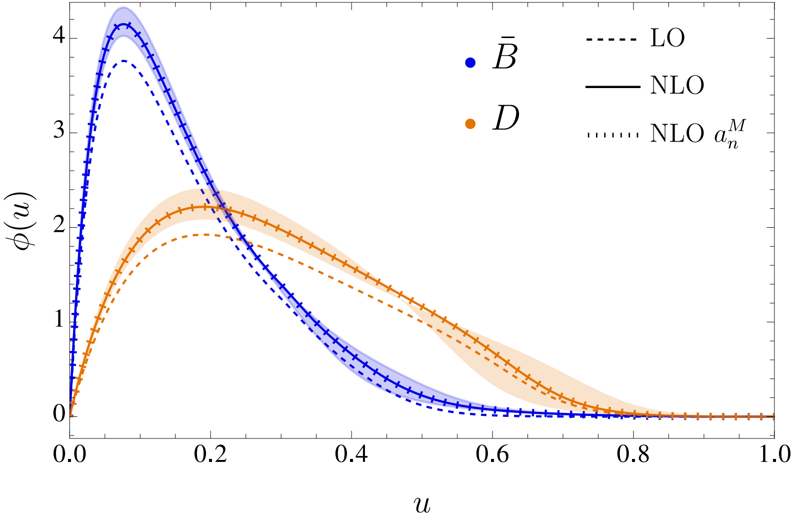
$\Rightarrow$ merge the functions with a "**merging function**" $\vartheta(u;\delta,\sigma)$

$$\phi(u) = \vartheta(u;\delta,\sigma)\phi_p(u) + (1 - \vartheta(u;\delta,\sigma))\phi_t(u)$$

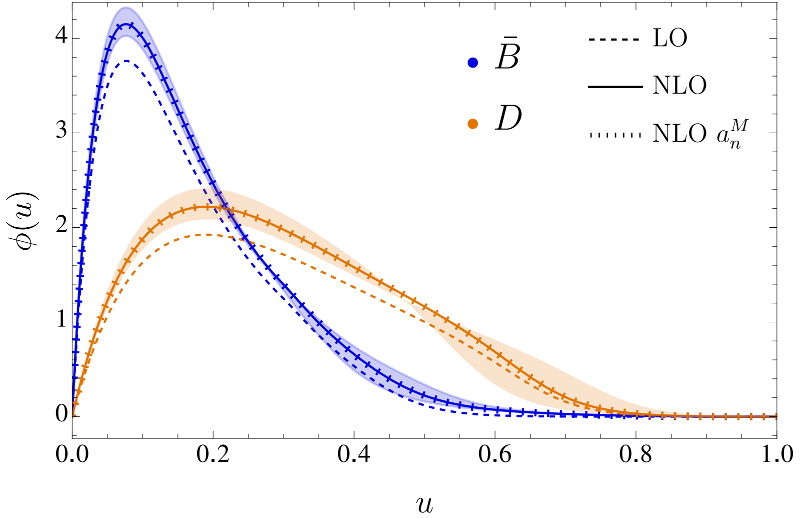
δ sets the threshold in the overlap region, $\sigma \sim 10^{-2}$ the smoothness

- $\vartheta(u;\delta,\sigma)|_{u \ll \delta} = 1,$
- $\vartheta(u;\delta,\sigma)|_{u \gg \delta} = 0,$
- $\vartheta(u;\delta,0) = \theta(\delta - u)$

$\bar{B}$ and D LCDAs at the Matching Scale

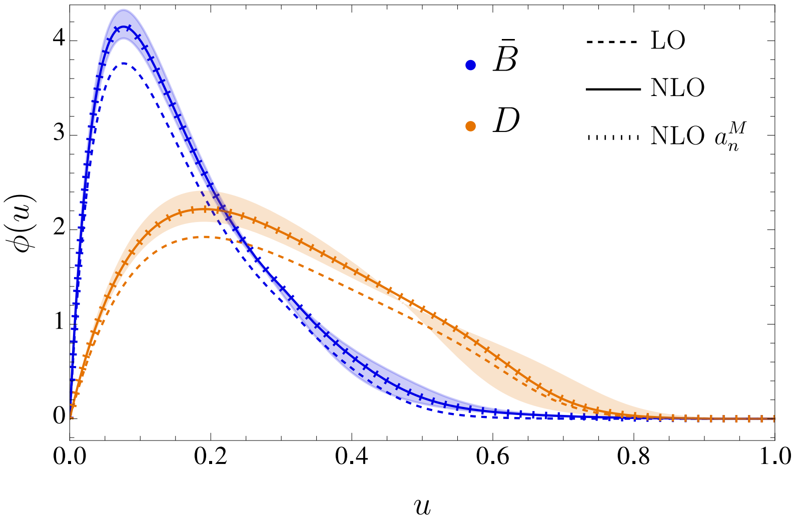


$\bar{B}$ and D LCDAs at the Matching Scale



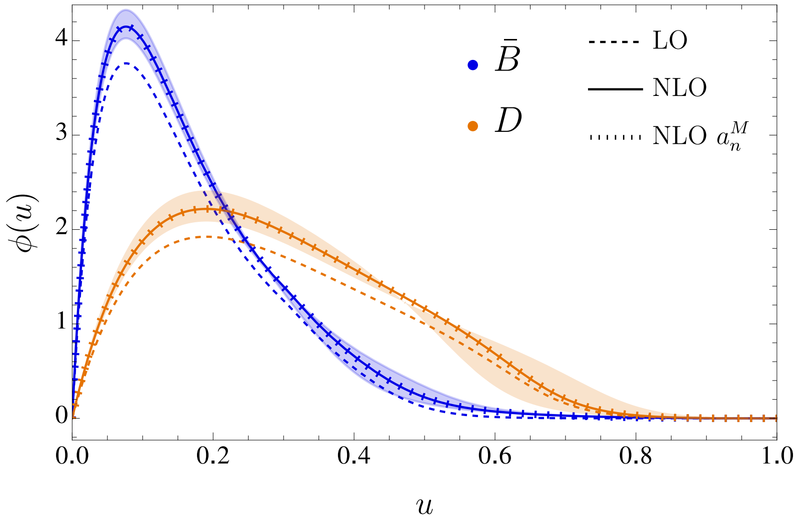
- NLO ~10% corrections

$\bar{B}$ and D LCDAs at the Matching Scale



- NLO $\sim$10% corrections
- Shaded bands from varying $\delta \pm 15\%$

$\bar{B}$ and D LCDAs at the Matching Scale



- NLO ~10% corrections
- Shaded bands from varying $\delta \pm 15\%$
- Perfect agreement with expansion up to 20 Gegenbauer moments $a_n^M(\mu)$

Branching Ratio $W \to B\gamma$: Numbers

- QCD Collinear factorization: LCDA was modelled at 1 GeV and evolved up to m_W with ERBL kernel [Grossman, König, Neubert 2015]

$$\mathrm{Br}_{[\mathrm{GKN15}]} = (1.99 \pm 0.17_{\mathrm{in}} \, {}^{+0.03}_{-0.06} \, {}_{\mu_h} \qquad {}^{+2.48}_{-0.80} \, {}_{\lambda_B}) \cdot 10^{-12}$$

