$$T = \max_{\mathbf{n}} \frac{\sum_i |\mathbf{p_i} \cdot \mathbf{n}|}{\sum_i |\mathbf{p_i}|}$$

Resummation of Sudakov Shoulder Logarithms in Heavy Jet Mass

$$\rho = \frac{1}{Q^2} \max\{m_L^2, m_H^2\}$$

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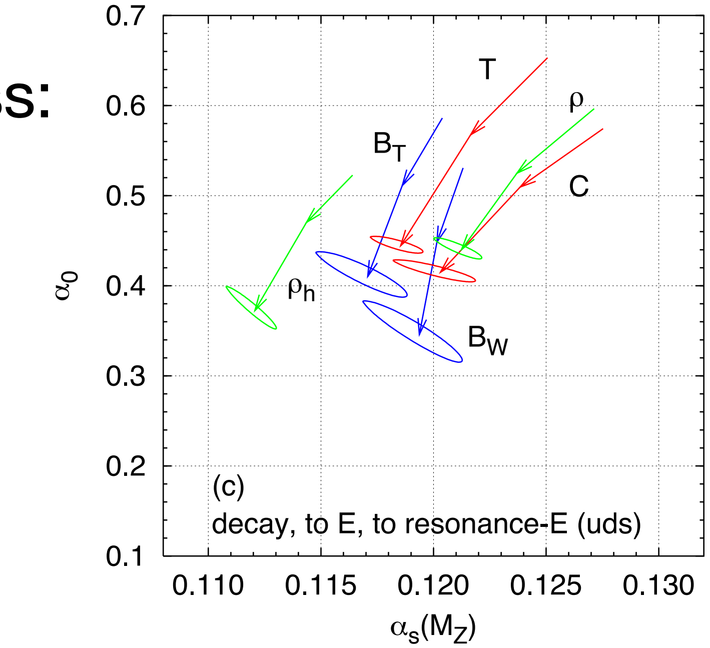
Based on:

- [Arindam Bhattacharya, Matthew Schwartz, XYZ, 2205.05702]
- [Arindam Bhattacharya, Johannes Michel, Matthew Schwartz, Iain Stewart, XYZ, in progress]

Motivation

- Remaining problem in α_s measurement with heavy jet mass:



20 years ago: [Salam, Wicke, hep-ph/0102343]

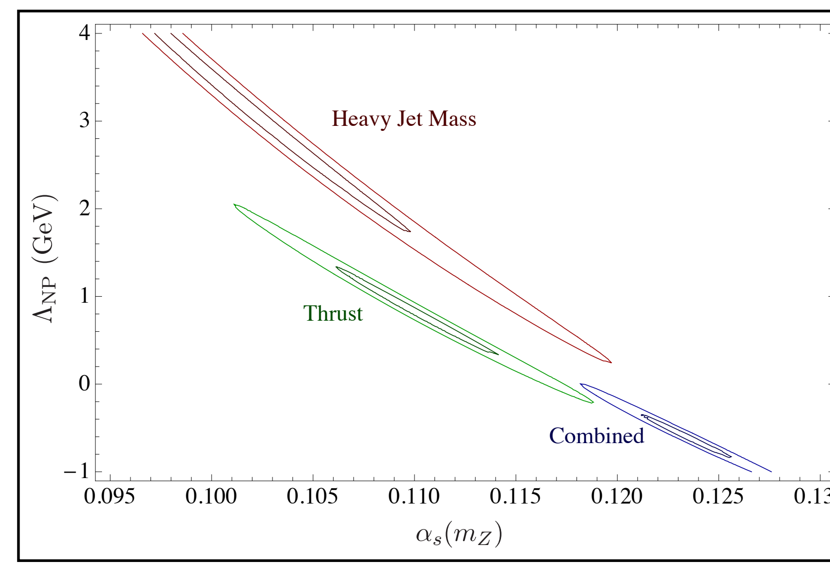
> Secondly fits for the heavy-jet mass (a very non-inclusive variable) lead to values for α_s which are about 10% smaller than for inclusive variables like the thrust or the mean jet mass. This needs to be understood. It could be due to a difference in the behaviour of the perturbation series at higher orders. But in appendix D there is evidence from Monte Carlo simulations that hadronisation corrections for ρ_h have unusual characteristics: in contrast to what is seen in more inclusive variables, the hadronisation depends strongly on the underlying hard configuration. There is therefore a need to develop techniques allowing a more formal approach to the study of such problems.

10 years ago: [Becher, Schwartz, 0803.0342]
[Chien, Schwartz, 1005.1644]

$\mathrm{N^3LL}$ dijet resummation + power correction:

Inconsistence between thrust and heavy jet mass

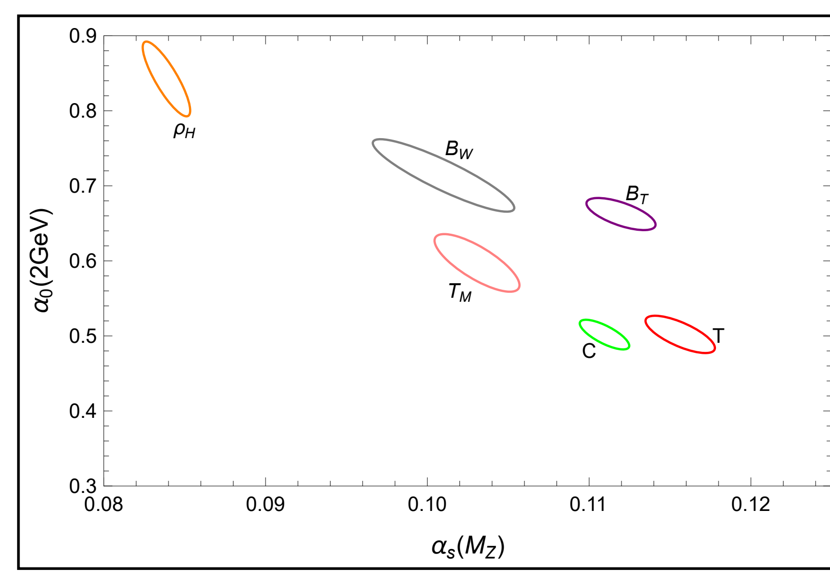
[Also Vicent Mateu's talk at SCET **2011**]



Today: [Banfi, El-Menoufi, Wood, 2303.01534]

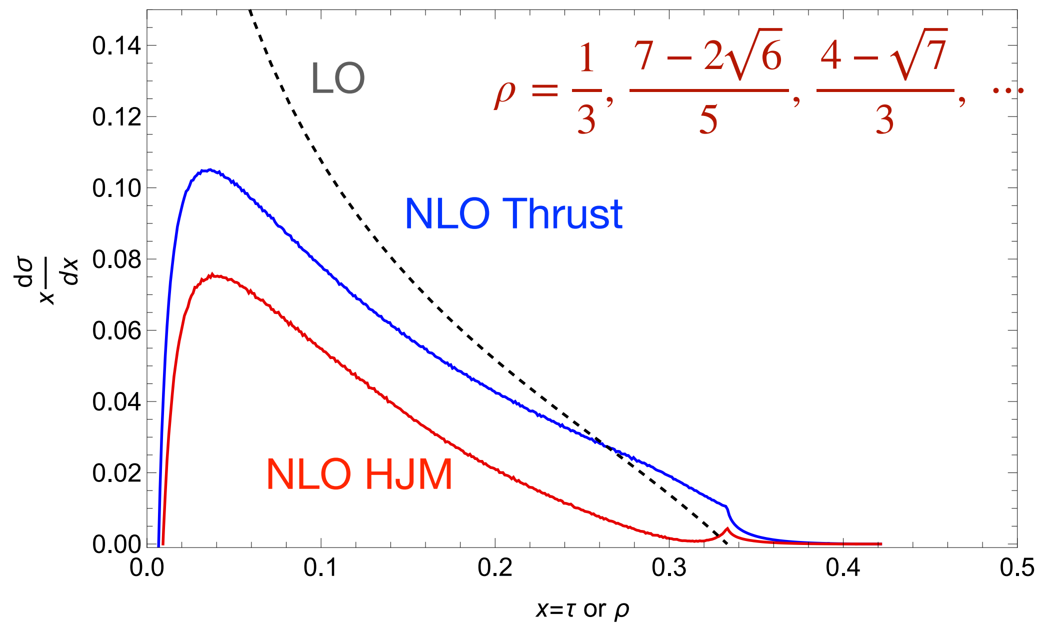
A new method to compute the leading non-perturbative corrections in dijet region:

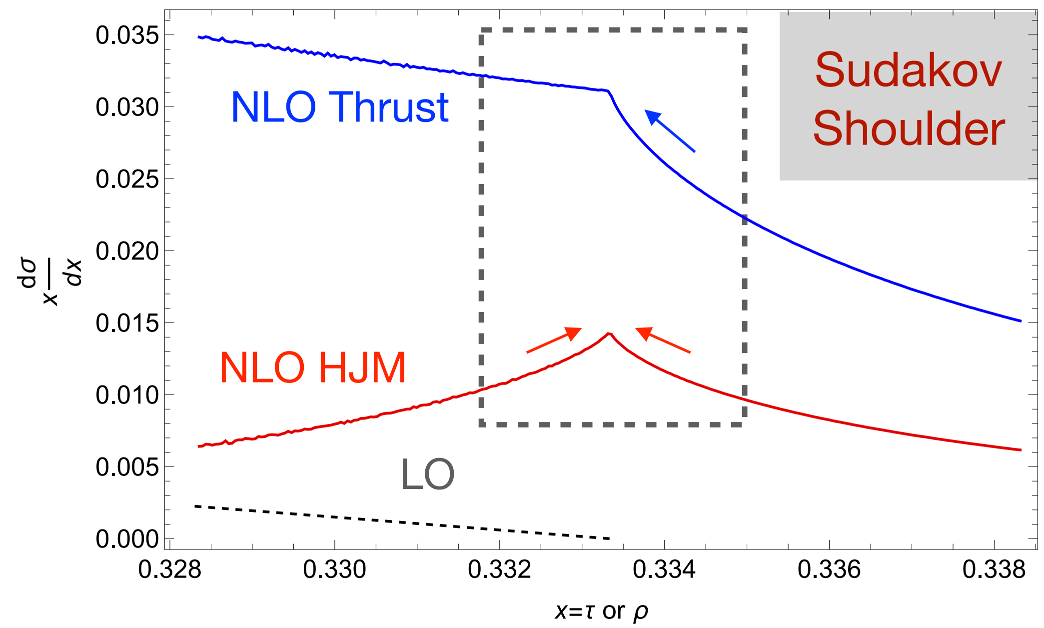
Result from HJM is still off



Thrust vs HJM

- Thrust and HJM have different kinks order by order in perturbation theory





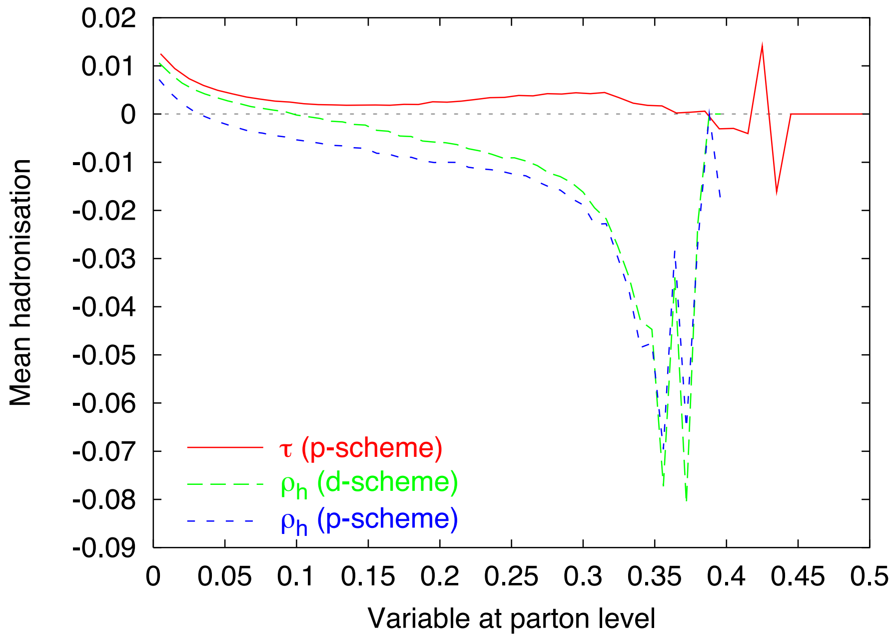
HJM left shoulder could have significant effect on the α_s measurement

- There are also non-perturbative effects:
  - Scheme dependence: E-scheme, p-scheme

    [Mateu, Stewart, Thaler, 1209.3781]
  - Hadronization corrections

    [Salam, Wicke, hep-ph/0102343]

    [Nason, Zanderighi, 2301.03607]

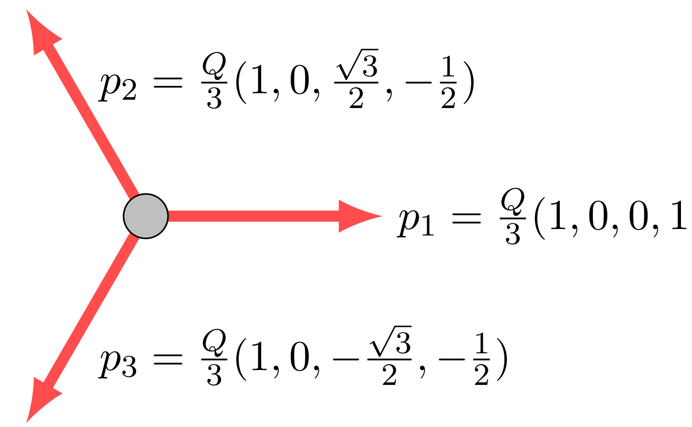


- Our first step is to understand the perturbative picture— Sudakov shoulders

Sudakov shoulders

- Sudakov shoulders arise from incomplete cancellations between the virtual corrections and real emissions, where the range of event shape grows order-by-order in perturbation theory. [Catani, Webber, hep-ph/9710333]
- Start with 3-parton configuration, the event shapes are restricted at each order:

Tree, one-loop virtual:	$C \leq \frac{3}{4}, \quad \tau, \rho \leq \frac{1}{3}$
Real emission:	$C \leq 1, \quad \tau, \rho \leq \frac{7-2\sqrt{6}}{5}$



Incomplete cancellation ⇒ divergence, kinks, etc. ⇒ large logarithms

- Fixed-order calculation gives
  - Thrust: only right shoulder

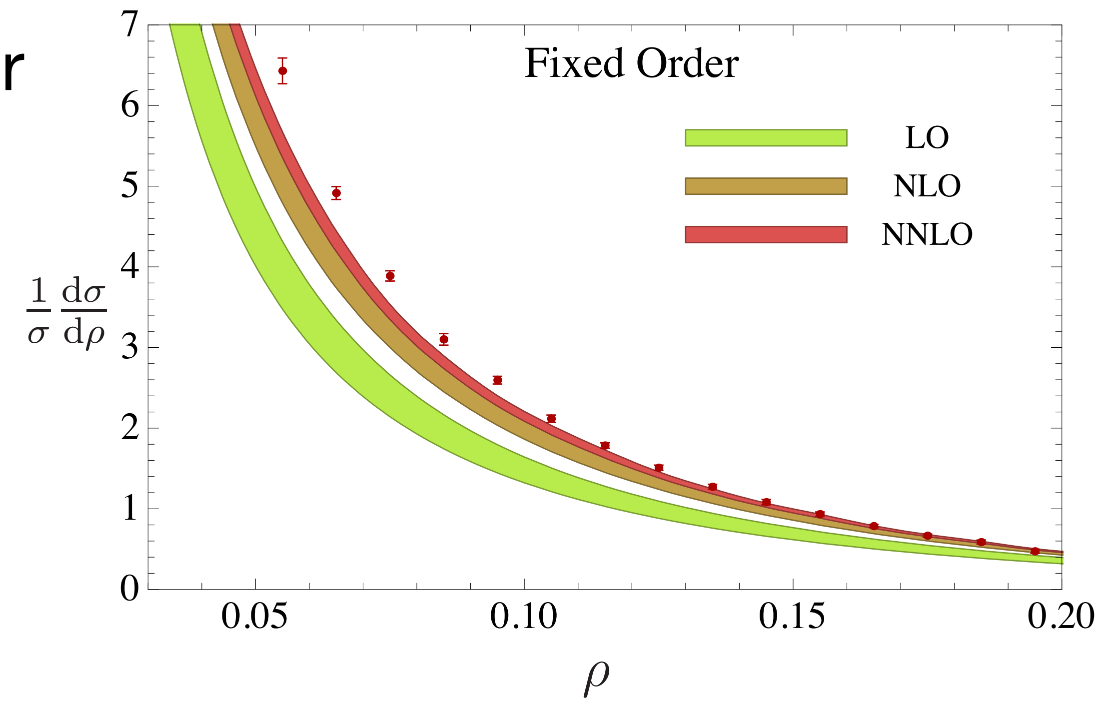
$$t = \tau - \frac{1}{3} \qquad \frac{1}{\sigma_{LO}}\frac{d\sigma}{d\tau} = \frac{\alpha_s}{4\pi}\theta(t)\left\{-6\left(2C_F + C_A\right) t\ln^2 t + \left[6C_F\left(1 - 4\ln 3\right) + C_A\left(1 - 12\ln 3\right) + 4n_f T_F\right] t\ln t\right\}$$

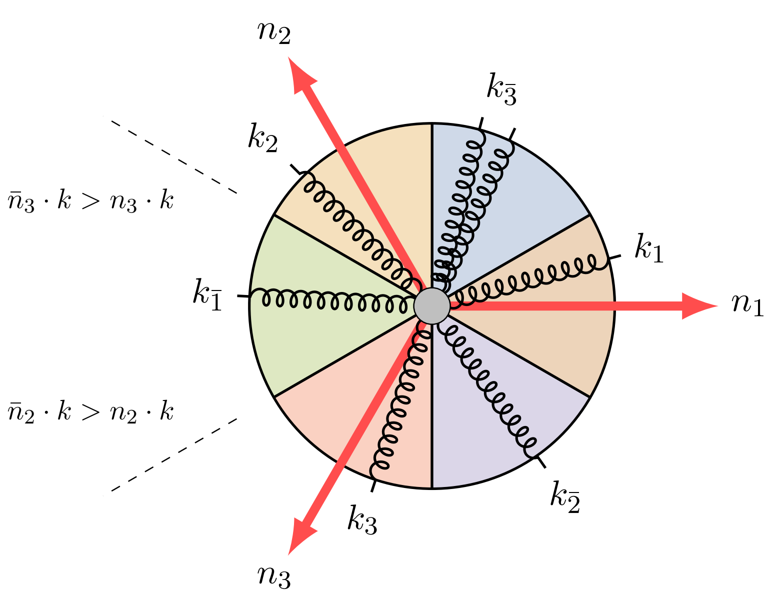
  - HJM: left shoulder (affects the α_s fit!) and right shoulder

$$r = \frac{1}{3} - \rho \qquad \frac{1}{\sigma_{LO}}\frac{d\sigma}{d\rho} = \frac{\alpha_s}{4\pi}\theta(r)\left\{-2\left(2C_F + C_A\right) r\ln^2 r + \left[2C_F\left(1 + 4\ln\frac{4}{3}\right) + C_A\left(\frac{1}{3} + 4\ln\frac{4}{3}\right) + \frac{4}{3}n_f T_F\right] r\ln r\right\}$$
$$+\frac{\alpha_s}{4\pi}\theta(-r)\left\{-4\left(2C_F + C_A\right)(-r)\ln^2(-r) + \left[4C_F\left(1 - 4\ln 6\right) + 2C_A\left(\frac{1}{3} - 4\ln 6\right) + \frac{8}{3}n_f T_F\right](-r)\ln(-r)\right\}$$

