Soft-collinear gravity with fermionic matter

Patrick Hager (MITP, JGU Mainz)

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Based on

[2212.02525]

[2210.09336]

[2112.04983]

[2110.02969]

in collaboration with

Martin Beneke (TUM), Dominik Schwienbacher (University of Bern),
Robert Szafron (BNL)

SCET Gravity at NLP?

- Similarities and differences between gauge theory and gravity.
- Soft gravity is similar to gauge theory: Soft theorem, LBK

$$\mathcal{A}^{\gamma}_{\text{rad}} = -\sum_i Q_i \left(\frac{\varepsilon_\mu p_i^\mu}{p_i \cdot k} + \frac{k_\nu \varepsilon_\mu J_i^{\mu\nu}}{p_i \cdot k} \right) \mathcal{A}_0$$

$$\mathcal{A}^{h}_{\text{rad}} = \frac{\kappa}{2}\sum_i \left(\frac{\varepsilon_{\mu\nu} p_i^\mu p_i^\nu}{p_i \cdot k} + \frac{k_\nu \varepsilon_{\mu\rho} p_i^\rho J_i^{\mu\nu}}{p_i \cdot k} + \frac{1}{2}\frac{k_\rho k_\sigma \varepsilon_{\mu\nu} J^{\mu\rho} J^{\nu\sigma}}{p_i \cdot k} \right) \mathcal{A}_0$$

- Collinear gravity is different: collinear divergences are absent
- For integer-spin representations: systematic construction of Lagrangian at subleading-power, based on diffeomorphism group.
  [Beneke, PH, Szafron 2210.09336, 2112.04983]
- How to include fermionic particles?

How to construct SCET

- Power-counting parameter $\lambda = |\frac{p_\perp}{n_+ p}| \ll 1$.
  [Bauer et al. hep-ph/0011336, hep-ph/0109045, hep-ph/0202088;]
- Introduce the mode split, implement gauge symmetry
- Control large field components

$$n_+ A_c \sim 1\,, \quad h_{++} \sim \lambda^{-1}\,.$$

- Multipole-expand the soft fields in soft-collinear products
  [Beneke, Chapovsky, Diehl, Feldmann hep-ph/0206152; Beneke, Feldmann hep-ph/0211358]

$$\phi_s(x) = \phi_s(x_-) + (x - x_-) \cdot \partial \phi_s + \ldots$$

- Ensure gauge symmetry respects multipole expansion

$$\phi_c(x) \to U_s(x_-)\phi_c(x)\,.$$

Gauge Transformation and "Wilson lines"

- Gauge transformations in gravity are local translations

$$x^\mu \to y^\mu(x) = x^\mu + \varepsilon^\mu(x)\,.$$

- Use active point of view: fields transform, not coordinates

$$[U\varphi] = \varphi - \varepsilon^\alpha \partial_\alpha \varphi + \frac{1}{2}\varepsilon^\alpha \varepsilon^\beta \partial_\alpha \partial_\beta \varphi + \varepsilon^\alpha \partial_\alpha \varepsilon^\beta \partial_\beta \varphi + \mathcal{O}(\varepsilon^3)\,.$$

- Gravitational "Wilson lines": translation operators

$$\left[W_\theta^{-1}\varphi\right] = \varphi + \theta^\alpha \partial_\alpha \varphi + \frac{1}{2}\theta^\alpha \theta^\beta \partial_\alpha \partial_\beta \varphi + \mathcal{O}(\theta^3)\,.$$

- Translate the field to a reference frame where certain gauge conditions hold.

Gauge Transformations

- Collinear fluctuations on top of a soft background field:
  [Beneke, PH, Szafron 2112.04983, 2210.09336]

collinear: $$h_{\mu\nu} \to \left[U_c\left({U_{c\mu}}^{\alpha}{U_{c\nu}}^{\beta}h_{\alpha\beta}\right)\right] + \left[U_c\left({U_{c\mu}}^{\alpha}{U_{c\nu}}^{\beta}g_{s\alpha\beta}\right)\right] - g_{s\mu\nu}\,,$$

$$g_{s\mu\nu} \to g_{s\mu\nu}\,,$$

,

soft: $$h_{\mu\nu} \to \left[U_s({U_{s\mu}}^{\alpha}{U_{s\nu}}^{\beta}h_{\alpha\beta})\right]\,,$$

$$s_{\mu\nu} \to \left[U_s({U_{s\mu}}^{\alpha}{U_{s\nu}}^{\beta}s_{\alpha\beta})\right] + \left[U_s({U_{s\mu}}^{\alpha}{U_{s\nu}}^{\beta}\eta_{\alpha\beta})\right] - \eta_{\mu\nu}\,,$$

,

.