Branching Ratio $W \to B\gamma$: Numbers

- QCD Collinear factorization: LCDA was modelled at 1 GeV and evolved up to m_W with ERBL kernel [Grossman, König, Neubert 2015]
- HQET factorization: considering $m_W \sim m_b \Rightarrow$ not resumming ln $\frac{m_b}{m_W}$ [Ishaq, Jia, Xiong, Yang 2019]

$$\text{Br}_{[\text{GKN15}]} = (1.99 \pm 0.17_{\text{in}} {}^{+0.03}_{-0.06}{}_{\mu_h} {}^{+2.48}_{-0.80}{}_{\lambda_B}) \cdot 10^{-12}$$

$$\text{Br}_{\text{HQET}} = (2.51 \pm 0.21_{\text{in}} {}^{+0.19}_{-0.69}{}_{\mu_b} {}^{+0.49}_{-0.40}{}_{\beta} {}^{+3.04}_{-0.95}{}_{\lambda_B}) \cdot 10^{-12}$$

Branching Ratio $W \to B\gamma$: Numbers

- QCD Collinear factorization: LCDA was modelled at 1 GeV and evolved up to m_W with ERBL kernel [Grossman, König, Neubert 2015]
- HQET factorization: considering $m_W \sim m_b \Rightarrow$ not resumming $\ln \frac{m_b}{m_W}$ [Ishaq, Jia, Xiong, Yang 2019]
- With our LCDA we merge the two frameworks and sum both $\ln \frac{m_b}{m_W}$ and $\ln \frac{\Lambda_{\text{QCD}}}{m_b}$

$$\text{Br}_{[\text{GKN15}]} = \left(1.99 \pm 0.17_{\text{in}} \,{}^{+0.03}_{-0.06}\,{}_{\mu_h} \,{}^{+2.48}_{-0.80}\,{}_{\lambda_B}\right) \cdot 10^{-12}$$

$$\text{Br}_{\text{HQET}} = \left(2.51 \pm 0.21_{\text{in}} \,{}^{+0.19}_{-0.69}\,{}_{\mu_b} \,{}^{+0.49}_{-0.40}\,{}_{\beta} \,{}^{+3.04}_{-0.95}\,{}_{\lambda_B}\right) \cdot 10^{-12}$$

$$\text{Br}_{[2023]} = \left(2.54 \pm 0.21_{\text{in}} \,{}^{+0.04}_{-0.07}\,{}_{\mu_h} \,{}^{+0.07}_{-0.09}\,{}_{\mu_b} \,{}^{+0.18}_{-0.13}\,{}_{\delta} \,{}^{+0.59}_{-0.33}\,{}_{\beta} \,{}^{+2.86}_{-0.95}\,{}_{\lambda_B}\right) \cdot 10^{-12}$$

Branching Ratio $W \to B\gamma$: Numbers

- QCD Collinear factorization: LCDA was modelled at 1 GeV and evolved up to m_W with ERBL kernel [Grossman, König, Neubert 2015]
- HQET factorization: considering $m_W \sim m_b \Rightarrow$ not resumming ln $\frac{m_b}{m_W}$ [Ishaq, Jia, Xiong, Yang 2019]
- With our LCDA we merge the two frameworks and sum both ln $\frac{m_b}{m_W}$ and ln $\frac{\Lambda_{\text{QCD}}}{m_b}$

$$\text{Br}_{[\text{GKN15}]} = \left(1.99 \pm 0.17_{\text{in}}\,{}^{+0.03}_{-0.06}\,{}_{\mu_h}\,{}^{+2.48}_{-0.80}\,{}_{\lambda_B}\right) \cdot 10^{-12}$$

$$\text{Br}_{\text{HQET}} = \left(2.51 \pm 0.21_{\text{in}}\,{}^{+0.19}_{-0.69}\,{}_{\mu_b}\,{}^{+0.49}_{-0.40}\,{}_{\beta}\,{}^{+3.04}_{-0.95}\,{}_{\lambda_B}\right) \cdot 10^{-12}$$

$$\text{Br}_{[2023]} = \left(2.54 \pm 0.21_{\text{in}}\,{}^{+0.04}_{-0.07}\,{}_{\mu_h}\,{}^{+0.07}_{-0.09}\,{}_{\mu_b}\,{}^{+0.18}_{-0.13}\,{}_{\delta}\,{}^{+0.59}_{-0.33}\,{}_{\beta}\,{}^{+2.86}_{-0.95}\,{}_{\lambda_B}\right) \cdot 10^{-12}$$

- central value enhanced by almost 30% w.r.t. [GKN15]

Branching Ratio $W \to B\gamma$: Numbers

- QCD Collinear factorization: LCDA was modelled at 1 GeV and evolved up to m_W with ERBL kernel [Grossman, König, Neubert 2015]
- HQET factorization: considering $m_W \sim m_b \Rightarrow$ not resumming $\ln \frac{m_b}{m_W}$ [Ishaq, Jia, Xiong, Yang 2019]
- With our LCDA we merge the two frameworks and sum both $\ln \frac{m_b}{m_W}$ and $\ln \frac{\Lambda_{\text{QCD}}}{m_b}$

$$\text{Br}_{[\text{GKN15}]} = \left(1.99 \pm 0.17_{\text{in}}\,{}^{+0.03}_{-0.06}\,{}_{\mu_h}\,{}^{+2.48}_{-0.80}\,{}_{\lambda_B}\right) \cdot 10^{-12}$$

$$\text{Br}_{\text{HQET}} = \left(2.51 \pm 0.21_{\text{in}}\,{}^{+0.19}_{-0.69}\,{}_{\mu_b}\,{}^{+0.49}_{-0.40}\,{}_{\beta}\,{}^{+3.04}_{-0.95}\,{}_{\lambda_B}\right) \cdot 10^{-12}$$

$$\text{Br}_{[2023]} = \left(2.54 \pm 0.21_{\text{in}}\,{}^{+0.04}_{-0.07}\,{}_{\mu_h}\,{}^{+0.07}_{-0.09}\,{}_{\mu_b}\,{}^{+0.18}_{-0.13}\,{}_{\delta}\,{}^{+0.59}_{-0.33}\,{}_{\beta}\,{}^{+2.86}_{-0.95}\,{}_{\lambda_B}\right) \cdot 10^{-12}$$

- central value enhanced by almost 30% w.r.t. [GKN15]
- poor convergence of HQET result denoted by large scale uncertainty

Branching Ratio $W \to B\gamma$: Numbers

- QCD Collinear factorization: LCDA was modelled at 1 GeV and evolved up to m_W with ERBL kernel [Grossman, König, Neubert 2015]
- HQET factorization: considering $m_W \sim m_b \Rightarrow$ not resumming ln $\frac{m_b}{m_W}$ [Ishaq, Jia, Xiong, Yang 2019]
- With our LCDA we merge the two frameworks and sum both ln $\frac{m_b}{m_W}$ and ln $\frac{\Lambda_{\text{QCD}}}{m_b}$

$$\text{Br}_{[\text{GKN15}]} = (1.99 \pm 0.17_{\text{in}}\,{}^{+0.03}_{-0.06}\,{}_{\mu_h}\,{}^{+2.48}_{-0.80}\,{}_{\lambda_B}) \cdot 10^{-12}$$

$$\text{Br}_{\text{HQET}} = (2.51 \pm 0.21_{\text{in}}\,{}^{+0.19}_{-0.69}\,{}_{\mu_b}\,{}^{+0.49}_{-0.40}\,{}_{\beta}\,{}^{+3.04}_{-0.95}\,{}_{\lambda_B}) \cdot 10^{-12}$$

$$\text{Br}_{[2023]} = (2.54 \pm 0.21_{\text{in}}\,{}^{+0.04}_{-0.07}\,{}_{\mu_h}\,{}^{+0.07}_{-0.09}\,{}_{\mu_b}\,{}^{+0.18}_{-0.13}\,{}_{\delta}\,{}^{+0.59}_{-0.33}\,{}_{\beta}\,{}^{+2.86}_{-0.95}\,{}_{\lambda_B}) \cdot 10^{-12}$$

