REFACTORIZATION IN SUBLEADING B DECAY INTO PHOTON

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Based on 2301.01739 with Tobias Hurth

INCLUSIVE B DECAY MODES

- $\bar{B} \to X_{s,d}\,\gamma$ and $\bar{B} \to X_{s,d}\,\ell^+\ell^-$ are very clean tests of the Standard Model
- They will be extremely well measured at Belle-II
- Well understood at leading power in Λ/m_b expansion

C. W. Bauer, et. al., 2000

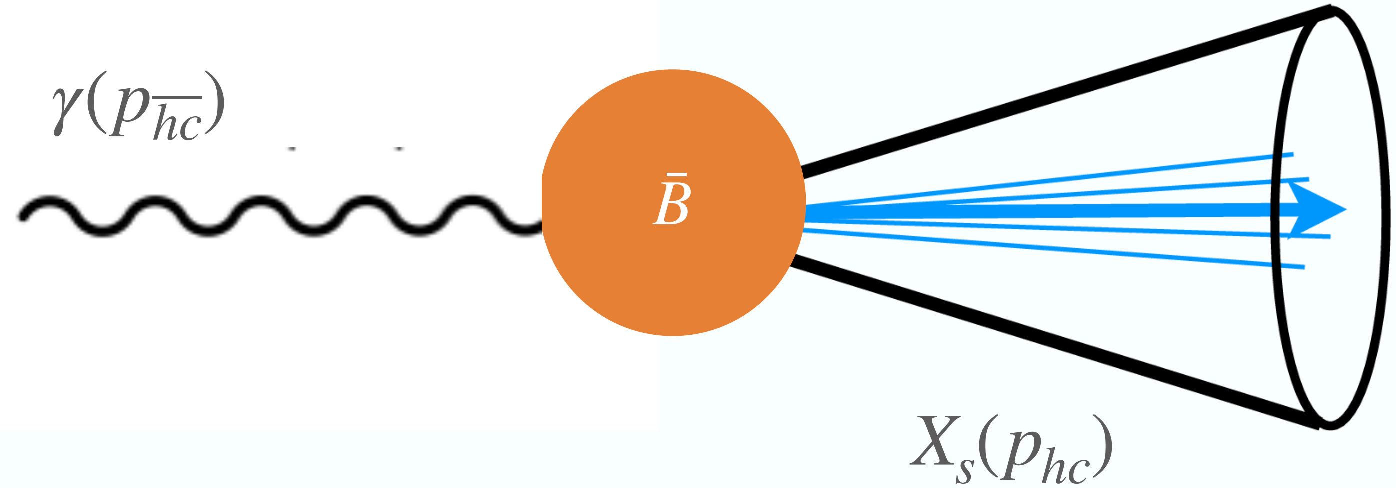
G. Korchemsky, et. al. 1994

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} \lambda_q \left(C_1\, O_1^q + C_2\, O_2^q + C_{7\gamma}\, O_{7\gamma} + C_{8g}\, O_{8g} + \sum_{i=3,\ldots,6} C_i\, O_i \right).$$

$$\hat{O}_7 = \frac{e}{16\pi^2} m_b\; \bar{s}\, \sigma^{\mu\nu} \frac{1}{2}(1+\gamma_5) b\; F_{\mu\nu}$$

POWER CORRECTIONS

I will focus on $\bar{B} \to X_s\gamma$



The naive factorization theorem

Benzke et. al. 2010

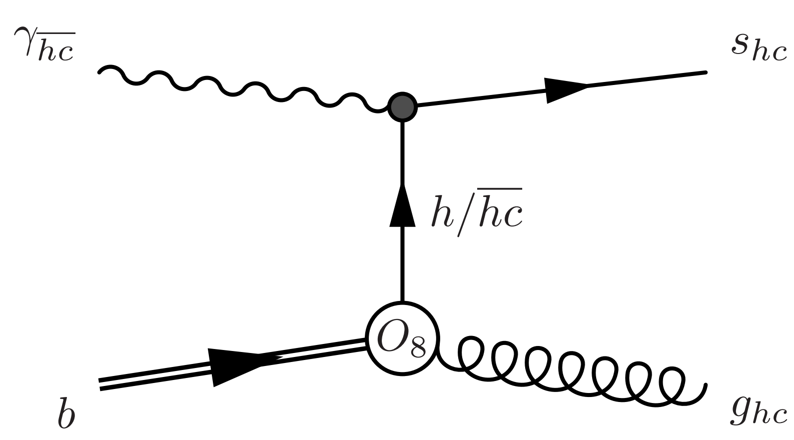
$$d\Gamma(\bar{B} \to X_s\gamma) = \sum_{n=0}^{\infty} \frac{1}{m_b^n} \sum_i H_i^{(n)} \otimes J_i^{(n)} \otimes S_i^{(n)}$$

$$+ \sum_{n=1}^{\infty} \frac{1}{m_b^n} \left[\sum_i H_i^{(n)} \otimes J_i^{(n)} \otimes S_i^{(n)} \otimes \bar{J}_i^{(n)} + \sum_i H_i^{(n)} \otimes J_i^{(n)} \otimes S_i^{(n)} \otimes \bar{J}_i^{(n)} \otimes \bar{J}_i^{(n)} \right]$$

We typically distinguish direct and resolved contributions

Resolved: photon couples to light quarks (non-local)

Kapustin et. al. 1995

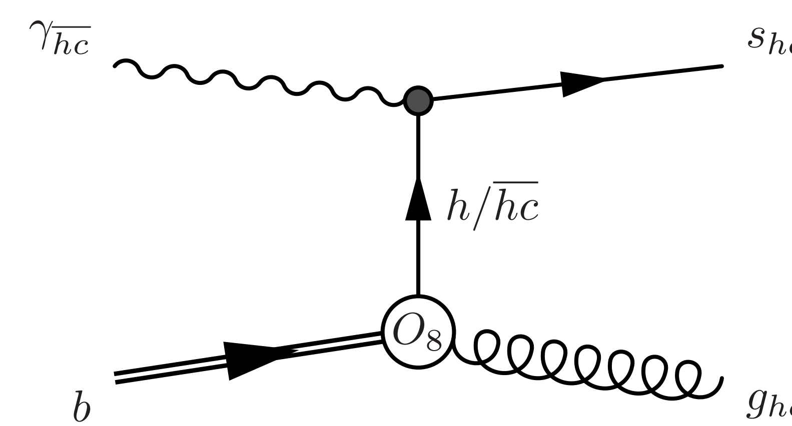


Contains anti-hard collinear radiate jet functions $\bar{J}$

$O_{8g} - O_{8g}$

$$O_{8g} = -\frac{g_s}{8\pi^2} m_b \bar{s}\sigma_{\mu\nu}(1+\gamma_5)G^{\mu\nu}b \,.$$

$$q^\mu = E_\gamma \bar{n}^\mu \quad \text{and} \quad p_B^\mu = M_B v^\mu \,,$$



- Contribution of the gluon dipole operator does not factorize — convolution integral is endpoint divergent
- Typical case of SCET I endpoint divergence
  - We can use methods developed for collider observables to reshuffle the divergent part
- First application with non-perturbative objects in flavor physics

ENDPOINT FACTORIZATION IN SCET

SCET I

Physical observable level

Operatorial rearrangements (KSZ)