- Gauge charge P^{μ} has λ-scaling.

Gauge Transformations

- Collinear fluctuations on top of a soft background field:
  [Beneke, PH, Szafron 2112.04983, 2210.09336]

$$\text{collinear:} \quad h_{\mu\nu} \to \left[U_c\left({U_{c\mu}}^{\alpha}{U_{c\nu}}^{\beta}h_{\alpha\beta}\right)\right] + \left[U_c\left({U_{c\mu}}^{\alpha}{U_{c\nu}}^{\beta}g_{s\alpha\beta}\right)\right] - g_{s\mu\nu}\,,$$

$$g_{s\mu\nu} \to g_{s\mu\nu}\,,$$

$$\text{QCD:} \quad A_c \to U_c A_c U_c^\dagger + \frac{i}{g}U_c\left[D_s\,,U_c^\dagger\right]\,,$$

$$A_s \to A_s\,,$$

$$\text{soft:} \quad h_{\mu\nu} \to \left[U_s({U_{s\mu}}^{\alpha}{U_{s\nu}}^{\beta}h_{\alpha\beta})\right]\,,$$

$$s_{\mu\nu} \to \left[U_s({U_{s\mu}}^{\alpha}{U_{s\nu}}^{\beta}s_{\alpha\beta})\right] + \left[U_s({U_{s\mu}}^{\alpha}{U_{s\nu}}^{\beta}\eta_{\alpha\beta})\right] - \eta_{\mu\nu}\,,$$

$$\text{QCD:} \quad A_c \to U_s A_c U_s^\dagger\,,$$

$$A_s \to U_s A_s U_s^\dagger + \frac{i}{g}U_s\left[\partial\,,U_s^\dagger\right]\,.$$

- Gauge charge P^μ has λ-scaling.

Collinear "Wilson line" W_c^{-1}

- Construction similar to QCD: fix collinear light-cone gauge.
  [Beneke, Chapovsky, Diehl, Feldmann hep-ph/0206152; Beneke, Feldmann hep-ph/0211358]

- Use collinear "Wilson line" to define $\mathfrak{h}_{\mu\nu}$ satisfying $\mathfrak{h}_{\mu+} = 0$:

$$\mathfrak{h}_{\mu\nu} = W_c{}^{\alpha}{}_{\mu} W_c{}^{\beta}{}_{\nu} \left[W_c^{-1} (\eta_{\alpha\beta} + h_{\alpha\beta}) \right] - \eta_{\mu\nu} ,$$
$$W_c^{-1} = T_{\theta_{\rm LC}} = 1 + \theta^{\alpha}_{\rm LC}(x) \partial_\alpha + \frac{1}{2} \theta^{\alpha}_{\rm LC}(x) \theta^{\beta}_{\rm LC}(x) \partial_\alpha \partial_\beta + \dots ,$$

- $\mathfrak{h}_{\mu\nu}$ is manifestly collinear gauge-invariant, $h_{\mu+}$ appears only in Wilson line.

- Similarity to QCD at linear level

$$\mathcal{A}_{c\mu} = A_{c\mu} - \partial_\mu \frac{n_+ A_c}{n_+ \partial} + \dots$$
$$\mathfrak{h}_{\mu\nu} = h_{\mu\nu} - \partial_\mu \left(\frac{h_{\nu+}}{n_+\partial} - \frac{1}{2} \partial_\nu \frac{h_{++}}{(n_+\partial)^2} \right) - \partial_\nu \left(\frac{h_{\mu+}}{n_+\partial} - \frac{1}{2} \partial_\mu \frac{h_{++}}{(n_+\partial)^2} \right) + \dots$$

Fixed-line Normal Coordinates

[Beneke, PH, Szafron 2112.04983]

- In QCD: fixed-line gauge $(x - x_-)^\mu A_\mu(x) = 0$.
- Gravitational analogue: Riemann Normal Coordinates.
- Generalise to light-front expansion $(x - x_-)^\alpha (x - x_-)^\beta \Gamma^\mu{}_{\alpha\beta}(x) = 0\,.$
- Construct normal coordinates only in the transverse directions $x_\perp$ and $n_- x$.
- Results in unconstrained components $g_{\mu\nu}(x_-)$, $\Gamma^\mu{}_{--}(x_-)$.
- Second and higher order are expressed via Riemann tensor.