- central value enhanced by almost 30% w.r.t. [GKN15]
- poor convergence of HQET result denoted by large scale uncertainty
- huge uncertainty due to poor knowledge of HQET LCDA

Summary

- Derived factorization for heavy meson LCDA at LP in Λ_{QCD}/m_Q ✓

Summary

- Derived factorization for heavy meson LCDA at LP in $\Lambda_{\mathrm{QCD}}/m_Q$ ✓
- Built concrete and practical models for $\bar{B}$ and D mesons ✓

Summary

- Derived factorization for heavy meson LCDA at LP in Λ_{QCD}/m_Q ✓
- Built concrete and practical models for $\bar{B}$ and D mesons ✓
- Applied to $W \to B\gamma$: new optimized theoretical predictions ✓

Summary

- Derived factorization for heavy meson LCDA at LP in Λ_{QCD}/m_Q ✓
- Built concrete and practical models for $\bar{B}$ and D mesons ✓
- Applied to $W \to B\gamma$: new optimized theoretical predictions ✓
- Ready to apply to exclusive $\bar{B} \to D$ decays in the future!

Summary

- Derived factorization for heavy meson LCDA at LP in Λ_{QCD}/m_Q ✓
- Built concrete and practical models for $\bar{B}$ and D mesons ✓
- Applied to $W \to B\gamma$: new optimized theoretical predictions ✓
- Ready to apply to exclusive $\bar{B} \to D$ decays in the future!

Thank You!

Backup Slides

Peak Matching ($u \sim \lambda$): Matching Equation

Matching equation

$$\mathcal{O}_C(u) = \int_0^\infty d\omega\, \mathcal{J}_p(u, \omega)\mathcal{O}_h(\omega)$$

Peak Matching ($u \sim \lambda$): Matching Equation

Matching equation

$$\mathcal{O}_C(u) = \int_0^\infty d\omega \, \mathcal{J}_p(u,\omega)\mathcal{O}_h(\omega)$$

such that

$$\langle H(p_H)|\mathcal{O}_C(u)|0\rangle = -i f_H \phi_p(u)$$
$$\langle H(p_H)|\mathcal{O}_h(\omega)|0\rangle = -i \tilde{f}_H \varphi_+(\omega)$$

Peak Matching ($u \sim \lambda$): Matching Equation

Matching equation

$$\mathcal{O}_C(u) = \int_0^\infty d\omega\, \mathcal{J}_p(u,\omega)\mathcal{O}_h(\omega)$$

such that

$$\langle H(p_H)|\mathcal{O}_C(u)|0\rangle = -i f_H \phi_p(u)$$
$$\langle H(p_H)|\mathcal{O}_h(\omega)|0\rangle = -i \tilde{f}_H \varphi_+(\omega)$$

taking $\langle Q(p_Q)\bar{q}(p_q)| \bullet |0\rangle$ on both sides we can extract $\mathcal{J}_p$ at $\mathcal{O}(\alpha_s)$ from

$$\langle Q\bar{q}|\mathcal{O}_C(u)|0\rangle \propto \left[\delta\left(u - \frac{n_+p_q}{n_+p_H}\right) + \frac{\alpha_s C_F}{4\pi} M^{(1)}\left(u, \frac{n_+p_q}{n_+p_H}\right)\right]$$

Peak Matching ($u \sim \lambda$): Matching Equation

Matching equation

$$\mathcal{O}_C(u) = \int_0^\infty d\omega\, \mathcal{J}_p(u,\omega)\mathcal{O}_h(\omega)$$

such that

$$\langle H(p_H)|\mathcal{O}_C(u)|0\rangle = -i f_H \phi_p(u)$$

$$\langle H(p_H)|\mathcal{O}_h(\omega)|0\rangle = -i \tilde{f}_H \varphi_+(\omega)$$

taking $\langle Q(p_Q)\bar{q}(p_q)| \bullet |0\rangle$ on both sides we can extract $\mathcal{J}_p$ at $\mathcal{O}(\alpha_s)$ from

$$\langle Q\bar{q}|\mathcal{O}_C(u)|0\rangle \propto \left[\delta\left(u - \frac{n_+p_q}{n_+p_H}\right) + \frac{\alpha_s C_F}{4\pi} M^{(1)}\left(u, \frac{n_+p_q}{n_+p_H}\right)\right]$$

$$\langle Q\bar{q}|\mathcal{O}_h(\omega)|0\rangle \propto \left[\delta\left(\omega - \frac{n_+p_q}{n_+v}\right) + \frac{\alpha_s C_F}{4\pi} N^{(1)}\left(\omega, \frac{n_+p_q}{n_+v}\right)\right]$$

where p_q is the external soft-collinear momentum of the spectator quark

Matching equation

$$\mathcal{O}_C(u) = \int_0^\infty d\omega \, \mathcal{J}_p(u,\omega)\mathcal{O}_h(\omega)$$

such that

$$\langle H(p_H)|\mathcal{O}_C(u)|0\rangle = -if_H\phi_p(u)$$
$$\langle H(p_H)|\mathcal{O}_h(\omega)|0\rangle = -i\tilde{f}_H\varphi_+(\omega)$$

taking $\langle Q(p_Q)\bar{q}(p_q)| \bullet |0\rangle$ on both sides we can extract $\mathcal{J}_p$ at $\mathcal{O}(\alpha_s)$ from

$$\langle Q\bar{q}|\mathcal{O}_C(u)|0\rangle \propto \left[\delta\left(u - \frac{n_+p_q}{n_+p_H}\right) + \frac{\alpha_s C_F}{4\pi} M^{(1)}\left(u, \frac{n_+p_q}{n_+p_H}\right)\right]$$

$$\langle Q\bar{q}|\mathcal{O}_h(\omega)|0\rangle \propto \left[\delta\left(\omega - \frac{n_+p_q}{n_+v}\right) + \frac{\alpha_s C_F}{4\pi} N^{(1)}\left(\omega, \frac{n_+p_q}{n_+v}\right)\right]$$

where p_q is the external soft-collinear momentum of the spectator quark

$$\mathcal{J}_p(u,\omega) = \theta(m_H - \omega)\left[\delta(u - \tfrac{\omega}{m_H}) + \frac{\alpha_s C_F}{4\pi}\left(M^{(1)}(u, \tfrac{\omega}{m_H}) - m_H N^{(1)}(u m_H, \omega)\right)\right]$$

Peak Matching ($u \sim \lambda$): Matching Equation

Matching equation

$$\mathcal{O}_C(u) = \int_0^\infty d\omega\, \mathcal{J}_p(u,\omega)\mathcal{O}_h(\omega)$$

such that

$$\langle H(p_H)|\mathcal{O}_C(u)|0\rangle = -if_H\phi_p(u)$$

$$\langle H(p_H)|\mathcal{O}_h(\omega)|0\rangle = -i\tilde{f}_H\varphi_+(\omega)$$

taking $\langle Q(p_Q)\bar{q}(p_q)| \bullet |0\rangle$ on both sides we can extract $\mathcal{J}_p$ at $\mathcal{O}(\alpha_s)$ from

$$\langle Q\bar{q}|\mathcal{O}_C(u)|0\rangle \propto \left[\delta\left(u - \frac{n_+p_q}{n_+p_H}\right) + \frac{\alpha_s C_F}{4\pi} M^{(1)}\left(u, \frac{n_+p_q}{n_+p_H}\right)\right]$$

$$\langle Q\bar{q}|\mathcal{O}_h(\omega)|0\rangle \propto \left[\delta\left(\omega - \frac{n_+p_q}{n_+v}\right) + \frac{\alpha_s C_F}{4\pi} N^{(1)}\left(\omega, \frac{n_+p_q}{n_+v}\right)\right]$$