M. Beneke et. al. 2019

Soft quark Sudakov conjecture

I. Moult et al. 2020

DIS off diagonal channel

M. Beneke et. al. 2020

Gluon thrust off-diagonal

M. Beneke et. al. 2022

} **Off-diagonal channels ↔ soft quark exchange**

** diagonal channels free from endpoint divergence at LL*

I. Moult et al., 2018
M. Beneke et. al., 2018

SCET II

Amplitude level

$h \to \gamma\gamma$

Z. Liu, et. al. 2019,2020

$gg \to H$

Z. Liu, et. al. 2022

QED in B decays

T. Feldmann et. al., 2022

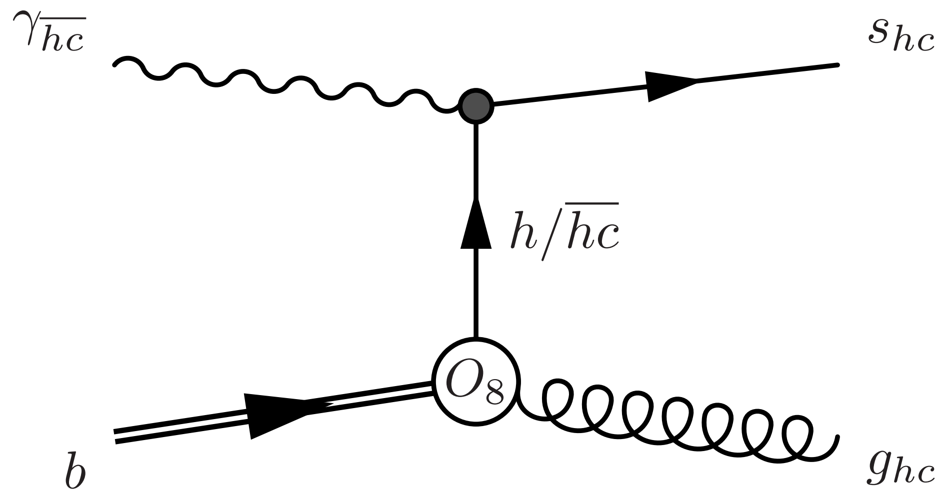
C. Cornella, et. al. 2022

See also:

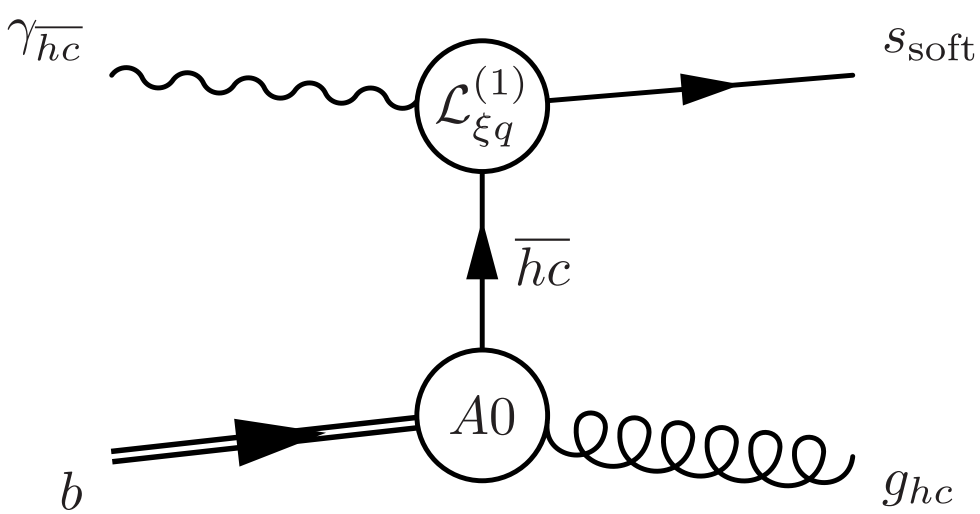
P. Böer, 2018

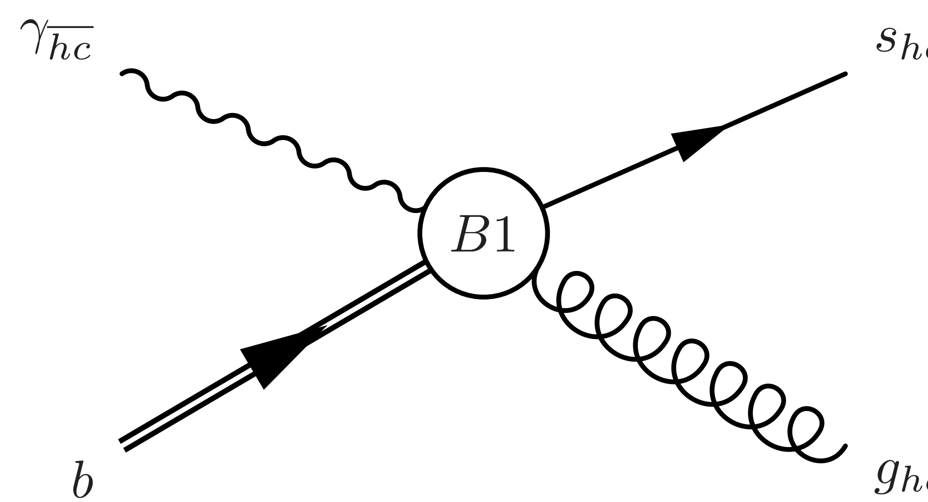
G. Bell et. al. 2022

$O_{8g} - O_{8g}$



Degeneracy in the EFT leads to divergences





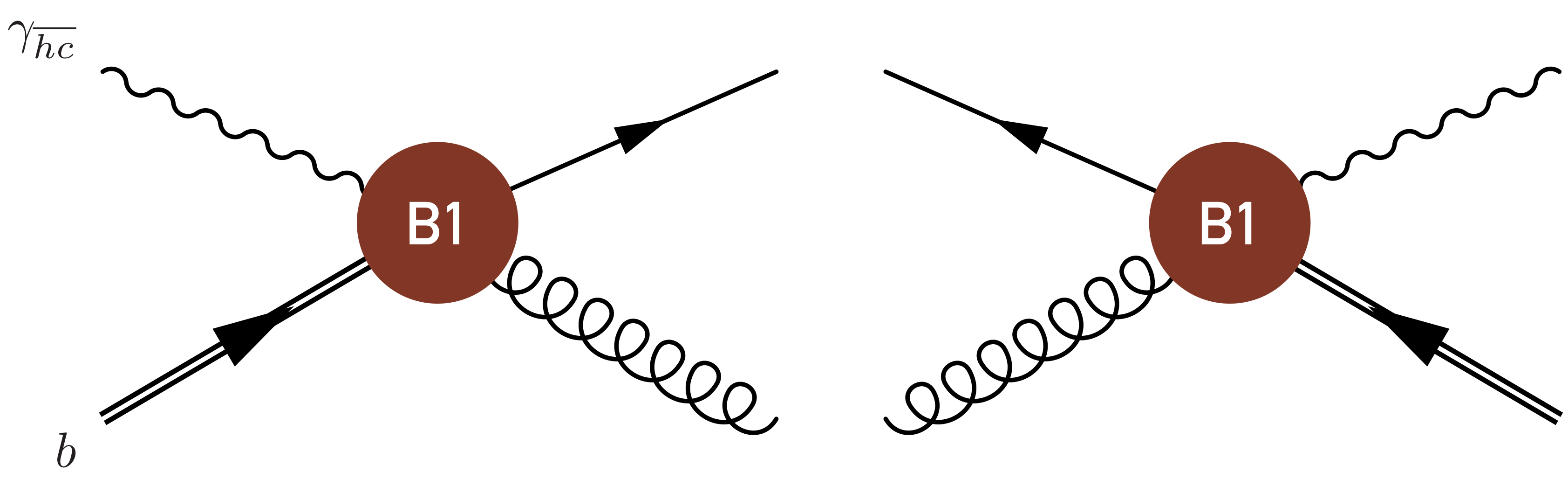
$$\mathcal{O}_{8g}^{A0}(0) = \overline{\chi}_{\overline{hc}}(0)\frac{\not{n}}{2}\gamma_{\mu\perp}\mathcal{A}^{\mu}{}_{hc\perp}(0)\left(1+\gamma_5\right)h(0)$$

$$\mathcal{O}_{8g}^{B1}(u) = \int \frac{dt}{2\pi} e^{-ium_b t}\overline{\chi}_{hc}(t\bar{n})\,\gamma_{\nu\perp}\,Q_s\,\mathcal{B}^{\nu}{}_{\overline{hc}\perp}(0)\,\gamma_{\mu\perp}\,\mathcal{A}^{\mu}{}_{hc\perp}(0)\left(1+\gamma_5\right)h(0)$$

DIRECT CONTRIBUTION

$$C_{LO}^{B1}\left(m_b, u\right) = (-1)\frac{\bar{u}}{u}\frac{m_b^2}{4\pi^2}\frac{G_F}{\sqrt{2}}\lambda_t C_{8g} = (-1)\frac{\bar{u}}{u}C_{LO}^{A0}\left(m_b\right)$$