Impact on metric tensor

- Can identify a residual dynamic soft background metric $\hat{g}_{s\mu\nu}$:

$$\begin{aligned}
\hat{g}_{s-+} &= e_{-+} - (x-x_-)^\alpha \left[\omega_-\right]_{\alpha+} \\
\hat{g}_{s-\mu_\perp} &= e_{-\mu_\perp} - (x-x_-)^\alpha \left[\omega_-\right]_{\alpha\mu_\perp} \\
\hat{g}_{s--} &= \left(e_-{}^\alpha - (x-x_-)^\rho \left[\omega_-\right]_\rho{}^\alpha\right)\left(e_-{}^\beta - (x-x_-)^\sigma \left[\omega_-\right]_\sigma{}^\beta\right)\eta_{\alpha\beta} \\
\hat{g}_{s\mu_\perp\nu_\perp} &= \eta_{\mu_\perp\nu_\perp}
\end{aligned}$$

- Determined in terms of vierbein $e_-{}^\alpha(x_-)$ and spin-connection $\left[\omega_-\right]_{\alpha\beta}(x_-)$.
- These fields are independent when viewed from the EFT.
- Can be arranged in a soft-covariant derivative

$$n_- D_s = \hat{E}_s{}^\mu{}_- \partial_\mu \,.$$

- Quadrupole and higher-pole terms: expressed via Riemann tensor.

Main Takeaway

"Homogeneous" symmetry in Gravity consists of local Poincaré transformations. This implies a covariant derivative that contains the momentum as well as the angular momentum. All other interaction terms due to multipole-expansion are expressed via Riemann tensor and its derivative.

Schematically, the scalar-soft graviton Lagrangian takes the form

$$\mathcal{L}_{\hat{\varphi}\hat{\varphi}s} = \frac{1}{2} n_+\partial\hat{\varphi} n_- D_s \hat{\varphi} + \frac{1}{2}\partial_\perp\hat{\varphi}\partial_\perp\hat{\varphi} - \frac{\kappa}{8} x_\perp^\alpha x_\perp^\beta R_{\alpha-\beta-} n_+\partial\hat{\varphi} n_+\partial\hat{\varphi} + \mathcal{O}(\lambda^3)\,,$$

where

$$n_- D_s = n_-\partial - \underbrace{\frac{\kappa}{2} s_{-\alpha}\partial^\alpha}_{\text{from vierbein}} - \underbrace{\frac{\kappa}{4}\left(\partial_\alpha s_{\beta-} - \partial_\beta s_{\alpha-}\right)}_{\text{from spin-connection}} J^{\alpha\beta} + \mathcal{O}(s^2)$$

$$J^{\alpha\beta} = (x - x_-)^\alpha \partial^\beta - (x - x_-)^\beta \partial^\alpha$$

What about Fermions?

[Beneke, PH, Schwienbacher 2212.02525]

- Half-integer spin fields come with additional local Lorentz symmetry.
- Takes the form of standard gauge transformation

$$\psi \to D_\Lambda(x)\psi \,.$$

- Standard Wilson line exists

$$W(z,x) = \mathbf{P}\exp\left[i\int_{\mathcal{C}(x,z)} \mathrm{d}y^\mu\, \Omega_\mu(y)\right] ,$$

$$W(z,x) \to D_\Lambda(z)W(z,x)D_\Lambda^{-1}(x) \,.$$

- Gravity is implemented via vierbein ${e^a}_\mu$ and spin-connection Ω_μ

$$D_\mu = \partial_\mu - i\Omega_\mu \,.$$

LLT transformations

- Collinear fluctuation on top of a soft background:

collinear:

$$\Omega_{c\mu} \to D_{\Lambda_c} \Omega_{c\mu} D_{\Lambda_c}^{-1} + i D_{\Lambda_c} \left[\partial_\mu - i\Omega_{s\mu}, D_{\Lambda_c}^{-1}\right] ,$$

$$\Omega_{s\mu} \to \Omega_{s\mu} ,$$

$$e_c{}^a{}_\mu \to \Lambda_c{}^a{}_b e_c{}^b{}_\mu + \left(\Lambda_c{}^a{}_b e_s{}^b{}_\mu - e_s{}^a{}_\mu\right) ,$$

$$e_s{}^a{}_\mu \to e_s{}^a{}_\mu ,$$

soft:

$$\Omega_{c\mu}(x) \to D_{\Lambda_s}(x) \Omega_{c\mu}(x) D_{\Lambda_s}^{-1}(x) ,$$

$$\Omega_{s\mu} \to D_{\Lambda_s} \Omega_{s\mu} D_{\Lambda_s}^{-1} + i D_{\Lambda_s} \left[\partial_\mu D_{\Lambda_s}^{-1}\right] ,$$

$$e_c{}^a{}_\mu(x) \to \Lambda_s{}^a{}_b(x) e_c{}^b{}_\mu(x) ,$$

$$e_s{}^a{}_\mu \to \Lambda_s{}^a{}_b e_s{}^b{}_\mu .$$

- Spin-connection Ω_μ transforms like a gauge field.
- Vierbein $e^a{}_\mu$ transforms similar to the graviton.