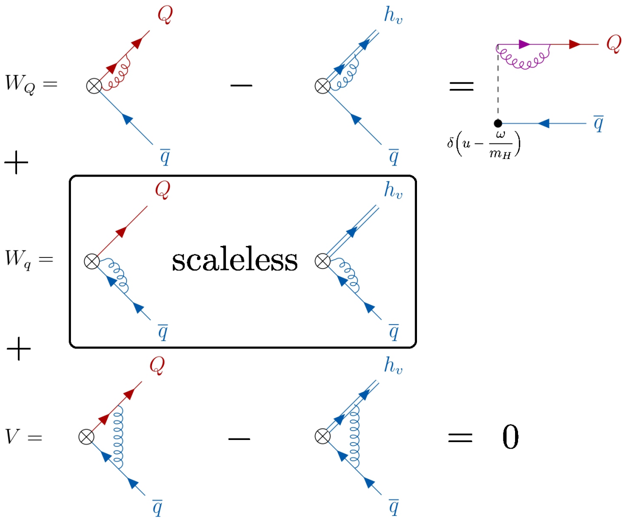
where p_q is the external soft-collinear momentum of the spectator quark

$$\mathcal{J}_p(u,\omega) = \theta(m_H - \omega)\left[\delta\left(u - \frac{\omega}{m_H}\right) + \frac{\alpha_s C_F}{4\pi}\left(M^{(1)}\left(u, \frac{\omega}{m_H}\right) - m_H N^{(1)}(u m_H, \omega)\right)\right]$$

the $\theta(m_H - \omega)$ comes from momentum conservation $p_H = p_Q + p_q$

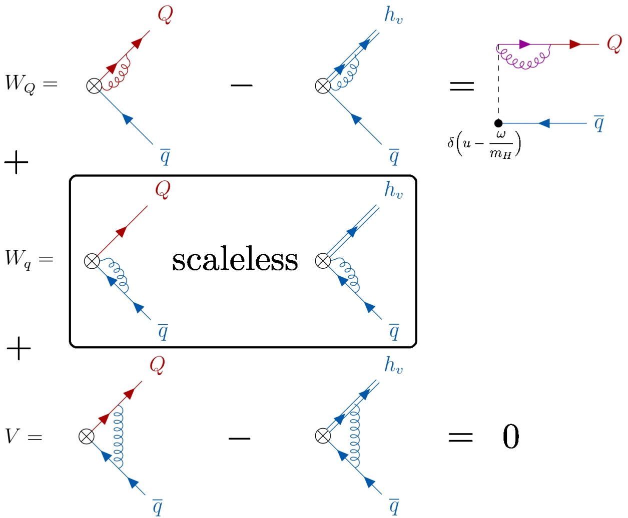
Peak Matching ($u \sim \lambda$): Result

$$M^{(1)}\left(u, \frac{\omega}{m_H}\right) - m_H N^{(1)}(u m_H, \omega) = \mathcal{J}_p^{(1)}(u, \omega)$$



Peak Matching ($u \sim \lambda$): Result

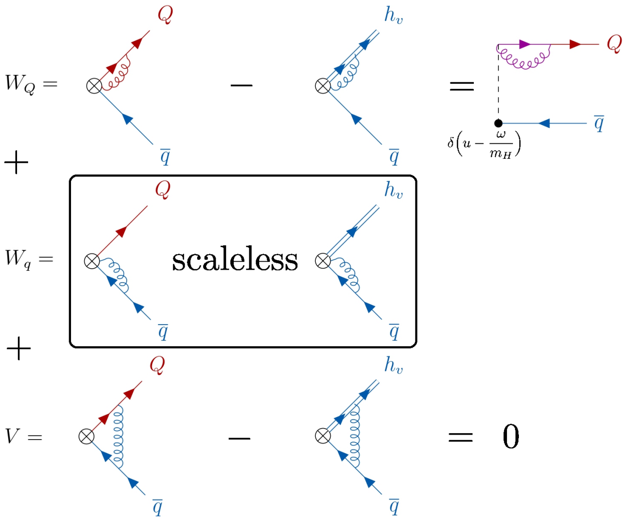
$$M^{(1)}\left(u, \frac{\omega}{m_H}\right) - m_H N^{(1)}(u m_H, \omega) = \mathcal{J}_p^{(1)}(u, \omega)$$



The one loop jet function turns out to be proportional to a delta function (as the tree level)

Peak Matching ($u \sim \lambda$): Result

$$M^{(1)}\left(u, \frac{\omega}{m_H}\right) - m_H N^{(1)}(u m_H, \omega) = \mathcal{J}_p^{(1)}(u, \omega)$$

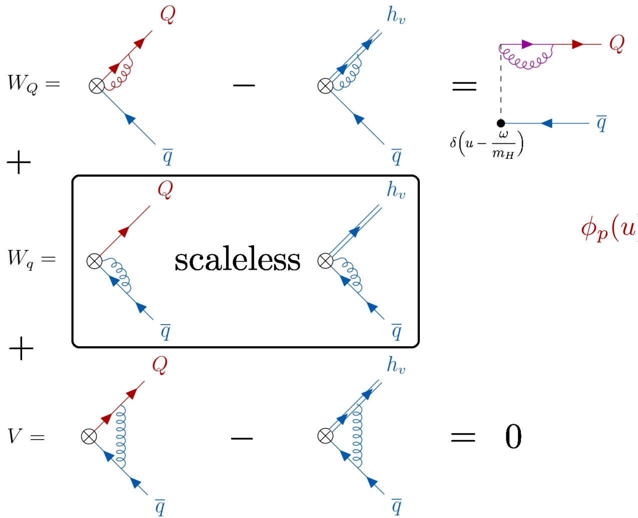


The one loop jet function turns out to be proportional to a delta function (as the tree level)

$\Rightarrow$ the LCDA in the peak region is very simple ($L \equiv \ln \frac{\mu^2}{m_H^2}$)

Peak Matching ($u \sim \lambda$): Result

$$M^{(1)}\left(u, \frac{\omega}{m_H}\right) - m_H N^{(1)}(u m_H, \omega) = \mathcal{J}_p^{(1)}(u, \omega)$$



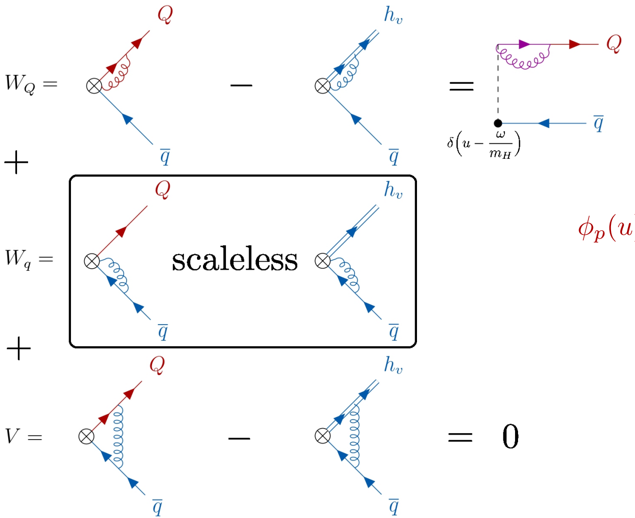
The one loop jet function turns out to be proportional to a delta function (as the tree level)