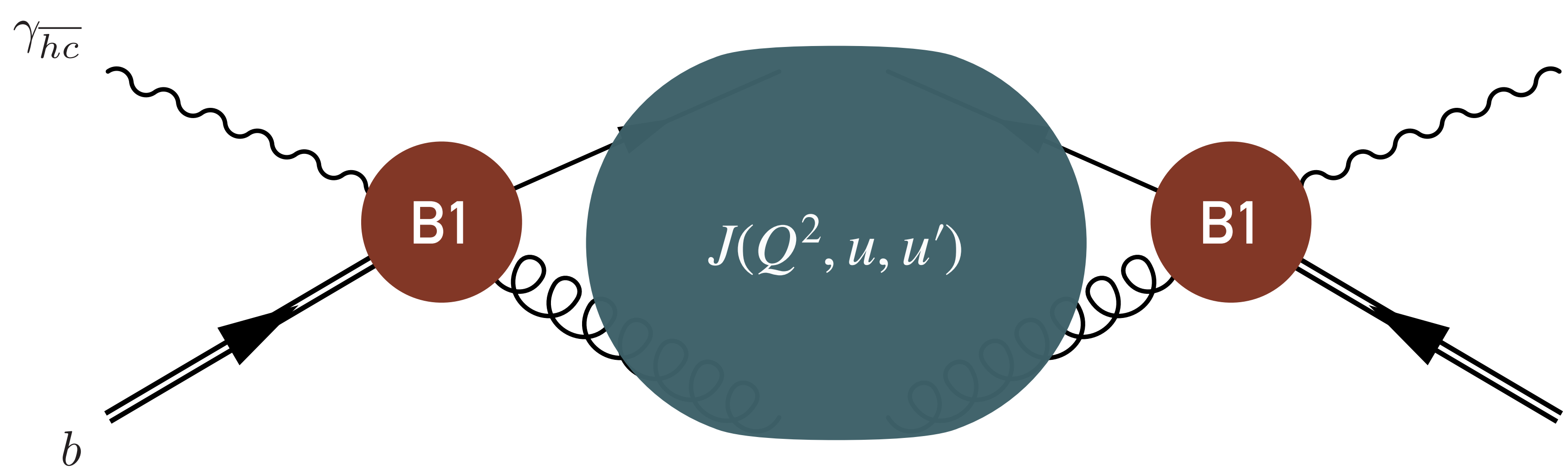
$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_B \int_0^1 du C^{B1}\left(m_b, u\right) \int_0^1 du' C^{B1*}\left(m_b, u'\right) \int_{-p_+}^{\bar{\Lambda}} d\omega J\left(M_B\left(p_+ + \omega\right), u, u'\right) \mathcal{S}\left(\omega\right)$$



DIRECT CONTRIBUTION

$$J\left(p^2,u,u'\right)=\frac{(-1)}{2N_c}\frac{1}{2\pi}\int\frac{dtdt'}{(2\pi)^2}d^4x\,e^{-im_b(ut-u't')+ipx}(d-2)^2$$
$$\text{Disc}\left[\langle 0|\,tr\left[\frac{\not{\bar{n}}}{4}(1-\gamma_5)\mathcal{A}^{\mu}{}_{hc\perp}(x)\,\chi_{hc}(t'\bar{n}+x)\,\overline{\chi}_{hc}(t\bar{n})\,\mathcal{A}^{hc\perp}_{\mu}(0)\,(1+\gamma_5)\right]|0\rangle\right]$$

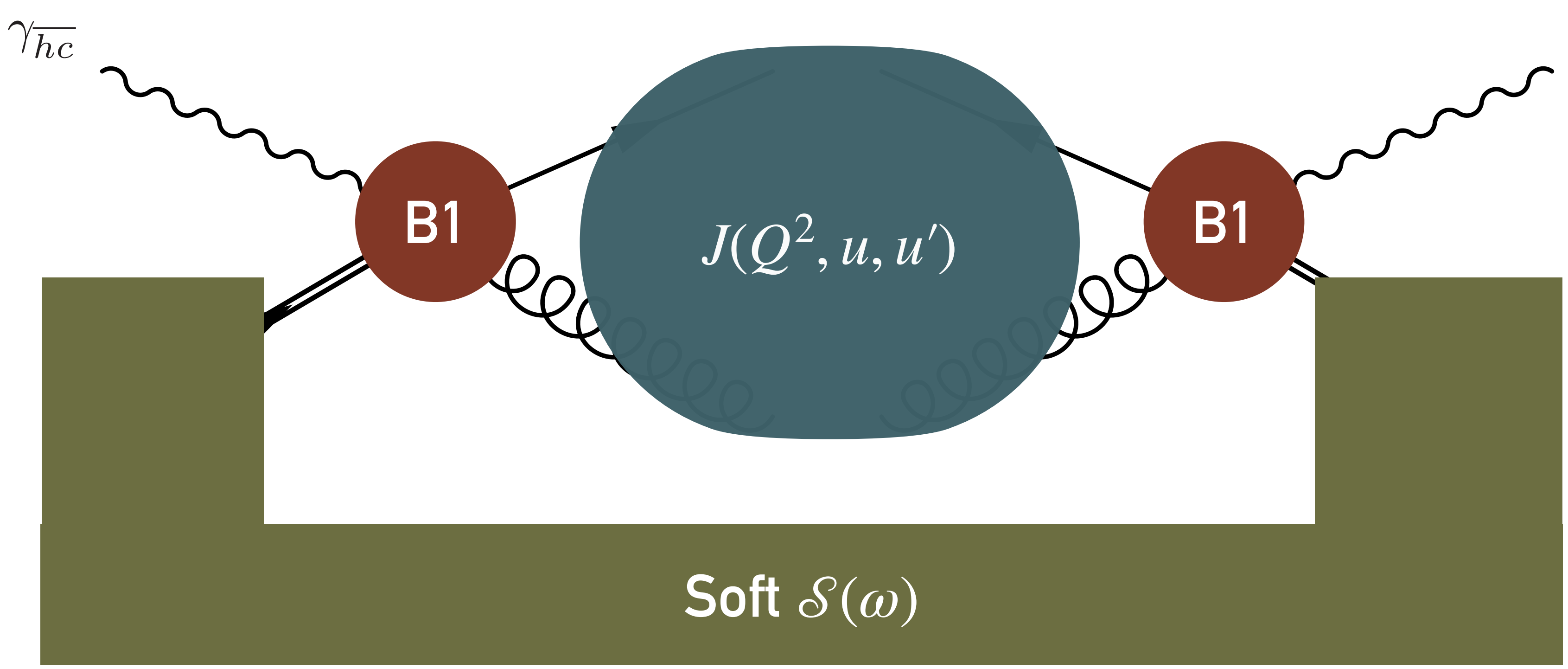
$$\frac{d\Gamma}{dE_\gamma}=\mathcal{N}_B\int_0^1 du\,C^{B1}(m_b,u)\int_0^1 du'\,C^{B1*}(m_b,u')\int_{-p_+}^{\overline{\Lambda}}d\omega\,J\left(M_B\left(p_++\omega\right),u,u'\right)\mathcal{S}(\omega)$$



DIRECT CONTRIBUTION

$$\mathcal{S}(\omega) = \frac{1}{2m_B}\int \frac{dt}{2\pi} e^{-i\omega t} \langle B \big| \, h(tn)\, S_n(tn)\, S_n^{\dagger}(0)\, h(0) \, \big| B \rangle$$

$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_B \int_0^1 du\, C^{B1}(m_b, u) \int_0^1 du'\, C^{B1*}(m_b, u') \int_{-p_+}^{\overline{\Lambda}} d\omega\, J\left(M_B(p_+ + \omega), u, u'\right) \mathcal{S}(\omega)$$



ENDPOINT DIVERGENCE IN DIRECT CONTRIBUTION

Hard matching coefficients

$$C_{\mathrm{LO}}^{B1}\left(m_b, u\right) = (-1)\frac{\bar{u}}{u}\,\frac{m_b^2}{4\pi^2}\,\frac{G_F}{\sqrt{2}}\lambda_t\,C_{8g} = (-1)\frac{\bar{u}}{u}C_{LO}^{A0}\left(m_b\right)$$

Convoluted with jet function

$$J_{\mathrm{LO}}\left(p^2, u, u'\right) = C_F\frac{\alpha_s}{4\pi\, m_b}\theta(p^2)\,A(\epsilon)\,\delta(u-u')u^{1-\epsilon}(1-u)^{-\epsilon}\left(\frac{p^2}{\mu^2}\right)^{-\epsilon}$$