Outline

- Previous work: [Arindam's talk at SCET 2022]
  - Fixed-order calculation near the shoulder
  - Factorization theorem
  - Trijet hemisphere soft function
  - Sudakov Landau poles
- Current work:
  - Resummation in Fourier space
  - ρ_L subtraction scheme
  - Shoulder profile and result
  - Matched to dijet resummation





Previous work:
shoulder factorization

With Arindam Bhattacharya and Matthew Schwartz

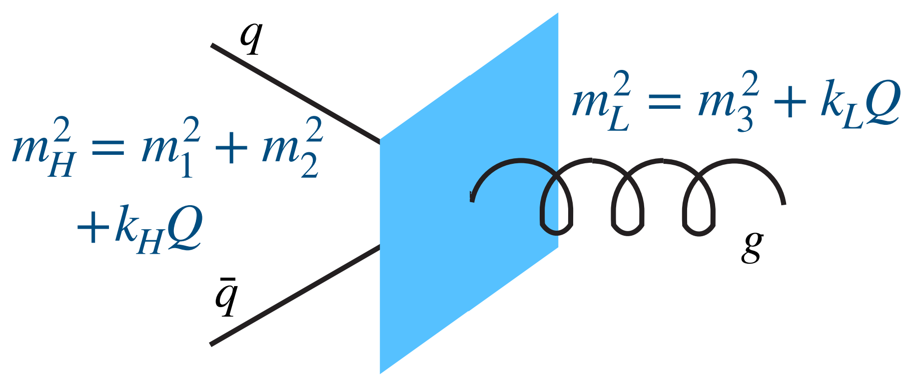
- Factorization theorem
- Trijet hemisphere soft function
- Sudakov Landau poles

Shoulder factorization theorem

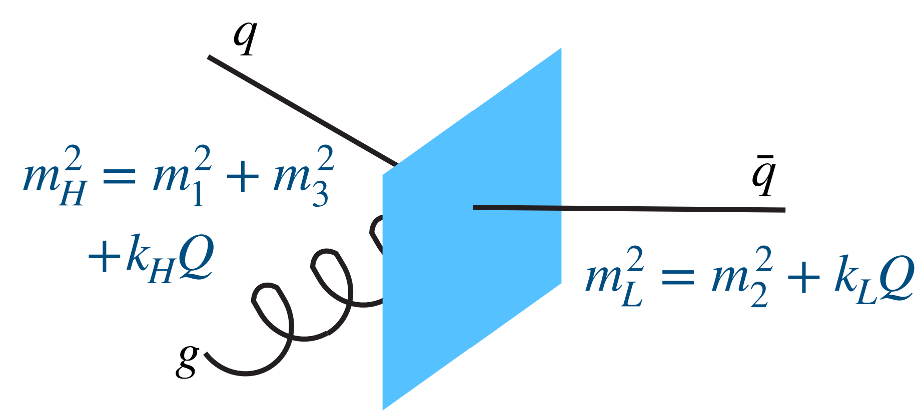
$$\frac{d\sigma_i}{dx} = \sigma_{LO} H(Q) \int dm_1^2 dm_2^2 dm_3^2 dk_L dk_H J_q(m_1^2) J_q(m_2^2) J_g(m_3^2) S_i^{(x)}(k_L, k_H) \times M_t^{(x)} \Theta\left(M_t^{(x)}\right)$$

- σ_{LO}: LO phase space
- $H(Q)$: Trijet hard function
- $J_q(m_1^2) J_q(m_2^2) J_g(m_3^2)$: Inclusive jet function
- $S_i^{(x)}(k_L, k_H)$: Trijet hemisphere soft function
- $M_t^{(x)} \Theta\left(M_t^{(x)}\right)$: Measurement

$i = g$:



$i = q, \bar{q}$:



HJM: $M_t^{(\rho)} = \frac{1}{3} - \rho - m_L^2 + m_H^2$

Thrust: $M_t^{(\tau)} = \tau - \frac{1}{3} - m_L^2 - m_H^2$

- The factorization theorem is derived from SCET_I and trijet kinematics
- For HJM, this measurement is valid for both left shoulder ($\rho < 1/3$) and right shoulder ($\rho > 1/3$)
- New ingredient needed: six-directional differential soft function, integrated to the trijet hemisphere soft function

Trijet hemisphere soft function

- Definition of differential soft function

$$S_{6i}(q_i) = 2g_s^2\mu^{2\epsilon}\int \frac{d^dk}{(2\pi)^{d-1}}\delta^+(k^2)\mathscr{H}(k,q_i)\times\left[C_{23}\frac{n_2\cdot n_3}{(n_2\cdot k)(n_3\cdot k)}+C_{12}\frac{n_1\cdot n_2}{(n_1\cdot k)(n_2\cdot k)}+C_{13}\frac{n_1\cdot n_3}{(n_1\cdot k)(n_3\cdot k)}\right]$$

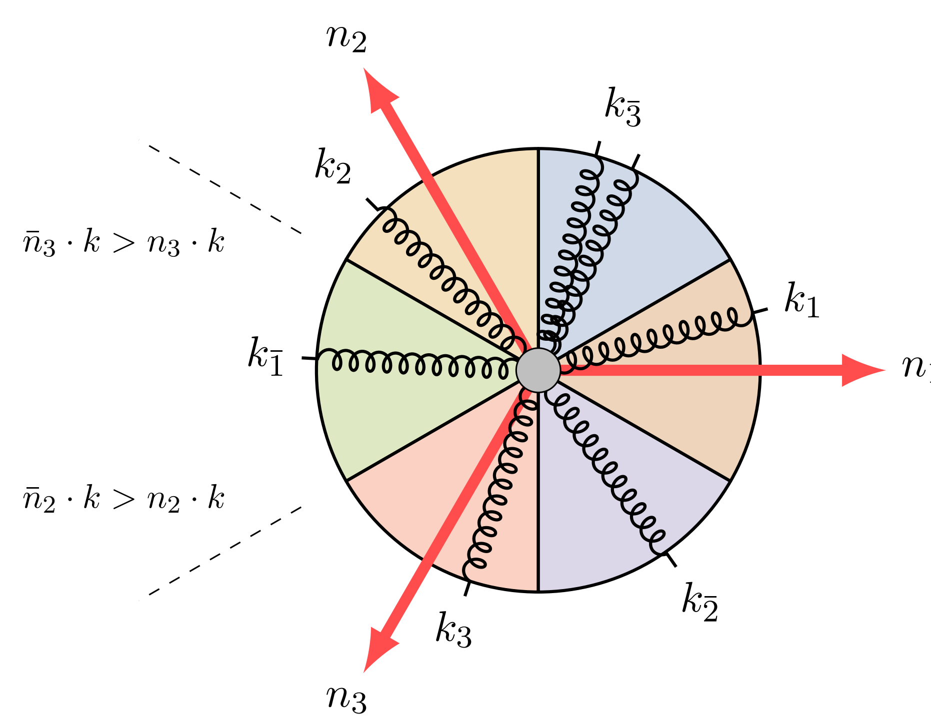
- From thrust axis constraint (trijet kinematics):

$$m_1^2+\frac{2Q}{3}\left(n_1\cdot k_1+N_2\cdot k_{\bar{2}}+N_3\cdot k_{\bar{3}}\right)<\frac{1}{3}-\rho+m_2^2+m_3^2+\frac{2Q}{3}\left(n_2\cdot k_2+n_3\cdot k_3+\bar{n}_1\cdot k_{\bar{1}}\right)$$

→ soft projections

$$\mathscr{H}(k,q_i)=\theta\left(n_2\cdot k-\bar{n}_2\cdot k\right)\theta\left(n_3\cdot k-\bar{n}_3\cdot k\right)\delta\left(q_1-\frac{2}{3}n_1\cdot k\right) + \text{other five terms}$$

- For HJM, $N_2=\left(1,0,+\frac{\sqrt{3}}{2},\frac{3}{2}\right)$, $N_3=\left(1,0,-\frac{\sqrt{3}}{2},\frac{3}{2}\right)$
- For thrust, $N_2=\bar{n}_2$, $N_3=\bar{n}_3$



Trijet hemisphere soft function

- One-loop calculation
  - Topology identification (7 master integrals)
  - Numerically evaluate them and analytically reconstruct some numbers with PSLQ

Pentagon functions [Chicherin, Sotnikov, 2209.07803]

Weight	Parity-even	Parity-odd
One	$\ln 2, \ln 3$	$i\pi$
Two	$Li_2\left(\frac{2}{3}\right)$	$iIm\left[Li_2 e^{\frac{i\pi}{3}}\right]$ Gieseking's constant
Three	$Li_3\left(\frac{2}{3}\right), Li_3\left(\frac{1}{4}\right), \zeta_3$	$iIm\left[Li_3\left(\frac{i}{\sqrt{3}}\right)\right], iIm\left[Li_3\left(1+i\sqrt{3}\right)\right]$

- Integrating the differential soft function:

Non-global logs beyond NNLL

$$S_i(q_L, q_H, \mu) = \int d^6 q_i S_{6i}(q_i, \mu)\delta\left(q_L - q_1 - q_{\bar{2}} - q_{\bar{3}}\right)\delta\left(q_H - q_{\bar{1}} - q_2 - q_3\right) = S_{iL}(q_L, \mu) S_{iH}(q_H, \mu) S_f(q_L - q_H)$$