$\Rightarrow$ the LCDA in the peak region is very simple ($L \equiv \ln \frac{\mu^2}{m_H^2}$)

$$\phi_p(u) = \frac{\tilde{f}_H}{f_H} m_H \varphi_+(u m_H) \times \left[1 + \frac{\alpha_s C_F}{4\pi}\left(\frac{L^2}{2} + \frac{L}{2} + \frac{\pi^2}{12} + 2\right)\right]$$

Peak Matching ($u \sim \lambda$): Result

$$M^{(1)}\left(u, \frac{\omega}{m_H}\right) - m_H N^{(1)}(u m_H, \omega) = \mathcal{J}_p^{(1)}(u, \omega)$$



The one loop jet function turns out to be proportional to a delta function (as the tree level)

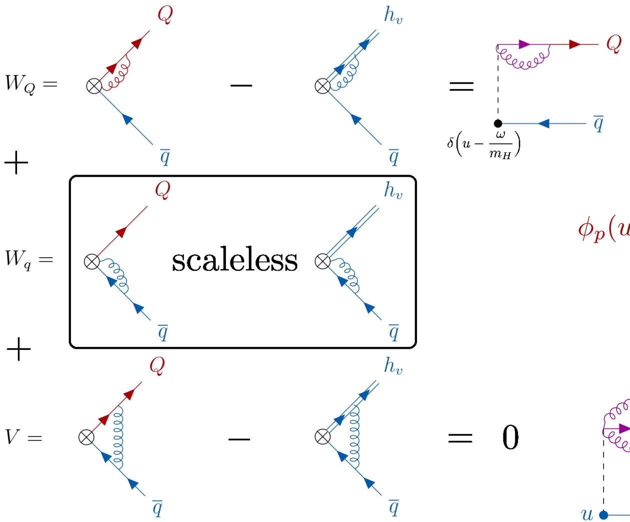
$\Rightarrow$ the LCDA in the peak region is very simple ($L \equiv \ln \frac{\mu^2}{m_H^2}$)

$$\phi_p(u) = \frac{\tilde{f}_H}{f_H} m_H \varphi_+(u m_H) \times \left[1 + \frac{\alpha_s C_F}{4\pi}\left(\frac{L^2}{2} + \frac{L}{2} + \frac{\pi^2}{12} + 2\right)\right]$$

This form holds to all orders in α_s:

Peak Matching ($u \sim \lambda$): Result

$$M^{(1)}\left(u, \frac{\omega}{m_H}\right) - m_H N^{(1)}(u m_H, \omega) = \mathcal{J}_p^{(1)}(u,\omega)$$

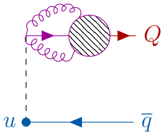


The one loop jet function turns out to be proportional to a delta function (as the tree level)

$\Rightarrow$ the LCDA in the peak region is very simple ($L \equiv \ln \frac{\mu^2}{m_H^2}$)

$$\phi_p(u) = \frac{\tilde{f}_H}{f_H} m_H \varphi_+(u m_H) \times \left[1 + \frac{\alpha_s C_F}{4\pi}\left(\frac{L^2}{2} + \frac{L}{2} + \frac{\pi^2}{12} + 2\right)\right]$$

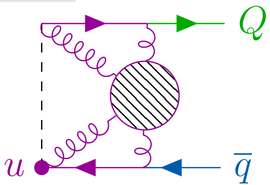
This form holds to all orders in α_s:



If a hard-collinear gluon is emitted by $\bar{q} \Rightarrow u \sim 1$, contribution to the tail!

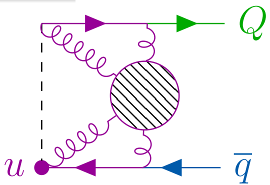
Tail Matching ($u \sim 1$): Matching Equation

The external momentum p_q is fixed to be soft-collinear



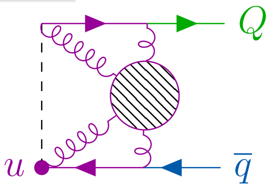
Tail Matching ($u \sim 1$): Matching Equation

The external momentum p_q is fixed to be soft-collinear but $u \sim 1 \Rightarrow \bar{q}$ internal line has to be hard-collinear (hc)



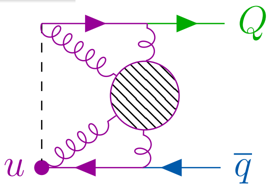
Tail Matching ($u \sim 1$): Matching Equation

The external momentum p_q is fixed to be soft-collinear
but $u \sim 1 \Rightarrow \bar{q}$ internal line has to be hard-collinear (hc)
$\Rightarrow$ the hc loop integral is insensitive to $p_q \ll p_H$



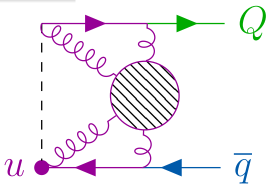
Tail Matching ($u \sim 1$): Matching Equation

The external momentum p_q is fixed to be soft-collinear but $u \sim 1 \Rightarrow \bar{q}$ internal line has to be hard-collinear (hc)
$\Rightarrow$ the hc loop integral is insensitive to $p_q \ll p_H$
$\Rightarrow$ the tail jet function cannot depend on ω!



Tail Matching ($u \sim 1$): Matching Equation

The external momentum p_q is fixed to be soft-collinear
but $u \sim 1 \Rightarrow \bar{q}$ internal line has to be hard-collinear (hc)
$\Rightarrow$ the hc loop integral is insensitive to $p_q \ll p_H$
$\Rightarrow$ the tail jet function cannot depend on ω!

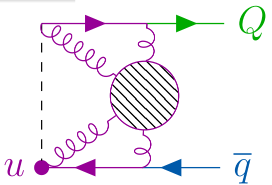


We match into **local** bHQET operators (OPE)

$$\mathcal{O}_C(u) = \mathcal{J}_+(u)\mathcal{O}_+ + \mathcal{J}_-(u)\mathcal{O}_-$$

Tail Matching ($u \sim 1$): Matching Equation

The external momentum p_q is fixed to be soft-collinear
but $u \sim 1 \Rightarrow \bar{q}$ internal line has to be hard-collinear (hc)
$\Rightarrow$ the hc loop integral is insensitive to $p_q \ll p_H$
$\Rightarrow$ the tail jet function cannot depend on ω!



We match into **local** bHQET operators (OPE)

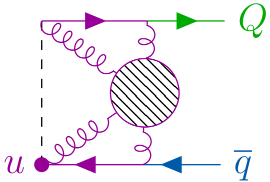
$$\mathcal{O}_C(u) = \mathcal{J}_+(u)\mathcal{O}_+ + \mathcal{J}_-(u)\mathcal{O}_-$$

$$\langle H(p_H)|\mathcal{O}_C(u)|0\rangle = -i f_H \phi_t(u)$$

$$\langle H(p_H)|\mathcal{O}_\pm|0\rangle = -i \tilde{f}_H$$

Tail Matching ($u \sim 1$): Matching Equation

The external momentum p_q is fixed to be soft-collinear
but $u \sim 1 \Rightarrow \bar{q}$ internal line has to be hard-collinear (hc)
$\Rightarrow$ the hc loop integral is insensitive to $p_q \ll p_H$
$\Rightarrow$ the tail jet function cannot depend on ω!