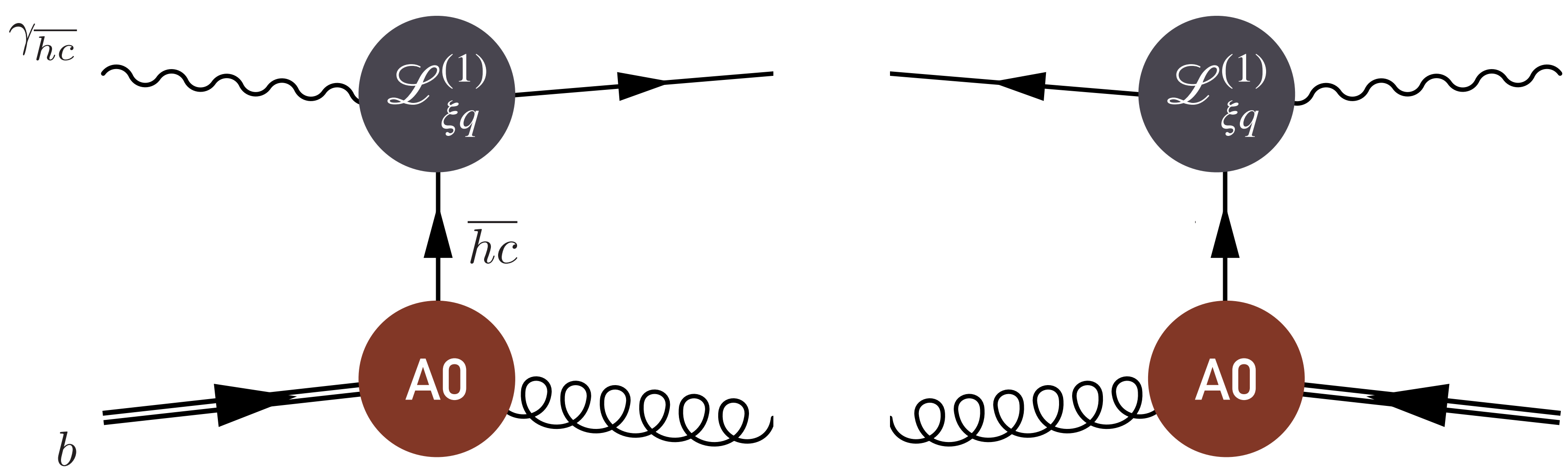
Lead to endpoint divergence in the $u \to 0$ limit

$$\int_0^1 \frac{du}{u}\int_0^1 \frac{du'}{u'}\delta(u-u')u^{1-\epsilon} \sim \int_0^1 \frac{du}{u^{1+\epsilon}}$$

Note that this part is perturbative

RESOLVED CONTRIBUTION

$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_A \left| C^{A0}\left(m_b\right)\right|^2 \int_{-p_+}^{\bar{\Lambda}} d\omega J_g\left(m_b\left(p_+ + \omega\right)\right)\int d\omega_1 \int d\omega_2 \bar{J}\left(\omega_1\right)\bar{J}^*\left(\omega_2\right)\mathcal{S}\left(\omega,\omega_1,\omega_2\right)$$



$$C^{A0}_{LO}\left(m_b\right) = \frac{m_b^2}{4\pi^2}\frac{G_F}{\sqrt{2}}\lambda_q C_{8g}$$

RESOLVED CONTRIBUTION

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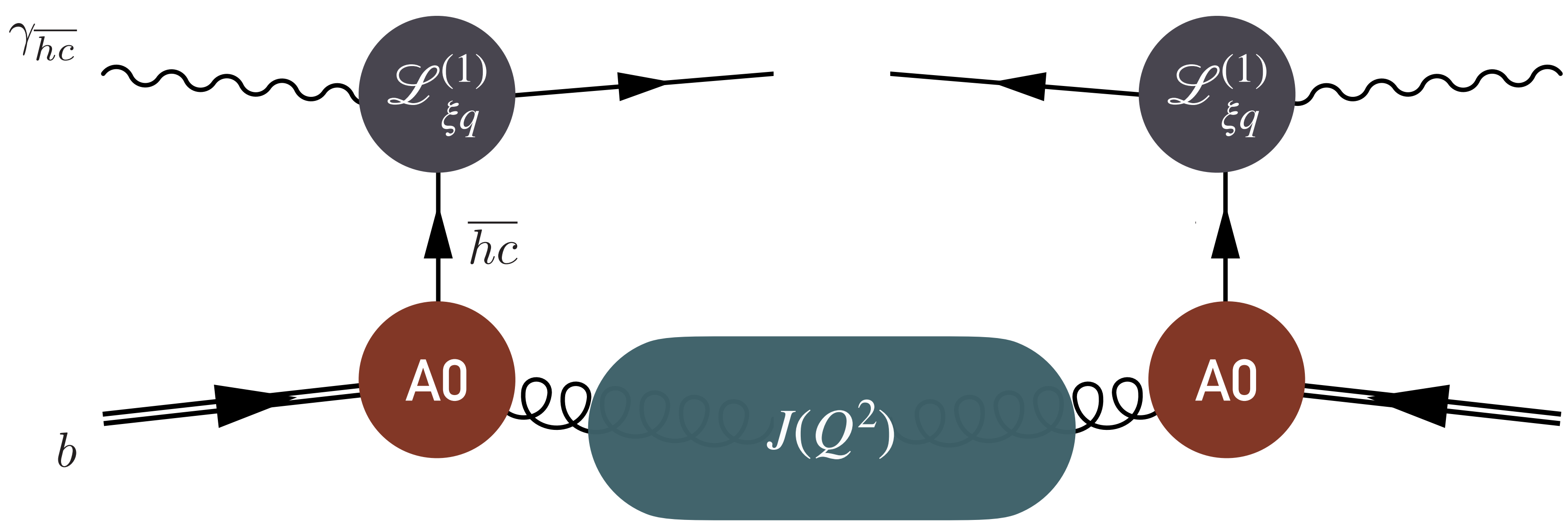
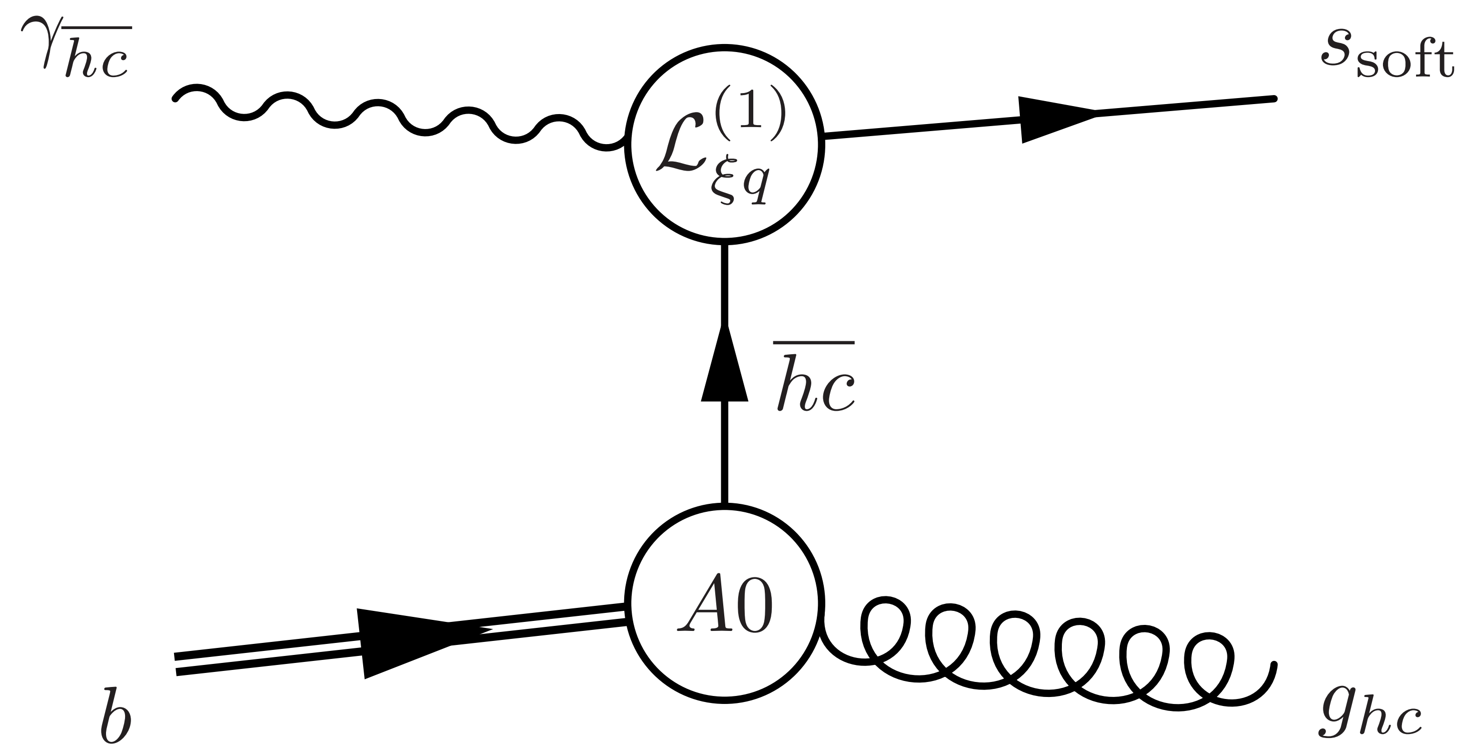