Similar to dijet resummation, at NNLL we only need $\mathcal{O}(\alpha_s)$ soft constant and we put it in $S_{iL}(q_L, \mu)$ and $S_{iH}(q_H, \mu)$

$$\frac{d\sigma_g}{d\rho} = \sigma_{LO} H(Q) \int dm_L^2 dm_H^2 \underbrace{\int dm_{1,2}^2 dk_H J_q(m_1^2) J_q(m_2^2) S_{iH}^{(\rho)}(k_H)\delta(m_H^2 - m_1^2 - m_2^2 - k_H Q)}_{K_H(m_H^2)}$$
$$\times \underbrace{\int dm_3^2 dk_L J_g(m_3^2) S_{iL}^{(\rho)}(k_L)\delta(m_L^2 - m_3^2 - k_L Q)}_{K_L(m_L^2)} \times \left(\frac{1}{3} - \rho - m_L^2 + m_H^2\right)\Theta\left(\frac{1}{3} - \rho - m_L^2 + m_H^2\right)$$

where $K_{L,H}(m^2)$ RGE can be solved in the Laplace space respectively

Sudakov Landau poles

- Resummation in the momentum space

Left shoulder: $r = \frac{1}{3} - \rho > 0$

$$\frac{1}{\sigma_{LO}}\frac{d\sigma_i}{d\rho} = \Pi_i(\partial_{\eta_l},\partial_{\eta_h}) r \left(\frac{rQ}{\mu_s e^{-\gamma_E}}\right)^{\eta_l+\eta_h} \frac{\sin(\pi\eta_l)}{\pi}\Gamma\left(-1-\eta_l-\eta_h\right)$$

Right shoulder: $s = \rho - \frac{1}{3} > 0$

$$\frac{1}{\sigma_{LO}}\frac{d\sigma_i}{d\rho} = \Pi_i(\partial_{\eta_l},\partial_{\eta_h}) s \left(\frac{sQ}{\mu_s e^{-\gamma_E}}\right)^{\eta_l+\eta_h} \frac{\sin(\pi\eta_h)}{\pi}\Gamma\left(-1-\eta_l-\eta_h\right)$$

$$\eta_l^{(g)} = 2C_A A_\Gamma(\mu_j,\mu_s)$$
$$\eta_h^{(g)} = 4C_F A_\Gamma(\mu_j,\mu_s)$$

With RG kernel following

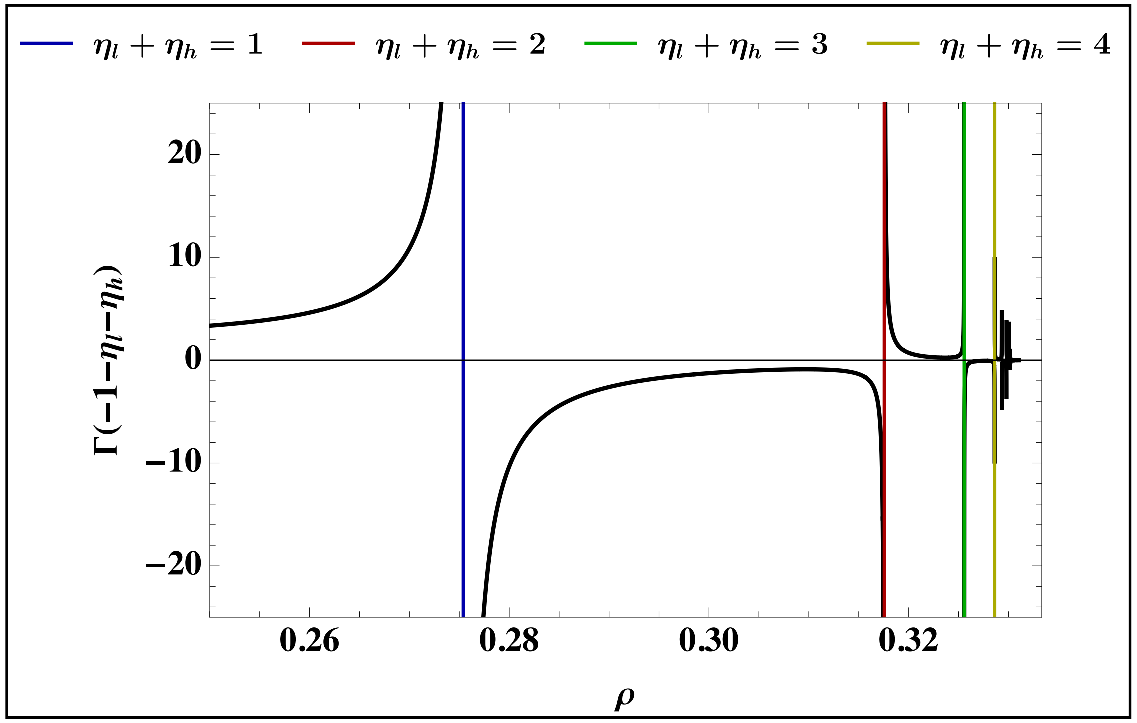
$$\Pi_g(\partial_{\eta_l},\partial_{\eta_h}) = \exp\left[4C_F S(\mu_h,\mu_j) + 4C_F S(\mu_s,\mu_j) + 2C_A S(\mu_h,\mu_j) + 2C_A S(\mu_s,\mu_j)\right]\exp\left[2A_{\gamma_{sg}}(\mu_s,\mu_h) + 2A_{\gamma_{sqq}}(\mu_s,\mu_h) + 2A_{\gamma_{jg}}(\mu_j,\mu_h) + 4A_{\gamma_{jq}}(\mu_j,\mu_h)\right]$$
$$\times\left(\frac{Q^2}{\mu_h^2}\right)^{-2A_\Gamma(\mu_h,\mu_j)} H(Q,\mu_h)\tilde{j}_q\left(\partial_{\eta_h} + \ln\frac{Q\mu_s}{\mu_j^2}\right)\tilde{j}_{\bar{q}}\left(\partial_{\eta_h} + \ln\frac{Q\mu_s}{\mu_j^2}\right)\tilde{j}_g\left(\partial_{\eta_l} + \ln\frac{Q\mu_s}{\mu_j^2}\right)\tilde{s}_{gL}(\partial_{\eta_l})\tilde{s}_{gH}(\partial_{\eta_h})$$

- The Γ function has an infinite number of poles in the r space (referred as Sudakov Landau pole):

$$-1-\eta_l-\eta_h = \underbrace{0,-1}_{\rho<0},\underbrace{-2,-3,\cdots}_{0<\rho<\frac{1}{3}}$$

- which is similar to the q_T resummation of Drell-Yan or Higgs production in the momentum space

[Catani et al., 9604351], [Frixione, Nason, Ridolfi, 9809367], [Becher, Neubert, 1007.4005], [Monni, Re, Torrielli, 1604.02191], etc.



Current work:
NNLL resummation

With Arindam Bhattacharya, Johannes Michel,
Matthew Schwartz and Iain Stewart

- Resummation in Fourier space
- ρ_L subtraction scheme
- Shoulder profile and result
- Matched to dijet resummation

Sudakov Landau poles

- There are non-log terms when expanding our resummation formula

$$\frac{1}{\sigma_{\rm LO}}\frac{d\sigma^{\rm resum}}{d\rho} \supset \alpha_s\left[\theta(r)\left(r\ln^2 r + r\ln r + r + c_1\right) + \theta(-r)\left(-r\ln^2(-r) - r\ln(-r) - r + c_2\right)\right] + \mathcal{O}(\alpha_s^2)$$

One way to remove these linear terms is to take the second derivative $\sigma_i(r) \equiv \frac{1}{\sigma_{LO}}\frac{d^3\sigma}{d\rho^3}$

Pros:

- $\sigma_i(r)$ shares the same form as dijet logarithms $\left(\frac{\ln^n r}{r}\right)_+$
- Our eyes are not blinded by the phase space factor

Cons:

- This only removes the first two poles $-1-\eta_l-\eta_h = 0, -1$ → $\rho < 0$

- Our shoulder HJM measurement $\theta\left(M_t^{(\rho)}\right) = \theta\left(r - m_L^2 + m_H^2\right), \quad r \sim \lambda^2$

Possible hierarchies between hemisphere masses:

where EFT is valid — irrelevant regions

$m_L^2 \sim \lambda^2,\ m_H^2 \sim \lambda^2;\ r \sim \lambda^2$	$m_L^2 \sim 1,\ m_H^2 \sim \lambda^2;\ r \sim 1$	
$m_L^2 \sim 1,\ m_H^2 \sim 1;\ r \sim \lambda^2$	$m_L^2 \sim \lambda^2,\ m_H^2 \sim 1;\ r \sim 1$	$m_L^2 \sim 1,\ m_H^2 \sim 1;\ r \sim 1$

our factorization includes

Our resummation contains non-EFT contributions

Sudakov Landau poles

- Another observation: from recoil sensitivity, each ρ receives contribution from both left shoulder and right shoulder

Laplace ≠ Fourier: Traditional resummation for event shapes (like thrust) in the Laplace space doesn't work for shoulder logarithms

Fourier space is the only space that diagonalizes the δ function and allows us to resum both shoulders together

- Recall r convolution for second derivative:

$$\sigma_i(r) \propto f_i(r) = \frac{1}{\Gamma(\eta_l)\Gamma(\eta_h)} \int_0^\infty dm_L^2 \int_0^\infty dm_H^2 \, (m_L^2)^{\eta_l - 1} (m_H^2)^{\eta_h - 1} \delta\left(r - m_L^2 + m_H^2\right)$$

$$= |r|^{\eta_l + \eta_h - 1} \Gamma\left(1 - \eta_l - \eta_h\right) \left[\theta(r) \frac{\sin(\eta_l \pi)}{\pi} + \theta(-r) \frac{\sin(\eta_h \pi)}{\pi}\right]$$