Implementing Fermions

- First implement the diffeomorphisms (GCT) consistently.
- The additional LLT must also be homogenised

$$\psi_c(x) \to D_{\Lambda_s}(x_-)\psi_c(x)\,.$$

- Employ two more Wilson lines, analogue of W_c and R

$$W_{c,\mathrm{LLT}}(x) = \mathbf{P}\exp\left(i\int_{-\infty}^{0} \mathrm{d}s\, n_+\hat{\Omega}_c(x+sn_+)\right)\,,$$
$$R_{\mathrm{LLT}}(x) = \mathbf{P}\exp\left(i\int_{0}^{1} \mathrm{d}s\,(x-x_-)^\mu \Omega_{s\mu}(x+s(x-x_-))\right)\,.$$

- Problem: GCT-Redefined fields do not transform with $D_\Lambda(x)$.

Proper Wilson lines

- GCT-redefined fields are translated by θ_{LC}

$$\left[W_c^{-1}\hat{\psi}_c\right] \to \left[W_c^{-1}D_{\Lambda_c}(x)\hat{\psi}_c\right] \equiv D_{\Lambda_c}(x+\theta_{\mathrm{LC}}(x))\left[W_c^{-1}\hat{\psi}_c\right] .$$

- Wilson lines must also be translated

$$V_c(x) = \mathbf{P}\exp\left(i\int_{-\infty}^{0} \mathrm{d}s\, n_+^\mu \hat{\Omega}_{c\mu}(x+sn_+ + \theta_{\mathrm{LC}}(x+sn_+))\right)$$
$$= \mathbf{P}\exp\left(i\int_{-\infty}^{0} \mathrm{d}s\, n_+^\mu W^\rho{}_\mu \left[W_c^{-1}\hat{\Omega}_{c\rho}\right](x+sn_+)\right).$$

- This is simply the collinear GCT-invariant spin-connection.
- Same for R: Ω_s in FLNC in exponent, yields soft Wilson line V_s.

Redefinitions

- Use all four Wilson lines to redefine matter and vierbein fields

$$\psi_c = \left[R V_s \, V_c{}^{-1} W_c^{-1} \left(\hat{\xi} + \hat{\zeta} \right) \right]$$

$$e_c{}^a{}_\mu(x) = \left[R R_\mu{}^\alpha [V_s]^a{}_b \left(\left[[V_c\,]_d{}^b \, W^\rho{}_\alpha \, W_c{}^{-1} \left(\hat{e}_s{}^d{}_\rho(x) + \hat{e}_c{}^d{}_\rho(x) \right) \right] - \hat{e}_s{}^b{}_\alpha(x) \right) \right]$$

- Residual background vierbein

$$\check{e}_s{}^a{}_\mu(x) = [V_s]_b{}^a R^\nu{}_\mu \left[R^{-1} e_s{}^b{}_\nu(x) \right]$$

contains emergent background and manifestly invariant Riemann terms.

- Precise form depends on FLNC construction.

Soft background

- The soft Wilson lines implement the emergent background vierbein

$$\hat{e}_s{}^a{}_\mu(x) = \delta_\perp{}^a{}_\mu + \frac{n_{-\mu}}{2} n_+^a + \frac{n_{+\mu}}{2}\left(e_-{}^a + (x - x_-)^\beta \omega_-{}^a{}_\beta\right) .$$

- This describes the same background metric $\hat{g}_{s\mu\nu}$ as found previously

$$\hat{g}_{s\mu\nu} = \hat{e}_s{}^a{}_\mu \hat{e}_s{}^b{}_\nu \eta_{ab} .$$

- Covariant derivative now contains the spin

$$n_- D_s = \partial_- - \frac{1}{2} s_-{}^\mu \partial_\mu + \frac{1}{2}\omega_{-\mu\rho}\left(L^{\mu\rho} + i\Sigma^{\mu\rho}\right) + \mathcal{O}(\lambda^4) .$$

- Framework consistently extended to arbitrary spin.