$$C^{A0}_{LO}(m_b) = \frac{m_b^2}{4\pi^2}\frac{G_F}{\sqrt{2}}\lambda_q C_{8g}$$

RADIATIVE JET FUNCTIONS

We integrate-out hard-anti-collinear QCD modes

$$\mathcal{O}_{T\xi q} = i\int d^d x T\left[\mathcal{L}^{(1)}_{\xi q}(x), \mathcal{O}^{A0}_{8g}(0)\right]$$



$\bar{J}(\omega)$ is formally identical to hard-collinear jet-functions typically defined when matching SCET I on SCET II

$$\mathcal{O}_{T\xi q} = \int d\omega \int \frac{dt}{2\pi} e^{-it\omega} \left[\overline{q_s}\right]_\alpha (tn) \left[\bar{J}(\omega)\right]^{a\,\nu\mu}_{\alpha\beta} Q_s \mathcal{B}^{\nu}{}_{\overline{hc}\perp}(0)\, \mathcal{A}^{\mu}_{hc\perp}(0) \left[h(0)\right]_\beta .$$

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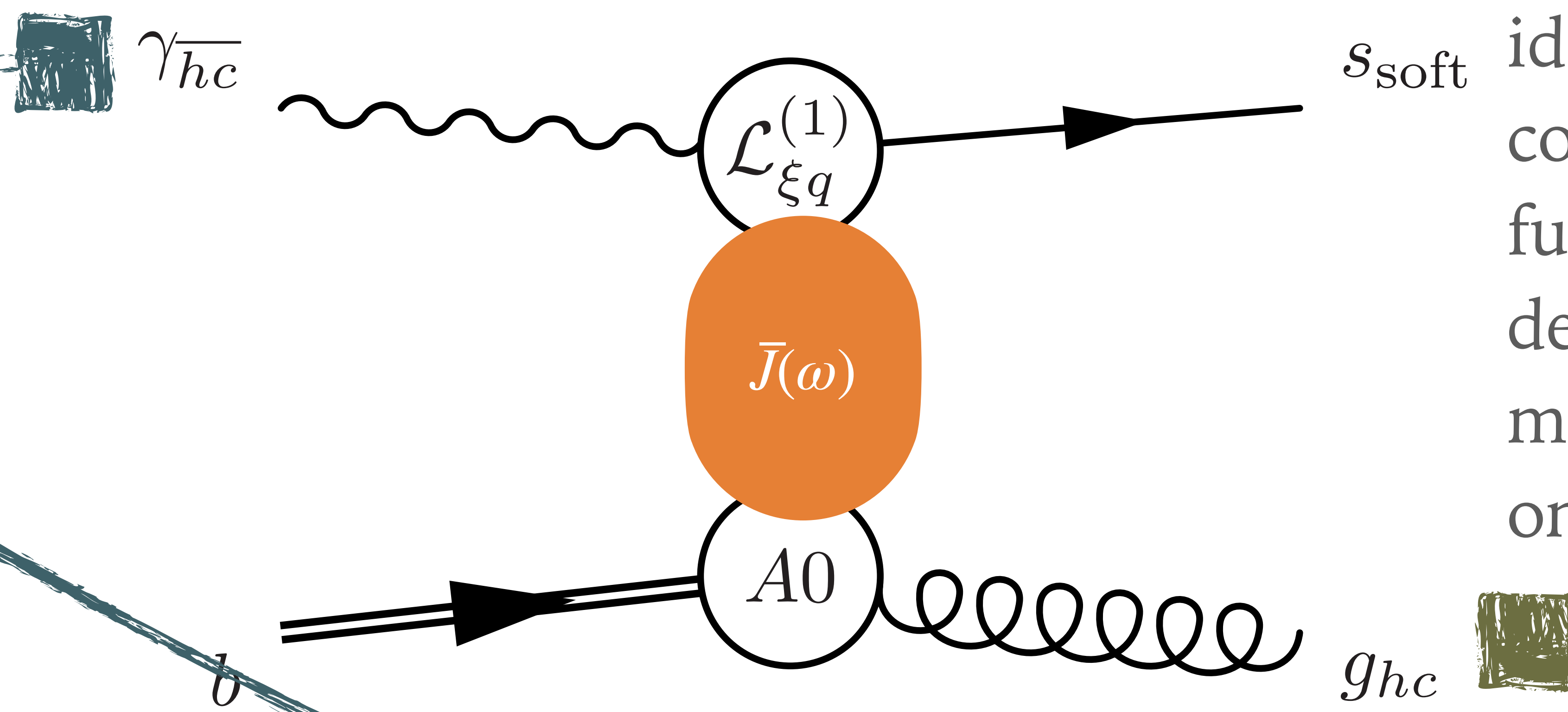

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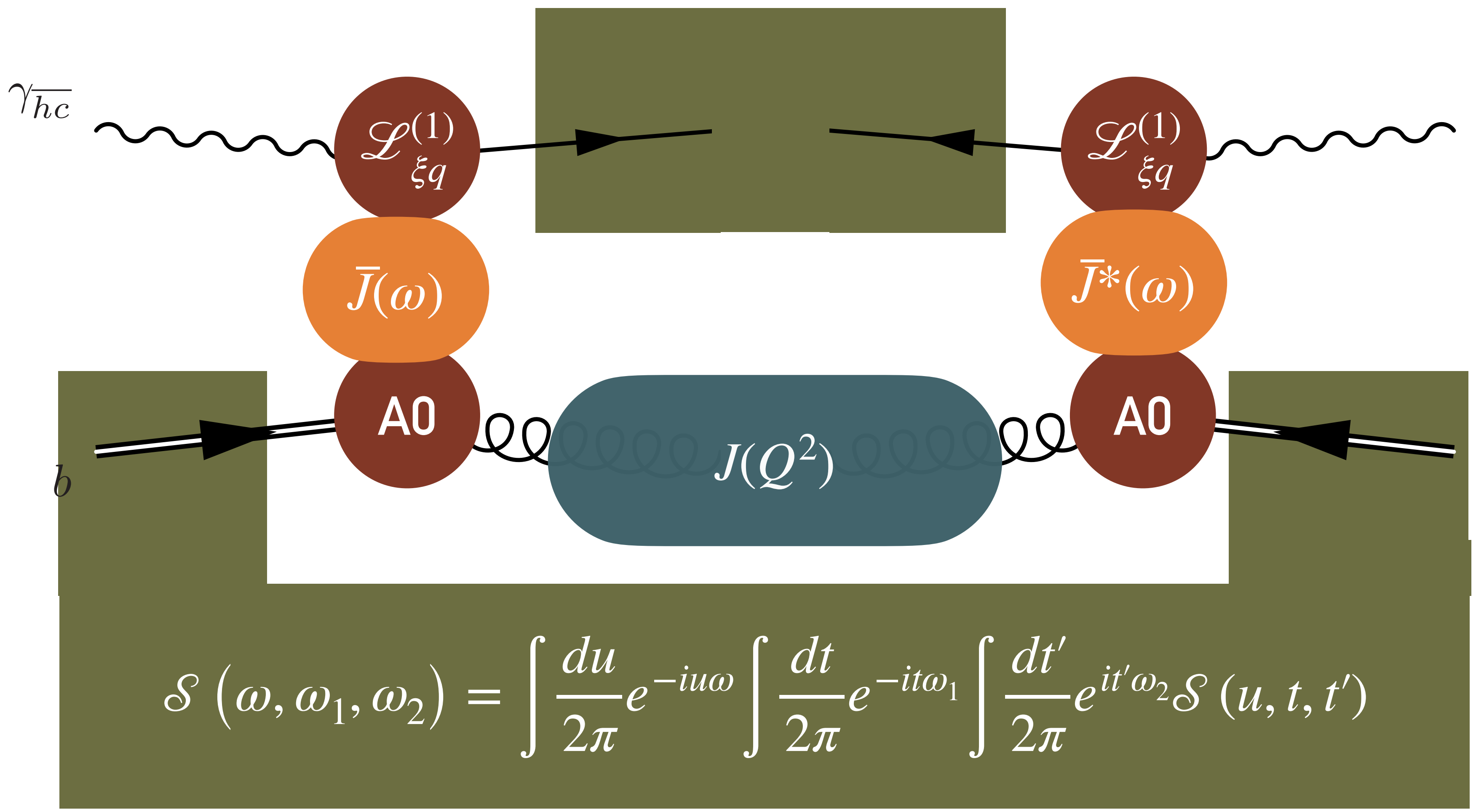
RESOLVED CONTRIBUTION