We match into **local** bHQET operators (OPE)

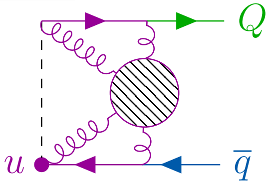
$$\mathcal{O}_C(u) = \mathcal{J}_+(u)\mathcal{O}_+ + \mathcal{J}_-(u)\mathcal{O}_-$$

two independent operators
as for the decay constant matching

$$\langle H(p_H)|\mathcal{O}_C(u)|0\rangle = -i f_H \phi_t(u)$$
$$\langle H(p_H)|\mathcal{O}_\pm|0\rangle = -i \tilde{f}_H$$

Tail Matching ($u \sim 1$): Matching Equation

The external momentum p_q is fixed to be soft-collinear
but $u \sim 1 \Rightarrow \bar{q}$ internal line has to be hard-collinear (hc)
$\Rightarrow$ the hc loop integral is insensitive to $p_q \ll p_H$
$\Rightarrow$ the tail jet function cannot depend on ω!



We match into **local** bHQET operators (OPE)

$$\langle H(p_H)|\mathcal{O}_C(u)|0\rangle = -if_H\phi_t(u)$$

$$\langle H(p_H)|\mathcal{O}_\pm|0\rangle = -i\tilde{f}_H$$

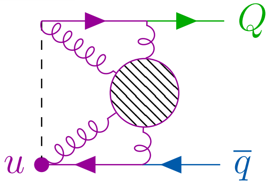
$$\mathcal{O}_C(u) = \mathcal{J}_+(u)\mathcal{O}_+ + \mathcal{J}_-(u)\mathcal{O}_-$$

two independent operators
as for the decay constant matching

$$\mathcal{O}_\pm \xrightarrow{\text{boost to RF}} \frac{1}{m_H n_\pm v}\bar{h}_v \not{n}_\pm \gamma^5 q_s$$

Tail Matching ($u \sim 1$): Matching Equation

The external momentum p_q is fixed to be soft-collinear
but $u \sim 1 \Rightarrow \bar{q}$ internal line has to be hard-collinear (hc)
$\Rightarrow$ the hc loop integral is insensitive to $p_q \ll p_H$
$\Rightarrow$ the tail jet function cannot depend on ω!



We match into **local** bHQET operators (OPE)

$$\langle H(p_H)|\mathcal{O}_C(u)|0\rangle = -i f_H \phi_t(u)$$

$$\langle H(p_H)|\mathcal{O}_\pm|0\rangle = -i\tilde{f}_H$$

$$\mathcal{O}_C(u) = \mathcal{J}_+(u)\mathcal{O}_+ + \mathcal{J}_-(u)\mathcal{O}_-$$

two independent operators
as for the decay constant matching

$$\mathcal{O}_\pm \xrightarrow{\text{boost to RF}} \frac{1}{m_H n_\pm v}\bar{h}_v \not{n}_\pm \gamma^5 q_s$$

Simple matching since SCET matrix el. starts at one-loop and is purely hc

$$\langle Q(p_Q)\bar{q}(p_q)|\mathcal{O}_C(u)|0\rangle \xrightarrow{u\sim 1} \mathcal{O}(\alpha_s) \propto \mathcal{J}_\pm(u)$$

Tail Matching ($u \sim 1$): Result

At one-loop we find

$$\mathcal{J}_{\text{tail}}(u) \equiv \mathcal{J}_+(u) + \mathcal{J}_-(u) = \frac{\alpha_s C_F}{4\pi}\frac{2\bar{u}}{u}\left[(1+u)(L - 2\ln u) - u + 1\right] + \mathcal{O}(\alpha_s^2)$$

Tail Matching ($u \sim 1$): Result

At one-loop we find

$$\mathcal{J}_{\text{tail}}(u) \equiv \mathcal{J}_+(u) + \mathcal{J}_-(u) = \frac{\alpha_s C_F}{4\pi} \frac{2\bar{u}}{u} \left[(1+u)(L - 2\ln u) - u + 1 \right] + \mathcal{O}(\alpha_s^2)$$

$\Rightarrow$ completely **perturbative** expression for the QCD LCDA in the tail

$$\phi_t(u) = \frac{\tilde{f}_H}{f_H} \mathcal{J}_{\text{tail}}(u)$$

Tail Matching ($u \sim 1$): Result

At one-loop we find

$$\mathcal{J}_{\text{tail}}(u) \equiv \mathcal{J}_{+}(u) + \mathcal{J}_{-}(u) = \frac{\alpha_s C_F}{4\pi}\frac{2\bar{u}}{u}\Big[(1+u)(L - 2\ln u) - u + 1\Big] + \mathcal{O}(\alpha_s^2)$$

$\Rightarrow$ completely **perturbative** expression for the QCD LCDA in the tail

$$\phi_t(u) = \frac{\tilde{f}_H}{f_H}\mathcal{J}_{\text{tail}}(u)$$

with the known HQET/QCD decay constant ratio [Eichten, Hill, '90]

$$\frac{\tilde{f}_H}{f_H} = 1 + \frac{\alpha_s C_F}{4\pi}\left(\frac{3}{2}\ln\frac{\mu^2}{m_Q^2} + 2\right) + \mathcal{O}(\alpha_s^2)$$

Tail Matching ($u \sim 1$): Result

At one-loop we find

$$\mathcal{J}_{\text{tail}}(u) \equiv \mathcal{J}_{+}(u) + \mathcal{J}_{-}(u) = \frac{\alpha_s C_F}{4\pi} \frac{2\bar{u}}{u} \Big[(1+u)(L - 2\ln u) - u + 1 \Big] + \mathcal{O}(\alpha_s^2)$$

⇒ completely **perturbative** expression for the QCD LCDA in the tail

$$\phi_t(u) = \frac{\tilde{f}_H}{f_H} \mathcal{J}_{\text{tail}}(u)$$

with the known HQET/QCD decay constant ratio [Eichten, Hill, '90]

$$\frac{\tilde{f}_H}{f_H} = 1 + \frac{\alpha_s C_F}{4\pi}\left(\frac{3}{2} \ln \frac{\mu^2}{m_Q^2} + 2 \right) + \mathcal{O}(\alpha_s^2)$$

notice $\phi_t(u) \propto \bar{u}$ ⇒ satisfies the QCD LCDA endpoint behaviour at $u \to 1$

Evolution from Λ_{QCD} to m_W: Strategy

1. Model for HQET LCDA at $\mu_s = 1\,\text{GeV}$

$$\varphi_+(\omega;\mu_s) = \left(1 + \frac{\alpha_s C_F}{4\pi}\left[\frac{1}{2} - \frac{\pi^2}{12}\right]\right)\varphi_+^{\text{mod}}(\omega;\mu_s) + \theta(\omega - \sqrt{e}\mu_s)\varphi_+^{\text{asy}}(\omega;\mu_s)$$

[Lee, Neubert '05]

Evolution from Λ_{QCD} to m_W: Strategy

1. Model for HQET LCDA at $\mu_s = 1\,\text{GeV}$

$$\varphi_+(\omega;\mu_s) = \left(1 + \frac{\alpha_s C_F}{4\pi}\left[\frac{1}{2} - \frac{\pi^2}{12}\right]\right)\varphi_+^{\text{mod}}(\omega;\mu_s)$$
$$+\,\theta(\omega - \sqrt{e}\mu_s)\varphi_+^{\text{asy}}(\omega;\mu_s)$$ [Lee, Neubert '05]

with $\quad \varphi_+^{\text{asy}}(\omega;\mu_s) \equiv \frac{\alpha_s C_F}{2\pi\omega}\left(\ln\frac{\mu_s^2}{\omega^2} + 1\right)$