Fourier transformation

$$\tilde{f}_i(y) = \int_{-\infty}^{+\infty} dr \; e^{iyr} f_i(r) = (-iy_+)^{-\eta_l} (+iy_-)^{-\eta_h}$$

$y_\pm = y \pm i\epsilon$

Alternatively, if we do the r integral first:

$$\int_{-\infty}^\infty dr \; e^{iyr} \int_0^\infty dm_L^2 \int_0^\infty dm_H^2 \, (m_L^2)^{\eta_l - 1} (m_H^2)^{\eta_h - 1} \delta\left(r - m_L^2 + m_H^2\right)$$

$$= \underbrace{\int_0^\infty dm_L^2 \, (m_L^2)^{\eta_l - 1} e^{i m_L^2 y}}_{\Im(y) > 0} \underbrace{\int_0^\infty dm_H^2 \, (m_H^2)^{\eta_h - 1} e^{-i m_H^2 y}}_{\Im(y) < 0}$$

$e^{-\epsilon(m_L^2 + m_H^2)}$ also suppresses the large mass region

Fourier space scale-setting

- Resumed second derivative

$$\tilde{\sigma}_i(y) = \int_{-\infty}^{+\infty} dr\ e^{iyr}\sigma_i(r) = \Pi_i(\partial_{\eta_l}, \partial_{\eta_h}) \times \left(-\frac{iye^{\gamma_E}\mu_s}{Q}\right)^{-\eta_l} \left(+\frac{iye^{\gamma_E}\mu_s}{Q}\right)^{-\eta_h}$$

- The canonical scale in the Fourier space is $\mu_h = Q, \quad \mu_j = \frac{Q}{\sqrt{|y|}}, \quad \mu_s = \frac{Q}{|y|}$
- To obtain the spectrum, we need to inverse Fourier transform $\tilde{\sigma}_i(y)$ and integrate over ρ (or r) twice

$$\frac{1}{\sigma_{LO}}\frac{d^3\sigma}{d\rho^3} = \int_{-\infty}^{+\infty}\frac{dy}{2\pi}e^{-iyr''}\tilde{\sigma}_i(y) = 2\Re\left[\int_0^{\infty}\frac{dy}{2\pi}e^{-iyr''}\tilde{\sigma}_i(y)\right], \quad \tilde{\sigma}_i^\star(-y) = \tilde{\sigma}_i(y)$$

$$\Rightarrow \frac{1}{\sigma_{LO}}\frac{d\sigma}{d\rho} = \int_{r_L}^{r} dr'\int_{r_L}^{r'} dr''\frac{1}{\sigma_{LO}}\frac{d^3\sigma}{d\rho^3} = 2\Re\left[\int_0^{\infty}\frac{dy}{2\pi}\left(\frac{e^{-ir_Ly} - e^{-iry}}{y^2} + i\frac{e^{-ir_Ly}(r_L - r)}{y}\right)\tilde{\sigma}_i(y)\right]$$

$$\equiv 2\Re\left[\int_0^{\infty}\frac{dy}{2\pi}K(y,r,r_L)\tilde{\sigma}_i(y)\right]$$

Kernel function

- The integration boundary $r_L = \frac{1}{3} - \rho_L$. This introduces a residual linear function.

Fixed-order matching

- Resummation matched to fixed-order: $\mu_{h,j,s}^{\rm FO} = Q$

$$\frac{d\sigma^{\rm match}}{d\rho} = \frac{d\sigma^{\rm FO}}{d\rho} + \sigma_{\rm LO}\int_{r_L}^{r} dr' \int_{r_L}^{r'} dr'' \left[\sigma^{\rm resum}(r'',\mu^{\rm res}) - \sigma^{\rm resum}(r'',\mu^{\rm FO})\right]$$
$$= \frac{d\sigma^{\rm FO}}{d\rho} + \sigma_{\rm LO} 2\Re\left\{\int_0^\infty \frac{dy}{2\pi} K(y,r,r_L)\left[\tilde{\sigma}_i(y,\mu^{\rm res}) - \tilde{\sigma}_i(y,\mu^{\rm FO})\right]\right\}$$

- But there is still residual ρ_L (or r_L) dependence
- In fact, picking a different boundary r'_L leads to an extra linear function $a_1(\alpha_s, r_L, r'_L) r + a_2(\alpha_s, r_L, r'_L)$ that starts at one order higher than fixed-order matching

- Question: can we subtract ρ_L-dependent terms to all orders in α_s?
- Our resummation gives rise to

$$\theta(r)\left[r \sum_{n\geq 0, m\geq 1} c_{n,m}\alpha_s^n \ln^m r + r l_1(\alpha_s, r_L)\right] + \theta(-r)\left[(-r)\sum_{n\geq 0, m\geq 1} d_{n,m}\alpha_s^n \ln^m(-r) + r l_2(\alpha_s, r_L)\right] + c(\alpha_s, r_L)$$

However, EFT only predicts $l_1(\alpha_s, r_L) - l_2(\alpha_s, r_L)$ so we need to subtract the artificial piece

ρ_L subtraction scheme

$$f(r, r_L) = \theta(r)\left[r \sum_{n\geq 0, m\geq 0} c_{n,m}\alpha_s^n \ln^m r\right] + \theta(-r)\left[(-r) \sum_{n\geq 0, m\geq 0} d_{n,m}\alpha_s^n \ln^m(-r)\right] + rl(\alpha_s, r_L) + c(\alpha_s, r_L)$$

linear background

- There are three special values that have access to all orders result

$$\begin{cases} f(1,r_L) = \sum_{n\geq 0} c_{n,0}\alpha_s^n + l(\alpha_s, r_L) + c(\alpha_s, r_L) \\ f(-1,r_L) = \sum_{n\geq 0} d_{n,0}\alpha_s^n - l(\alpha_s, r_L) + c(\alpha_s, r_L) \\ f(0,r_L) = c(\alpha_s, r_L) \end{cases}$$

- There are various ways to control the slope in each shoulder.
- The simplest one is to introduce a uniform parameter ξ (referred as ξ-scheme)

$$\xi \equiv \frac{\sum_{n\geq 0} c_{n,0}\alpha_s^n}{\sum_{n\geq 0}(c_{n,0} + d_{n,0})\alpha_s^n}, \quad 0 \leq \xi \leq 1$$

LO singular suggests $\xi = 1$

- It turns out that doing the subtraction is equivalent to modifying the kernel

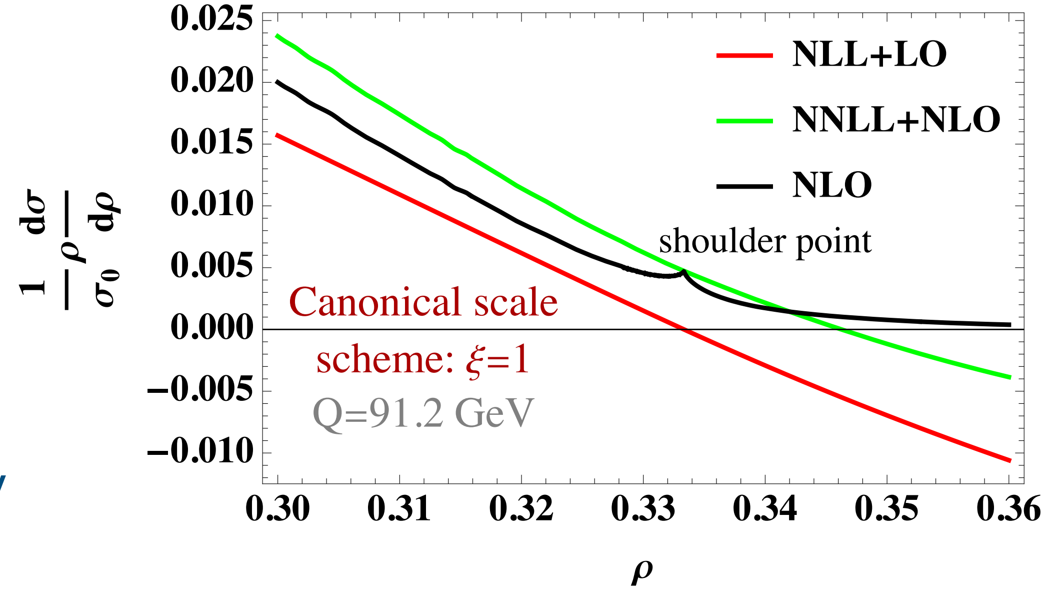
$$\tilde{K}(r, \xi) = \frac{1}{2\pi y^2}\left[1 - e^{-iyr} + (1-\xi)re^{-iy} - \xi re^{iy} + r(2\xi - 1)\right]$$

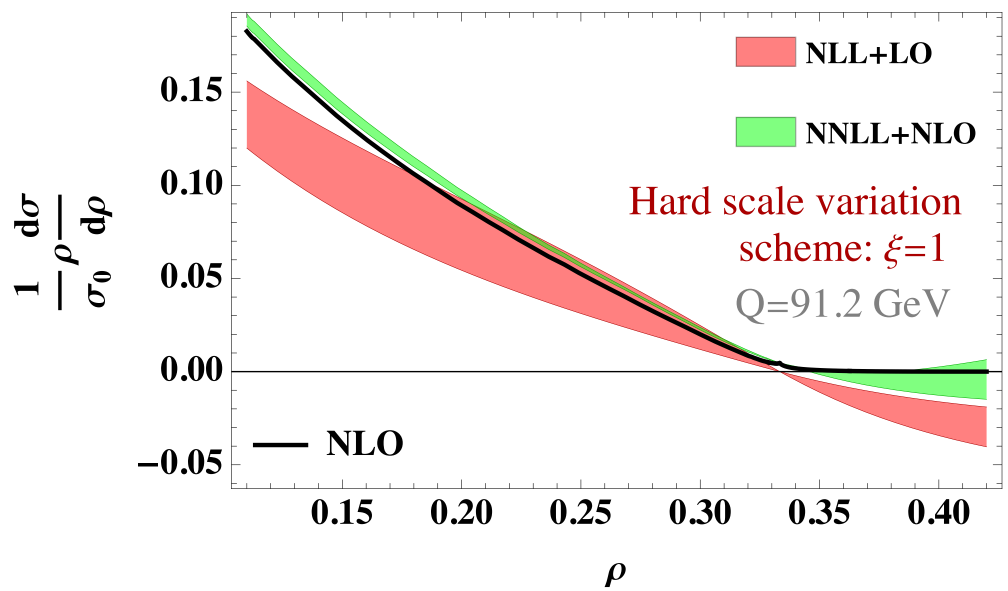
- Eventually, ξ becomes an additional source of uncertainty.