$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_A \left| C^{A0}(m_b) \right|^2 \int_{-p_+}^{\bar\Lambda} d\omega J_g\left(m_b\left(p_+ + \omega\right)\right) \int d\omega_1 \int d\omega_2 \bar{J}(\omega_1) \bar{J}^*(\omega_2) \mathcal{S}(\omega, \omega_1, \omega_2)$$



$$\mathcal{S}(\omega, \omega_1, \omega_2) = \int \frac{du}{2\pi} e^{-iu\omega} \int \frac{dt}{2\pi} e^{-it\omega_1} \int \frac{dt'}{2\pi} e^{it'\omega_2} \mathcal{S}(u, t, t')$$

$$\mathcal{S}(u,t,t') = (d-2)^2 g_s^2 \langle B| \bar{h}(un)(1-\gamma_5)\left[S_n(un)\, t^a S_n^\dagger(un)\right] S_{\bar n}(un)\, S_{\bar n}^\dagger(t'\bar n + un)$$
$$\frac{\not{n}\not{\bar n}}{4} q_s(t'\bar n + un)\, \bar{q}_s(t\bar n) \frac{\not{\bar n}\not{n}}{4} S_{\bar n}(t\bar n)\, S_{\bar n}^\dagger(0)\left[S_n(0)\, t^a S_n^\dagger(0)\right](1+\gamma_5)\, h(0) |B\rangle \; / \; (2m_B)$$

ENDPOINT DIVERGENCE IN RESOLVED CONTRIBUTION

$$\left[\bar{J}(\omega)\right]^{a\,\nu\mu}_{\alpha\beta} = \frac{t^a}{(\omega + i\,\epsilon)} \left[\gamma_\perp^\nu \gamma_\perp^\mu \frac{\bar{\not n}\not n}{4}\right]_{\alpha\beta}$$

$$\frac{d\Gamma}{dE_\gamma} = 2\mathcal{N} \left|C^{A0}_{LO}(m_b)\right|^2 \int_{-p_+}^{\bar{\Lambda}} d\omega\delta\left(m_b(p_+ + \omega)\right) \int_{-\infty}^{\infty} d\omega_1 \int_{-\infty}^{\omega_1} d\omega_2 \frac{1}{(\omega_1 - i\epsilon)} \frac{1}{(\omega_2 + i\epsilon)} \mathcal{S}(\omega, \omega_1, \omega_2)$$

For $\omega, \omega' \sim \Lambda$, the integral is well behaved

For $\omega, \omega' \gg \Lambda$ light quarks become hard-collinear and we can decouple soft gluons from them

$$\mathcal{S}(u,t,t') = (d-2)^2 g_s^2 \langle B|\,\bar{h}(un)(1-\gamma_5)\left[S_n(un)\,t^a S_n^\dagger(un)\right] S_{\bar{n}}(un)\,S_{\bar{n}}^\dagger(t'\bar{n}+un)$$
$$\frac{\not n\bar{\not n}}{4} q_s(t'\bar{n}+un)\,\bar{q}_s(t\bar{n})\,\frac{\bar{\not n}\not n}{4} S_{\bar{n}}(t\bar{n})\,S_{\bar{n}}^\dagger(0)\left[S_n(0)\,t^a S_n^\dagger(0)\right](1+\gamma_5)\,h(0)\,|B\rangle\;/\;(2m_B)$$

$$\mathcal{S}(u,t,t') \to \mathcal{S}(u) = \frac{1}{2m_B} \langle B \Big|\; h(un)\,S_n(un)\,S_n^\dagger(0)\,h(0)\;\Big| B\rangle$$

ENDPOINT DIVERGENCE IN RESOLVED CONTRIBUTION

$$\left[\bar{J}(\omega)\right]^{a\,\nu\mu}_{\alpha\beta} = \frac{t^a}{(\omega + i\,\epsilon)}\left[\gamma_\perp^\nu \gamma_\perp^\mu \frac{\not{\bar{n}}\not{n}}{4}\right]_{\alpha\beta}$$

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$$\frac{\not{n}\not{\bar{n}}}{4} q_s\,(t'\bar{n}+un)\,\bar{q}_s\,(t\bar{n})\,\frac{\not{\bar{n}}\not{n}}{4} S_{\bar{n}}\,(t\bar{n})\,S_{\bar{n}}^\dagger\,(0)\left[S_n\,(0)\,t^a S_n^\dagger\,(0)\right](1+\gamma_5)\,h\,(0)\,|B\rangle\;/\;(2m_B)$$

Schematically: $t, t' \to 0$

$$\mathcal{S}(u,t,t') \to \mathcal{S}\,(u) = \frac{1}{2m_B}\langle B\,|\;h\,(un)\,S_n\,(un)\,S_n^\dagger\,(0)\,h\,(0)\;|\,B\rangle$$

ENDPOINT DIVERGENCE IN RESOLVED CONTRIBUTION

$$\left[\bar{J}(\omega)\right]^{a\,\nu\mu}_{\alpha\beta} = \frac{t^a}{(\omega + i\,\epsilon)}\left[\gamma_\perp^\nu \gamma_\perp^\mu \frac{\not{\bar{n}}\not{n}}{4}\right]_{\alpha\beta}$$

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$$\frac{\not{n}\not{\bar{n}}}{4} q_s(t'\bar{n}+un)\,\bar{q}_s(t\bar{n})\,\frac{\not{\bar{n}}\not{n}}{4} S_{\bar{n}}(t\bar{n})\,S_{\bar{n}}^\dagger(0)\left[S_n(0)\,t^a S_n^\dagger(0)\right](1+\gamma_5)\,h(0)\,|B\rangle\;/\;(2m_B)$$

Schematically: $t, t' \to 0$ $\quad q_s(un) \to S_n(un)$ $\quad \bar{q}_s(0) \to S_n^\dagger(0)$

$$\mathcal{S}(u,t,t') \to \mathcal{S}(u) = \frac{1}{2m_B}\langle B\left|\,h(un)\,S_n(un)\,S_n^\dagger(0)\,h(0)\,\right|B\rangle$$

ENDPOINT FACTORIZATION OF THE SOFT FUNCTION

$$\mathcal{S}\left(\omega, \omega_1, \omega_2\right) \xrightarrow{\omega_1 \sim \omega_2 \gg \omega} \widetilde{\mathcal{S}}\left(\omega, \omega_1, \omega_2\right)$$