Evolution from Λ_{QCD} to m_W: Strategy

1. Model for HQET LCDA at $\mu_s = 1\,\text{GeV}$

$$\varphi_+(\omega;\mu_s) = \left(1 + \frac{\alpha_s C_F}{4\pi}\left[\frac{1}{2} - \frac{\pi^2}{12}\right]\right)\varphi_+^{\text{mod}}(\omega;\mu_s)$$
$$+\,\theta(\omega - \sqrt{e}\mu_s)\varphi_+^{\text{asy}}(\omega;\mu_s)$$ [Lee, Neubert '05]

with $\varphi_+^{\text{asy}}(\omega;\mu_s) \equiv \frac{\alpha_s C_F}{2\pi\omega}\left(\ln\frac{\mu_s^2}{\omega^2} + 1\right)$

$\varphi_+^{\text{mod}}(\omega,\beta;\mu_s)$ three generalizations of the exp. model ($\beta = 0$) [Grozin, Neubert '96]

[Beneke, Braun, Ji, Wei '18]

Evolution from Λ_{QCD} to m_W: Strategy

1. Model for HQET LCDA at $\mu_s = 1\,\mathrm{GeV}$

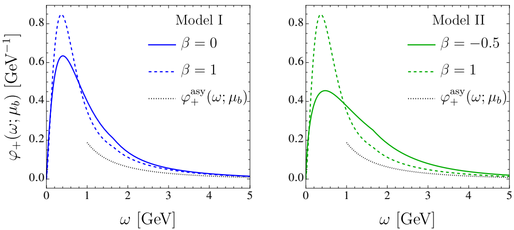
$$\varphi_+(\omega;\mu_s) = \left(1 + \frac{\alpha_s C_F}{4\pi}\left[\frac{1}{2} - \frac{\pi^2}{12}\right]\right)\varphi_+^{\text{mod}}(\omega;\mu_s)$$
$$+\,\theta(\omega - \sqrt{e}\mu_s)\varphi_+^{\text{asy}}(\omega;\mu_s)$$ [Lee, Neubert '05]

with $\varphi_+^{\text{asy}}(\omega;\mu_s) \equiv \frac{\alpha_s C_F}{2\pi\omega}(\ln\frac{\mu_s^2}{\omega^2} + 1)$

$\varphi_+^{\text{mod}}(\omega,\beta;\mu_s)$ three generalizations of the exp. model ($\beta = 0$) [Grozin, Neubert '96]

[Beneke, Braun, Ji, Wei '18]

2. $\varphi_+(\omega;\mu_s) \xrightarrow{\text{RGE}} \varphi_+(\omega;\mu \sim m_Q)$ [Lange, Neubert '03]



Evolution from Λ_{QCD} to m_W: Strategy

1. Model for HQET LCDA at $\mu_s = 1\,\text{GeV}$

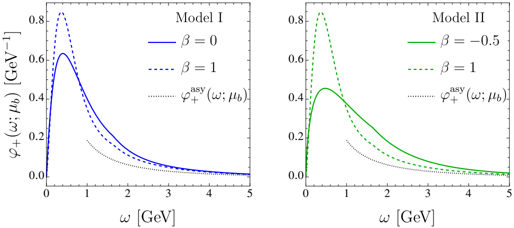
$$\varphi_+(\omega;\mu_s) = \left(1 + \frac{\alpha_s C_F}{4\pi}\left[\frac{1}{2} - \frac{\pi^2}{12}\right]\right)\varphi_+^{\text{mod}}(\omega;\mu_s) + \theta(\omega - \sqrt{e}\mu_s)\varphi_+^{\text{asy}}(\omega;\mu_s)$$ [Lee, Neubert '05]

with $\varphi_+^{\text{asy}}(\omega;\mu_s) \equiv \frac{\alpha_s C_F}{2\pi\omega}\left(\ln\frac{\mu_s^2}{\omega^2} + 1\right)$

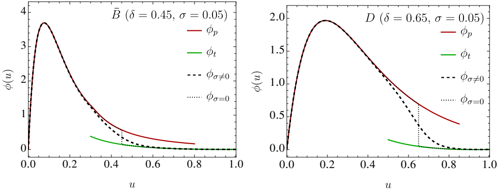
$\varphi_+^{\text{mod}}(\omega,\beta;\mu_s)$ three generalizations of the exp. model ($\beta = 0$) [Grozin, Neubert '96]

[Beneke, Braun, Ji, Wei '18]

2. $\varphi_+(\omega;\mu_s) \xrightarrow{\text{RGE}} \varphi_+(\omega;\mu\sim m_Q)$ [Lange, Neubert '03]



3. Matching obtaining $\phi(u;\mu)$



Evolution from Λ_{QCD} to m_W: Strategy

1. Model for HQET LCDA at $\mu_s = 1$ GeV

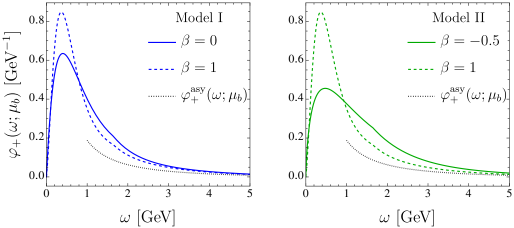
$$\varphi_+(\omega;\mu_s) = \left(1 + \frac{\alpha_s C_F}{4\pi}\left[\frac{1}{2} - \frac{\pi^2}{12}\right]\right)\varphi_+^{\text{mod}}(\omega;\mu_s) + \theta(\omega - \sqrt{e}\mu_s)\varphi_+^{\text{asy}}(\omega;\mu_s)$$ [Lee, Neubert '05]

with $\varphi_+^{\text{asy}}(\omega;\mu_s) \equiv \frac{\alpha_s C_F}{2\pi\omega}(\ln\frac{\mu_s^2}{\omega^2} + 1)$

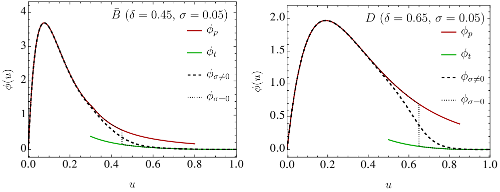
$\varphi_+^{\text{mod}}(\omega,\beta;\mu_s)$ three generalizations of the exp. model ($\beta = 0$) [Grozin, Neubert '96]

[Beneke, Braun, Ji, Wei '18]

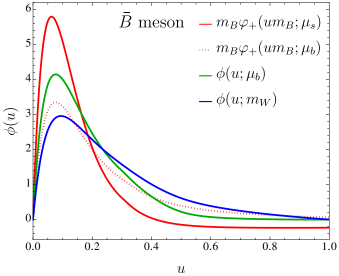
2. $\varphi_+(\omega;\mu_s) \overset{\text{RGE}}{\longrightarrow} \varphi_+(\omega;\mu \sim m_Q)$ [Lange, Neubert '03]