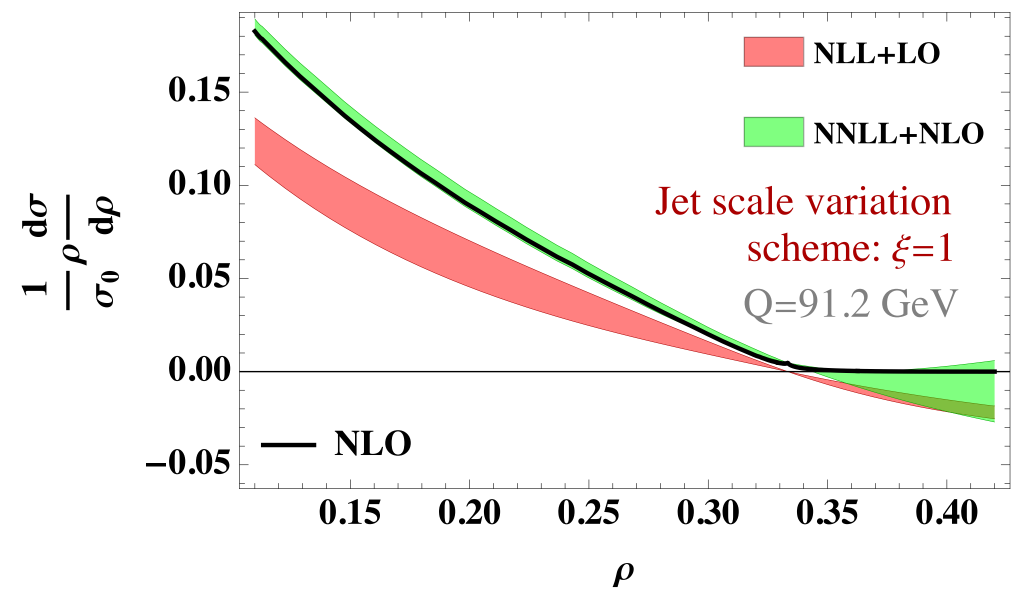
NNLL result with canonical scale

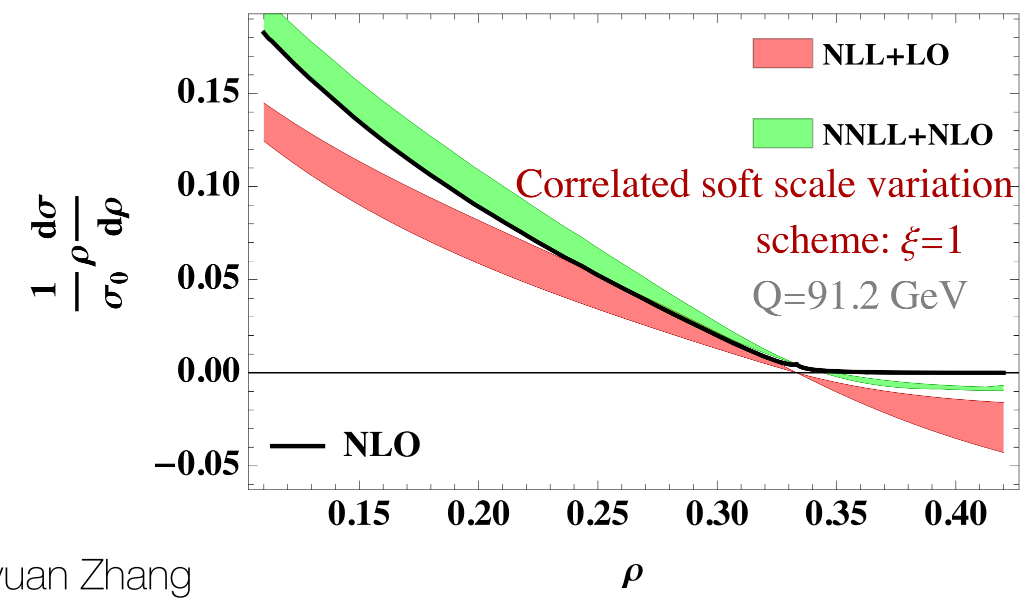
- Frozen soft scale: $\mu_s = \sqrt{(\mu_s^{\min})^2 + Q^2/y^2}, \quad \mu_s^{\min} = 2\text{GeV}$
- Central value: the kink is smoothed by resummation
- Band variation: μ_h, μ_j, correlated μ_s and ξ

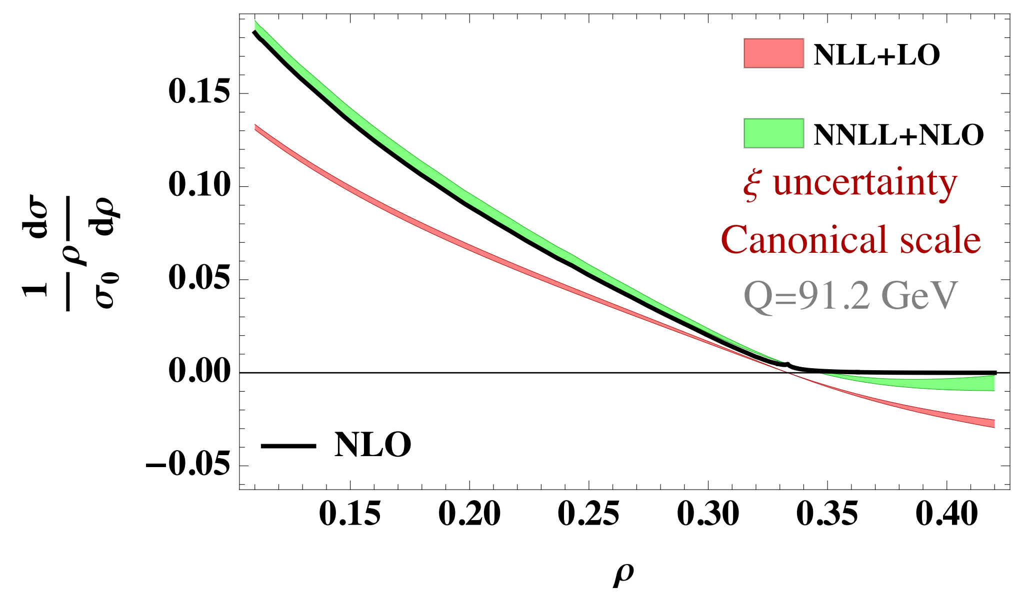
  Non-overlapping comes from fixed-order discrepancy









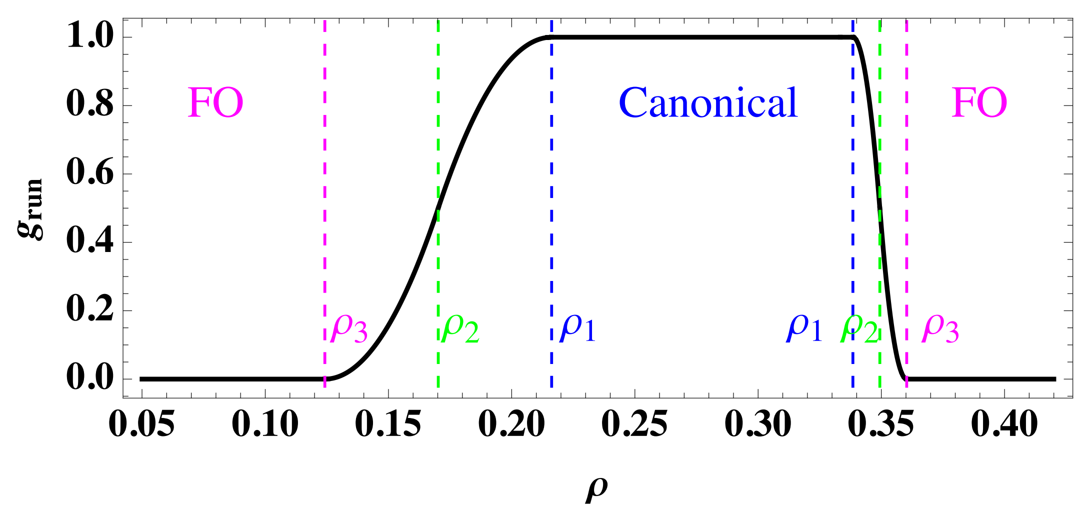


Shoulder profile scale

- We choose the q_T profile function: [Lustermans, Michel, Tackmann, Waalewijn, 1901.03331]