Endpoint soft (shape) function for resolved contribution is a convolution of a perturbative kernel $K(\omega, \omega'; \omega_1, \omega_2)$ *and leading power soft function*

$$\widetilde{\mathcal{S}}\left(\omega, \omega_1, \omega_2\right) = \int d\omega' K(\omega, \omega'; \omega_1, \omega_2) \mathcal{S}(\omega')$$

$$\widetilde{\mathcal{S}}\left(\omega, \omega_1, \omega_2\right) = C_F A(\epsilon) \frac{\alpha_s}{(4\pi)}\, \omega_1^{1-\epsilon} \delta(\omega_1 - \omega_2) \int_{\omega}^{\bar{\Lambda}} d\omega'\, \mathcal{S}(\omega') \left(\frac{(\omega' - \omega)}{\mu^2}\right)^{-\epsilon}$$

Compare Benzke et. al. 2010

ENDPOINT CONTRIBUTION

Evaluating every perturbative ingredient at LO near the endpoint

$$\frac{d\Gamma}{dE_\gamma}\Big|_B^{u,u'\to 0} = -\,\mathcal{N}\,\left|C_{LO}^{A0}\left(m_b\right)\right|^2 \frac{\alpha_s C_F}{(4\pi)\,m_b}\,\frac{1}{\epsilon}\,A(\epsilon)\int_{-p_+}^{\bar\Lambda} d\omega\,\mathcal{S}_{LO}(\omega)\left(\frac{m_b(\omega+p_+)}{\mu^2}\right)^{-\epsilon}$$

$$\frac{d\Gamma}{dE_\gamma}\Big|_A^{\text{asy}} = \mathcal{N}\,|\,C_{LO}^{A0}\left(m_b\right)|^2 \frac{\alpha_s C_F}{(4\pi)\,m_b}\,\frac{1}{\epsilon}A(\epsilon)\int_{-p_+}^{\bar\Lambda} d\omega\,\mathcal{S}_{LO}(\omega')\left(\frac{m_b(\omega+p_+)}{\mu^2}\right)^{-\epsilon}$$

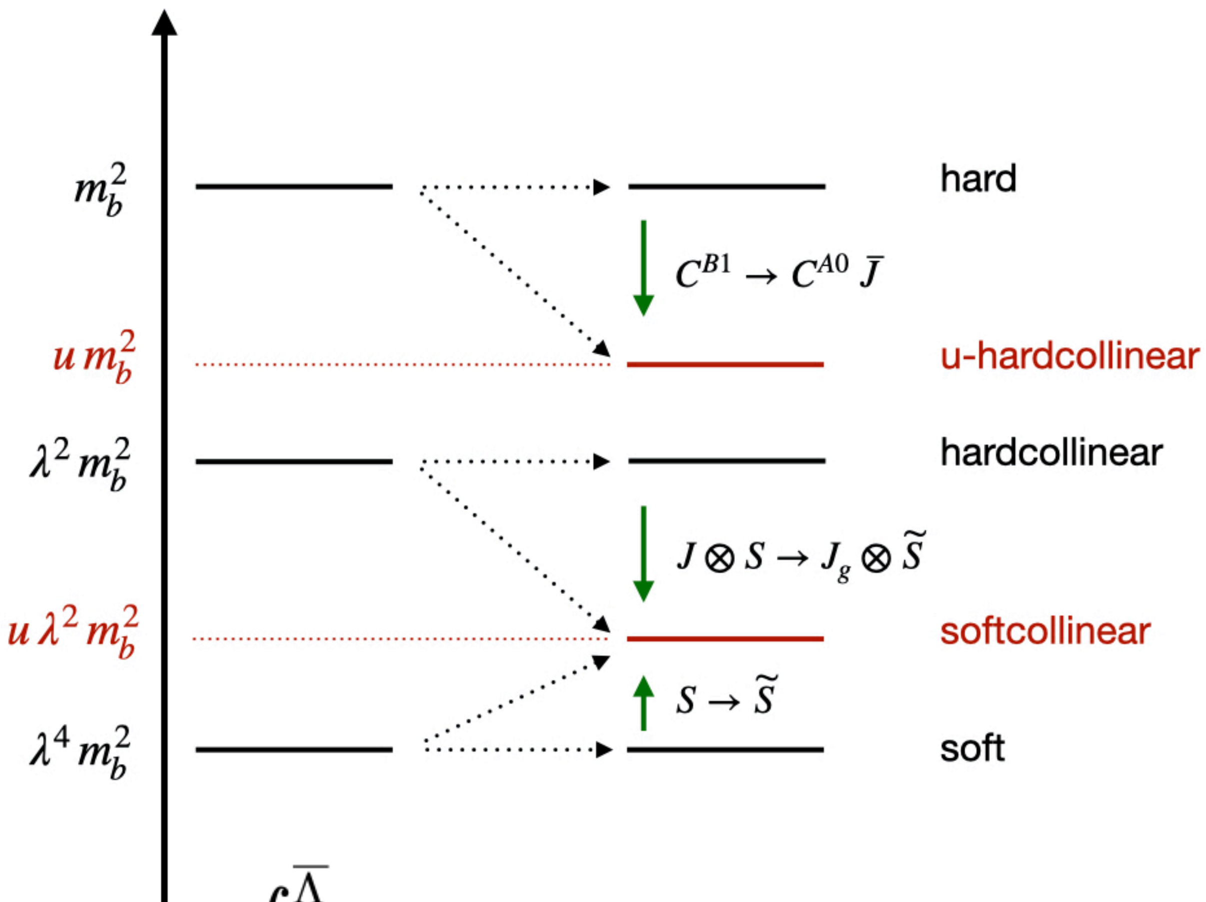
We verify that

$$\frac{d\Gamma}{dE_\gamma}\Big|_A^{\text{asy}} = (-1)\frac{d\Gamma}{dE_\gamma}\Big|_B^{u,u'\to 0}$$

ENDPOINT RESHUFFLING

Endpoint relations can be formulated as operator statements

$$C^{B1}\left(m_b, u\right)\left\langle \mathcal{O}_{8g}^{B1}(u)\right\rangle\Big|_{u\to 0} = C^{A0}\left(m_b\right) i\int d^d x e^{-i(nx/2)\,um_b}\left\langle T\left\{\mathcal{L}^{(1)}_{\xi q_{sc}}(x), \mathcal{O}_{8g}^{A0-u}(0)\right\}\right\rangle$$



Factorization conditions

$$\lim_{u\to 0} C^{B1}\left(m_b, u\right) = (-1)C^{A0}\left(m_b\right) m_b \bar{J}\left(um_b\right)$$

Z. Liu, et. al., 2019,2020

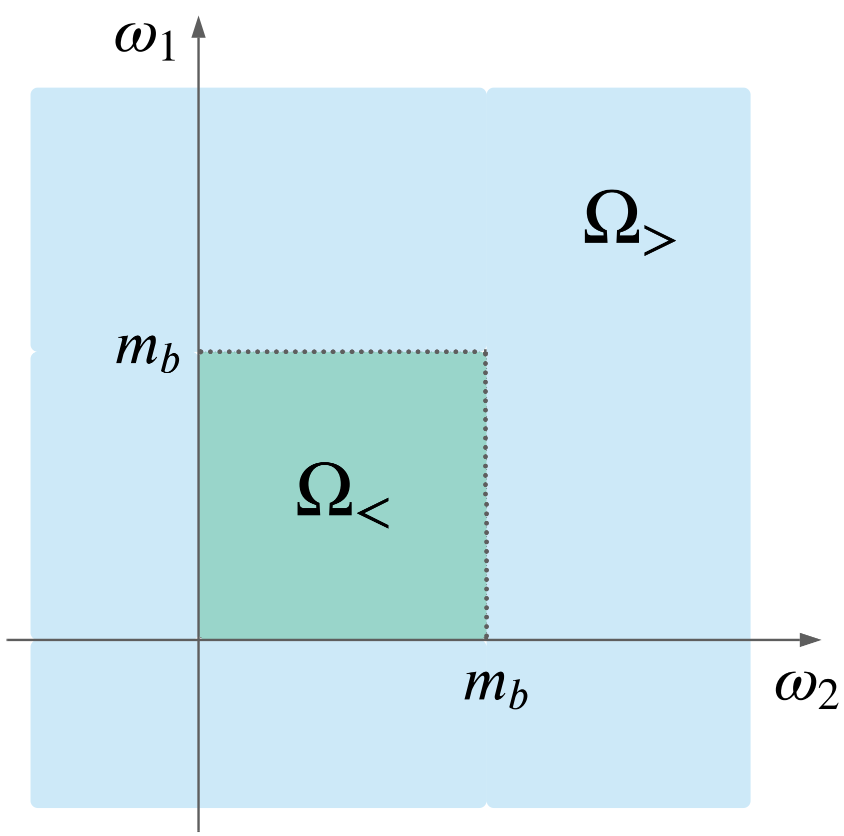
M. Beneke et. al., 2020

$$\int_{-p_+}^{\bar{\Lambda}} d\omega \left[\!\left[J\left(m_b\left(p_+ + \omega\right), u, u'\right) \mathcal{S}(\omega)\right]\!\right] = \int_{-p_+}^{\bar{\Lambda}} d\omega J_g(m_b(p_+ + \omega))\widetilde{\mathcal{S}}(\omega, m_b u, m_b u')$$