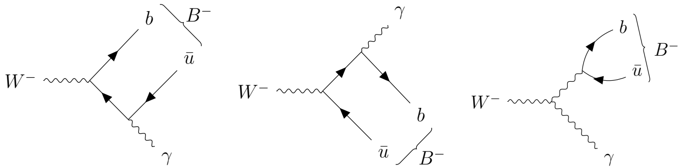
3. Matching obtaining $\phi(u;\mu)$



4. ERBL evolution $\phi(u;\mu) \to \phi(u;\mu_h = m_W)$



QCD Factorization Summary [Grossman, König, Neubert 2015]



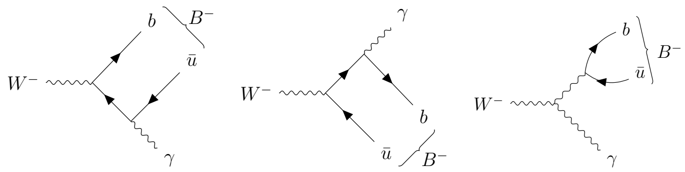
$1/x$ contr. $1/\bar{x}$ contr. Local contr.

at LP in $m_b/m_W \ll 1$

$$I_\pm^B = \int_0^1 dx\, H_\pm(x, \mu_h)\phi_B(x; \mu_h)$$

$$\bar{I}_\pm^B = \int_0^1 dx\, H_\pm(\bar{x}, \mu_h)\phi_B(x; \mu_h)$$

QCD Factorization Summary [Grossman, König, Neubert 2015]



$1/x$ contr. | $1/\bar{x}$ contr. | Local contr.

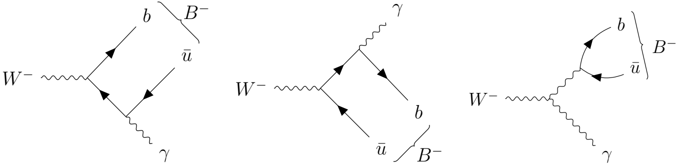
at LP in $m_b/m_W \ll 1$

$$I_\pm^B = \int_0^1 dx\, H_\pm(x, \mu_h)\phi_B(x; \mu_h)$$

$$\bar{I}_\pm^B = \int_0^1 dx\, H_\pm(\bar{x}, \mu_h)\phi_B(x; \mu_h)$$

with $H_\pm(x) = \frac{1}{x}(1 + \mathcal{O}(\alpha_s))$

QCD Factorization Summary [Grossman, König, Neubert 2015]



$1/x$ contr. $1/\bar{x}$ contr. Local contr.

at LP in $m_b/m_W \ll 1$

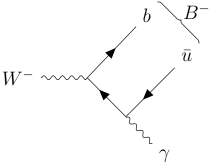
$$I_\pm^B = \int_0^1 dx\, H_\pm(x,\mu_h)\phi_B(x;\mu_h)$$

$$\bar{I}_\pm^B = \int_0^1 dx\, H_\pm(\bar{x},\mu_h)\phi_B(x;\mu_h)$$

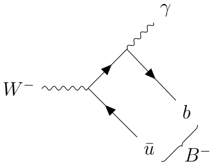
with $H_\pm(x) = \frac{1}{x}(1+\mathcal{O}(\alpha_s))$

$$\text{Br}(W\to B\gamma) = \frac{\alpha_{\text{em}} m_W f_B^2}{48 v^2 \Gamma_W} |V_{ub}|^2 \left(|F_1^B|^2 + |F_2^B|^2\right)$$

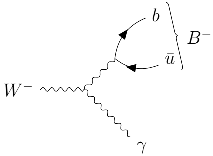
QCD Factorization Summary [Grossman, König, Neubert 2015]



$1/x$ contr.



$1/\bar{x}$ contr.



Local contr.

at LP in $m_b/m_W \ll 1$

$$I_\pm^B = \int_0^1 dx\, H_\pm(x,\mu_h)\phi_B(x;\mu_h)$$

$$\bar{I}_\pm^B = \int_0^1 dx\, H_\pm(\bar{x},\mu_h)\phi_B(x;\mu_h)$$

with $H_\pm(x) = \frac{1}{x}(1+\mathcal{O}(\alpha_s))$

$$\mathrm{Br}(W \to B\gamma) = \frac{\alpha_{\text{em}} m_W f_B^2}{48 v^2 \Gamma_W} |V_{ub}|^2 \left(|F_1^B|^2 + |F_2^B|^2 \right)$$

$$F_1^B = Q_u I_+^B + Q_d \bar{I}_+^B \qquad F_2^B = 2(Q_u - Q_d) - Q_u I_-^B + Q_d \bar{I}_-^B$$

Our task is to simply use our evolved LCDA for $\phi_B(x;\mu_h)$ in the convolutions

We will compare with the model from [GKN15] (with our inputs)

HQET Factorization [Ishaq, Jia, Xiong, Yang 2019]

The process can be studied at fixed-order in HQET **considering** $m_W \sim m_b$
$\Rightarrow |F_1^B| = |F_2^B| = Q_u I_\pm^B \sim \frac{m_b}{\Lambda_{\text{QCD}}} \gg \bar{I}_\pm^B$, Local ~ 1

HQET Factorization [Ishaq, Jia, Xiong, Yang 2019]

The process can be studied at fixed-order in HQET **considering** $m_W \sim m_b$
$\Rightarrow |F_1^B| = |F_2^B| = Q_u I_\pm^B \sim \frac{m_b}{\Lambda_{\text{QCD}}} \gg \bar{I}_\pm^B$, Local ~ 1

$$|F_{1,2}^B|_{\text{HQET}} = Q_u \frac{\tilde{f}_B(\mu_b)}{f_B} \int_0^\infty d\omega\, T(\omega, m_b, m_W, \mu_b)\varphi_+(\omega; \mu_b)$$

HQET Factorization [Ishaq, Jia, Xiong, Yang 2019]

The process can be studied at fixed-order in HQET **considering** $m_W \sim m_b$

$\Rightarrow |F_1^B| = |F_2^B| = Q_u I_\pm^B \sim \frac{m_b}{\Lambda_{\text{QCD}}} \gg \bar{I}_\pm^B$, Local ~ 1

$$|F_{1,2}^B|_{\text{HQET}} = Q_u \frac{\tilde{f}_B(\mu_b)}{f_B} \int_0^\infty d\omega\, T(\omega, m_b, m_W, \mu_b) \varphi_+(\omega; \mu_b)$$

we used our inputs and model for $\varphi_+(\omega; \mu_b)$ to have a fair comparison

HQET Factorization [Ishaq, Jia, Xiong, Yang 2019]

The process can be studied at fixed-order in HQET **considering** $m_W \sim m_b$
$\Rightarrow |F_1^B| = |F_2^B| = Q_u I_\pm^B \sim \frac{m_b}{\Lambda_{\text{QCD}}} \gg \bar{I}_\pm^B$, Local ~ 1

$$|F_{1,2}^B|_{\text{HQET}} = Q_u \frac{\tilde{f}_B(\mu_b)}{f_B} \int_0^\infty d\omega \, T(\omega, m_b, m_W, \mu_b) \varphi_+(\omega; \mu_b)$$

we used our inputs and model for $\varphi_+(\omega; \mu_b)$ to have a fair comparison

we checked by re-expanding the resummed result that

$$T(\omega, m_b, m_W, \mu_b)\Big|_{m_b \ll m_W} = \underbrace{H_\pm(x, m_W, \mu_h)}_{\text{hard scattering kernel}} \otimes_x \underbrace{f_{\text{ERBL}}(x, u, \mu_h, \mu_b)}_{\text{LCDA evolution}} \otimes_u \underbrace{\mathcal{J}_p(u, \omega, m_b, \mu_b)}_{\text{jet function}}$$