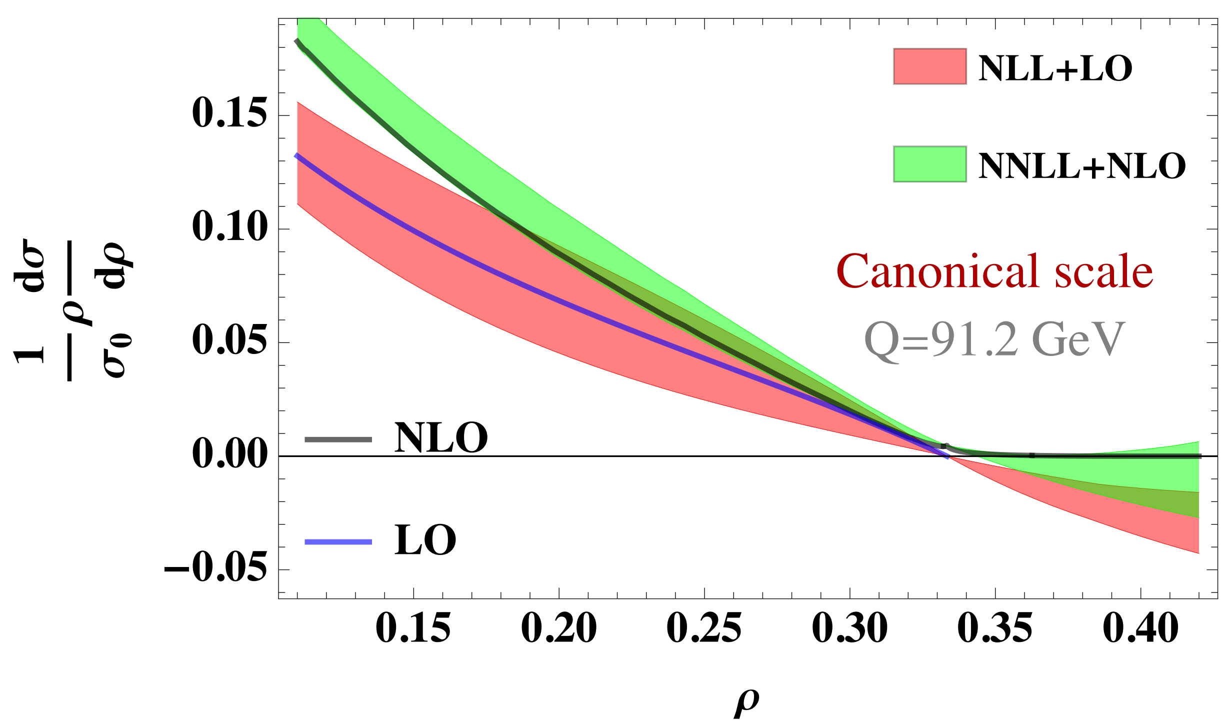
$$\mu_{j/s}^{\text{res}} = \left[\mu_{j/s}^{\text{can}}(y)\right]^{g_{\text{run}}(r)} \left[\mu_{j/s}^{\text{FO}}\right]^{1-g_{\text{run}}(r)} = \begin{cases} \mu_{j/s}^{\text{can}}(y) & \text{if } |r| < r_1^{(L,R)} \\ \mu_{j/s}^{\text{FO}} = Q & \text{if } |r| > r_3^{(L,R)} \\ \text{smooth function} & \text{else} \end{cases}$$

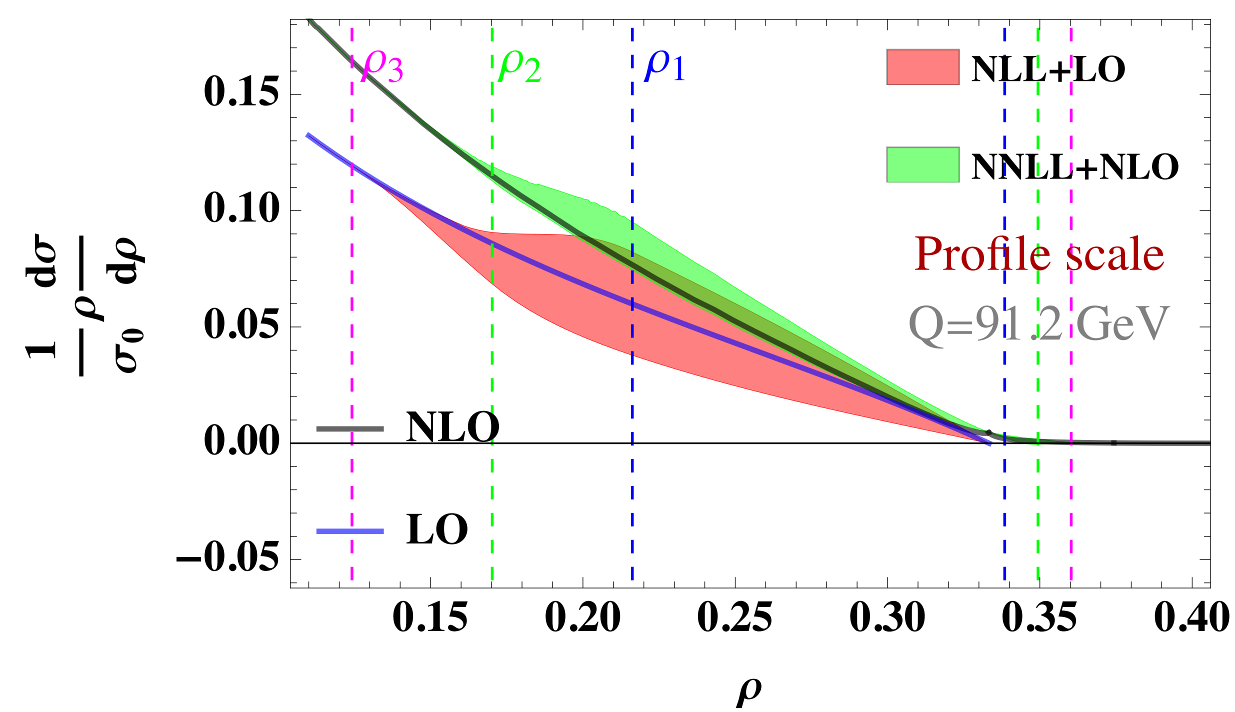
$$r_i^{(L,R)} = \frac{1}{3} - \rho_i^{(L,R)}$$



The left profile is determined by

$$\rho_3^{(L)} : \frac{d\sigma_{\text{S}}^{\text{NNLL}}}{d\rho} = \frac{1}{2}\frac{d\sigma_{\text{NS}}^{\text{NNLL}}}{d\rho}, \quad \rho_1^{(L)} : \frac{d\sigma_{\text{S}}^{\text{NNLL}}}{d\rho} = 2\frac{d\sigma_{\text{NS}}^{\text{NNLL}}}{d\rho}$$





The uncertainty is the envelope of μ_h, μ_j, correlated μ_s and ξ variations

Review: dijet resummation

[Becher, Schwartz, 0803.0342], [Chien, Schwartz, 1005.1644]

[Kelley, Schabinger, Schwartz, Zhu, 1105.3676]

- The $\mathrm{N^3LL}$ resummation

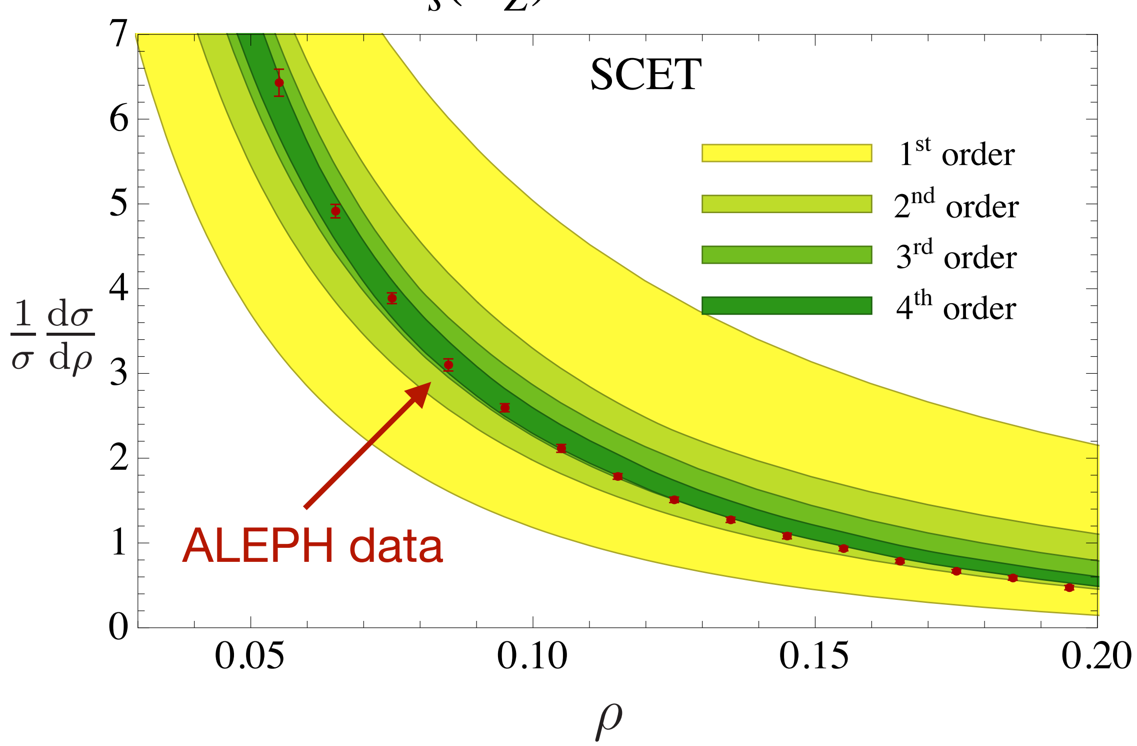
$$\frac{1}{\sigma_0}R_2^{(\rho)}(\rho) = \frac{1}{\sigma_0}\int_0^\rho d\rho' \frac{d\sigma_2}{d\rho'} = \exp\left[4C_F S(\mu_h,\mu_j) + 4C_F S(\mu_s,\mu_j) - 2A_H(\mu_h,\mu_s) + 4A_J(\mu_j,\mu_s)\right]\left(\frac{Q^2}{\mu_h^2}\right)^{-2A_\Gamma(\mu_h,\mu_j)}$$

$$\times H(Q^2,\mu_h^2) j_q\left(\partial_{\eta_h} + \ln\frac{Q\mu_s}{\mu_j^2}\right)\tilde{j}_q\left(\partial_{\eta_l} + \ln\frac{Q\mu_s}{\mu_j^2}\right)\tilde{s}_\mu(\partial_{\eta_1})\tilde{s}_\mu(\partial_{\eta_2})\tilde{s}_f\left(\partial_{\eta_1} - \partial_{\eta_2}\right)\left(\frac{\rho Q}{\mu_s}\right)^{\eta_1+\eta_2}\frac{e^{-\gamma_E\eta_1}}{\Gamma(\eta_1+1)}\frac{e^{-\gamma_E\eta_2}}{\Gamma(\eta_2+1)}$$