ENDPOINT SUBTRACTION

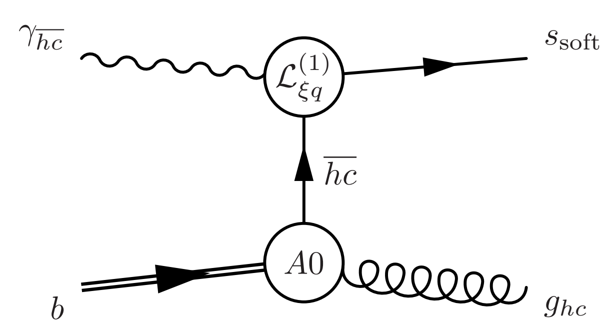
$$\int d\omega_1 d\omega_2 \widetilde{S}(\omega_1, \omega_2, \omega) \bar{J}(\omega_1) \bar{J}^*(\omega_2) = 0$$

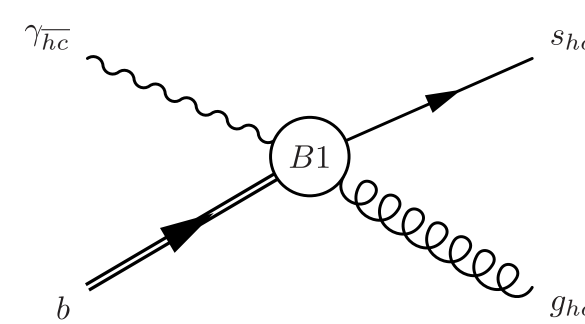
$$0 = \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 = 2\int_0^\infty d\omega_1 \int_0^{\omega_1} d\omega_2 = 2\left(\int_0^{m_b} d\omega_1 + \int_{m_b}^\infty d\omega_1 \right) \int_0^{\omega_1} d\omega_2$$



We can subtract the endpoint contribution from both terms in the factorization theorem

Note, when $\omega_1 \gg \omega_2$ or $\omega_2 \gg \omega_1$, there is no divergence in SCET I problems





FACTORIZATION THEOREM

We subtract the following expression

$$0 = 2\mathcal{N}\left|C^{A0}\left(m_b\right)\right|^2 \int_{-p_+}^{\bar{\Lambda}} d\omega J_g\left(m_b\left(p_+ + \omega\right)\right) \int_{m_b}^{\infty} d\omega_1 \overline{J}\left(\omega_1\right) \int_0^{\omega_1} d\omega_2 \overline{J}^*\left(\omega_2\right) \widetilde{\mathcal{S}}\left(\omega, \omega_1, \omega_2\right)$$

$$+ 2\mathcal{N} \int_0^1 du \, [\![C^{B1}\left(m_b, u'\right)]\!] \int_u^1 du' \, [\![C^{B1*}\left(m_b, u'\right)]\!] \int_{-p_+}^{\bar{\Lambda}} d\omega \, [\![J\left(m_b\left(p_+ + \omega\right), u, u'\right) \mathcal{S}(\omega)]\!]$$

And we obtain endpoint finite factorization theorem for NLP $B \to X_s\gamma$

$$\frac{d\Gamma}{dE_\gamma}|_{A+B} = 2\mathcal{N} \int_{-p_+}^{\bar{\Lambda}} d\omega \Big\{ J_g(m_b(p_+ + \omega)) \left|C^{A0}\left(m_b\right)\right|^2$$
$$\times \int_{-\infty}^{\infty} d\omega_1 \int_{-\infty}^{\omega_1} d\omega_2 \overline{J}(\omega_1)\, \overline{J}^*(\omega_2) \left[\mathcal{S}\left(\omega, \omega_1, \omega_2\right) - \theta(\omega_1 - m_b)\theta(\omega_2)\widetilde{\mathcal{S}}(\omega, \omega_1, \omega_2)\right]$$
$$+ \int_0^1 du \int_u^1 du' \left[C^{B1}_{LO}\left(m_b, u\right) \, C^{B1*}\left(m_b, u'\right) \, J\left(m_b\left(p_+ + \omega\right), u, u'\right) \mathcal{S}\left(\omega\right)\right.$$
$$\left. - [\![C^{B1}\left(m_b, u\right)]\!] \, [\![C^{B1*}\left(m_b, u'\right)]\!] \, [\![J\left(m_b\left(p_+ + \omega\right), u, u'\right) \mathcal{S}(\omega)]\!] \right] \Big\},$$

RENORMALIZATION

LP objects have well known renormalization kernels

$$C^{A0}_{\text{bare}}(m_b) = Z^{A0}(\mu)\, C^{A0}_{\text{ren}}(\mu, m_b) \qquad S^{\text{bare}}(\omega) = \int d\omega'\, Z_S(\mu, \omega - \omega')\, S^{\text{ren}}(\mu, \omega')$$

$$J_g^{\text{bare}}(p^2) = \int_0^{p^2} dp'^2\, Z_{J_g}(\mu, p^2 - p'^2)\, J_g^{\text{ren}}(\mu, p'^2)$$

B-type current

$$C^{B1}_{\text{bare}}(u) = \int_0^1 du'\, Z^{B1}(\mu, u, u')\, C^{B1}_{\text{ren}}(\mu\, .\, u')$$

M. Beneke et. al. 2018

The time-like $\omega > 0$ and space-like $\omega < 0$ radiative jet functions do not mix under renormalisation *S. W. Bosch et. al., 2003*

$$\bar{J}^{+}_{\text{bare/ren}}(\omega) = \theta(\omega)\bar{J}_{\text{bare/ren}}(\omega) \qquad \bar{J}^{+}_{\text{bare}}(\omega) = \int_0^{\infty} d\omega'\, Z^{+}_{\bar{J}}(\mu, \omega, \omega')\,,\, \bar{J}^{+}_{\text{ren}}(\mu, \omega')$$

$$\bar{J}^{-}_{\text{bare/ren}}(\omega) = \theta(-\omega)\bar{J}_{\text{bare/ren}}(\omega) \qquad \bar{J}^{-}_{\text{bare}}(\omega) = \int_{-\infty}^{0} d\omega'\, Z^{-}_{\bar{J}}(\mu, \omega, \omega')\, \bar{J}^{-}_{\text{ren}}(\mu, \omega')$$

RENORMALIZATION

Genuinely new NLP objects

$$S_{\text{bare}}(\omega, \omega_1, \omega_2) = \int d\omega' d\omega_1' d\omega_2' \, Z_S(\mu, \omega, \omega', \omega_1, \omega_1', \omega_2, \omega_2') \, S_{\text{ren}}(\mu, \omega', \omega_1', \omega_2')$$

$$J_{\text{bare}}(p^2, u_1, u_2) = \int dp'^2 \int_0^1 du_1' \int_0^1 du_2' \, Z_J(\mu, p^2 - p'^2, u_1, u_1', u_2, u_2') \, J_{\text{ren}}(p'^2, u_1', u_2')$$

Endpoint reshuffling and renormalization effectively commute in our case

The leftover piece cancels between the direct and resolved contributions

see also the thrust case *M. Beneke et. al. 2022*

SUMMARY

- First factorization theorem for inclusive observable in flavor physics cured from endpoint divergences
- Involves refactorization condition for non-perturbative soft function
- Similar approach to inclusive observables in collider application

SCET returns to flavor physics

See our paper for more details 2301.01739

and also recent papers on endpoint divergences in QED corrections for exclusive leptonic decays

T. Feldmann et. al., 2022

C. Cornella, et. al. 2022