Fermionic Lagrangian

$$\mathcal{L}^{(0)} = \bar{\hat{\chi}}_c \frac{\not{n}_+}{2} \left(i n_- D_s + i\not{\partial}_\perp \frac{1}{i n_+ \partial} i\not{\partial}_\perp \right) \hat{\chi}_c \,,$$

$$\mathcal{L}^{(1)} = -\frac{1}{4} \bar{\hat{\chi}}_c \frac{\not{n}_+}{2} \left(\left\{ n_-^a \hat{\mathfrak{h}}_a{}^\mu \,, i\partial_\mu \right\} + \gamma_\perp^a \left\{ \hat{\mathfrak{h}}_a{}^\rho \,, i\partial_\rho \right\} \frac{1}{i n_+ \partial} i\not{\partial}_\perp + i\not{\partial}_\perp \frac{1}{i n_+ \partial} \gamma_\perp^a \left\{ \hat{\mathfrak{h}}_a{}^\rho \,, i\partial_\rho \right\} \right) \hat{\chi}_c \,.$$

- Covariant derivative contains the full angular momentum $L^{\mu\nu} + S^{\mu\nu}$.
- $\mathcal{L}^{(1)}$ contains subleading purely-collinear interactions.
- Starting at $\mathcal{O}(\lambda^2)$, the Riemann tensor appears.

Lagrangian at $\mathcal{O}(\lambda^2)$

$$\mathcal{L}^{(2)} = \bar{\hat{\chi}}_c \frac{\not{n}_+}{2}\Bigg(\frac{3}{16}\{n_-^a \hat{\mathfrak{h}}_{a\alpha}\hat{\mathfrak{h}}^{\alpha\mu}, i\partial_\mu\} + \frac{3}{16}\Big\{i\not{\partial}_\perp \frac{1}{in_+\partial}, \{\gamma_\perp^a \hat{\mathfrak{h}}_{a\alpha}\hat{\mathfrak{h}}^{\alpha\mu}, i\partial_\mu\}\Big\}$$

$$+ \frac{1}{16}\gamma_\perp^a\{\hat{\mathfrak{h}}_a{}^\mu, i\partial_\mu\}\frac{1}{in_+\partial}\gamma_\perp^b\{\hat{\mathfrak{h}}_b{}^\nu, i\partial_\nu\}$$

$$- \frac{1}{32}\Bigg(\hat{\mathfrak{h}}^\lambda{}_a[i\partial_b\hat{\mathfrak{h}}_{c\lambda}](n_-^c\,[\gamma_\perp^a, \gamma_\perp^b] + n_-^b\,[\gamma_\perp^c, \gamma_\perp^a] + n_-^a\,[\gamma_\perp^b, \gamma_\perp^c])$$

$$+ \Big[i\not{\partial}_\perp \frac{1}{in_+\partial}, \hat{\mathfrak{h}}^\lambda{}_a[in_+\partial\hat{\mathfrak{h}}_{c\lambda}](\gamma_\perp^c n_-^a - \gamma_\perp^a n_-^c)\Big]$$

$$+ i\not{\partial}_\perp \frac{1}{in_+\partial}\hat{\mathfrak{h}}^\lambda{}_a[in_+\partial\hat{\mathfrak{h}}_{c\lambda}]\,[\gamma_\perp^a, \gamma_\perp^c]\,\frac{1}{in_+\partial}i\not{\partial}_\perp\Bigg)\Bigg)\hat{\chi}_c$$

$$+ \frac{1}{4}x_\perp^\alpha x_\perp^\beta R_{-\alpha-\beta}\bar{\hat{\chi}}_c\frac{\not{n}_+}{2}in_+\partial\hat{\chi}_c - \left(\frac{1}{4}\bar{q}\left\{\hat{\mathfrak{h}}^\mu{}_a, i\partial_\mu\right\}\gamma_\perp^a\hat{\chi}_c + \text{h.c.}\right).$$

Conclusion

- Derived rigorously the SCET for Gravity to subleading order for half-integer fields.
- Same framework also consistently implements other internal gauge symmetries.
- Effective theory is covariant with respect to the emergent gauge symmetry described by ${e_-}^{\alpha}$ and $\omega_{-\alpha\beta}$.
- Covariant derivative naturally generalises to include spin of fermion fields.
- Explains form and origin of the subleading term of the soft theorem for arbitrary spin.
- This concludes the construction of the effective theory.