Four-loop cusp: [Henn, Korchemsky, Mistlberger, 1911.10174]

[Manteuffel, Panzer, Schabinger, 2002.04617]

$\alpha_s(m_Z) = 0.1214$



Resummation order:

Order	resum.	Γ_{cusp}	γ_n	c_n	matching
1st order	NLL	2-loop	1-loop	tree	–
2nd order	NNLL	3-loop	2-loop	1-loop	LO
3rd order	$\mathrm{N^3LL}$	4-loop	3-loop	2-loop	NLO
4th order	$\mathrm{N^3LL}$	4-loop	3-loop	3-loop	NNLO

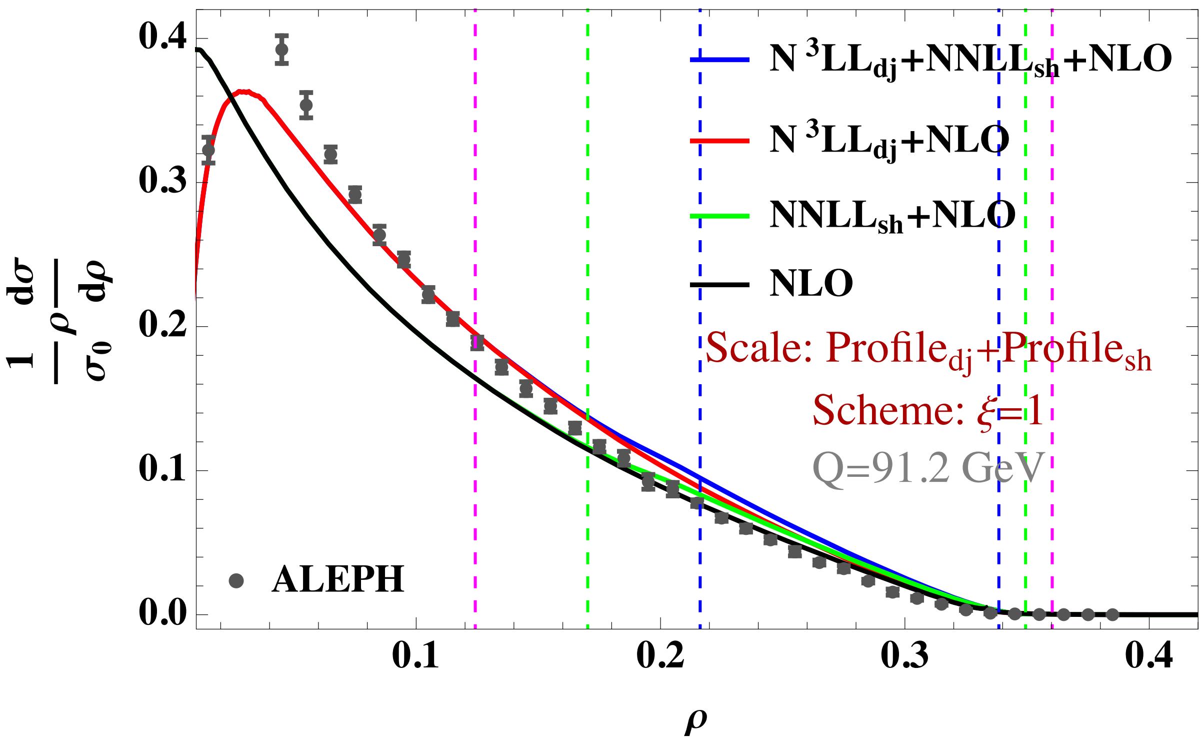
[A. Heister et al. [ALEPH Collaboration], 2004]

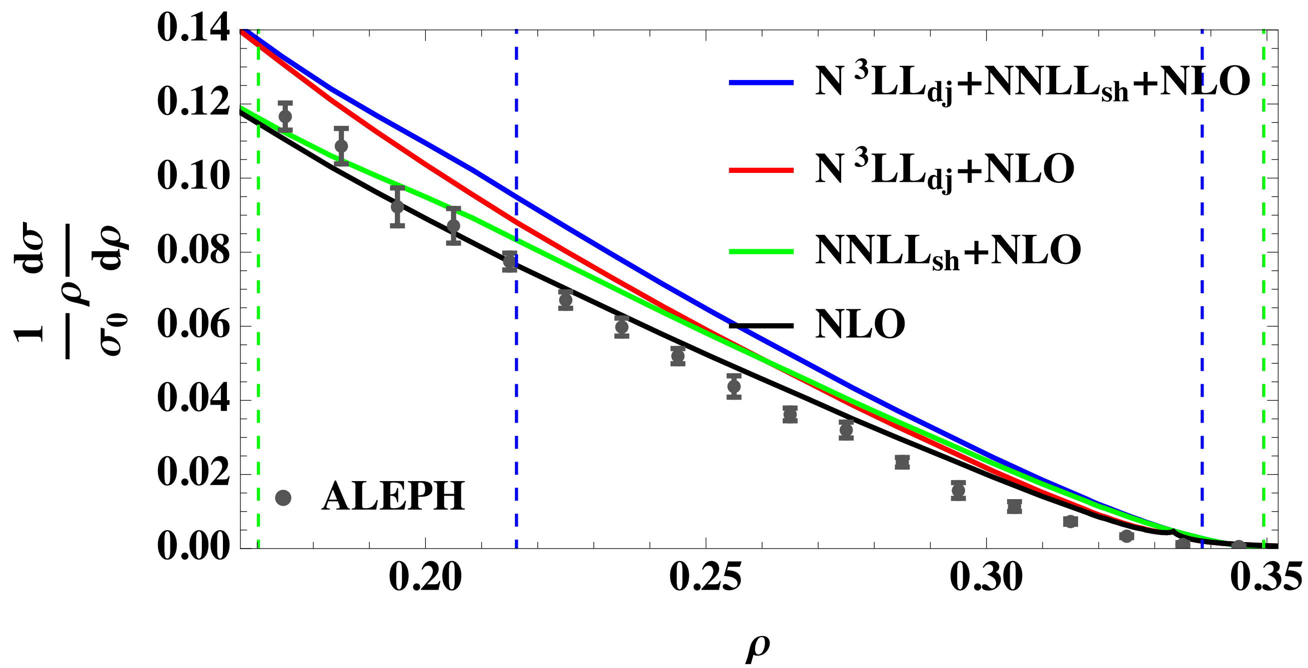
N3LL dijet + NNLL shoulder

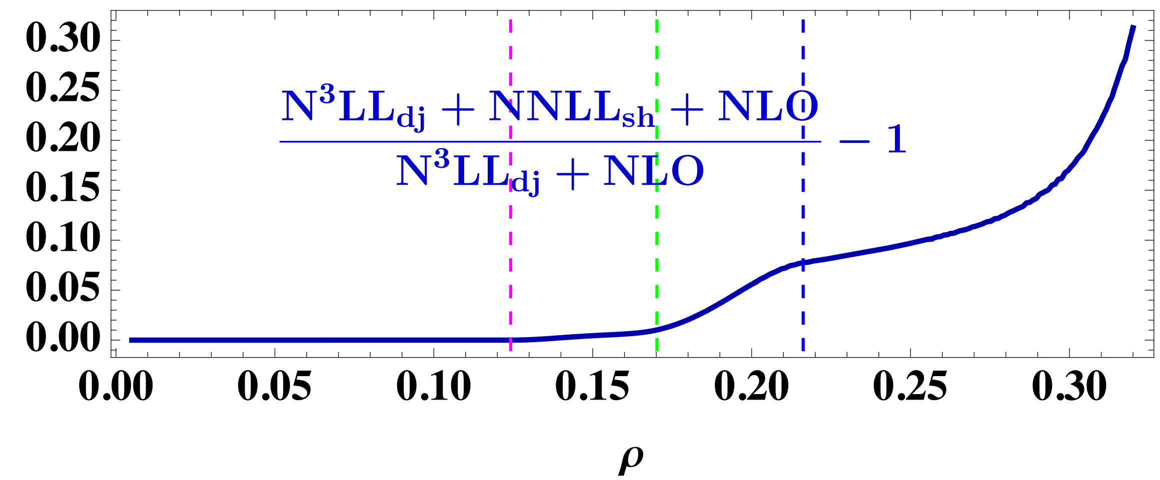
- Joint resummation of dijet and shoulder logarithms:

$$\frac{d\sigma^{\rm match}}{d\rho} = \frac{d\sigma^{\rm FO}}{d\rho} + \left(\frac{d\sigma^{\rm res}_{\rm sh}}{d\rho} - \frac{d\sigma^{\rm S}_{\rm sh}}{d\rho}\right) + \left(\frac{d\sigma^{\rm res}_{\rm dj}}{d\rho} - \frac{d\sigma^{\rm S}_{\rm dj}}{d\rho}\right)$$

- Preliminarily, we will use thrust profile for dijet resummation and canonical/q_T profile for shoulder resummation. [Abbate, Fickinger, Hoang, Mateu, Stewart, 1006.3080]







- Shoulder resummation provides significant corrections:
- Discrepancy between theory and data: We will need to account for power corrections

Conclusion

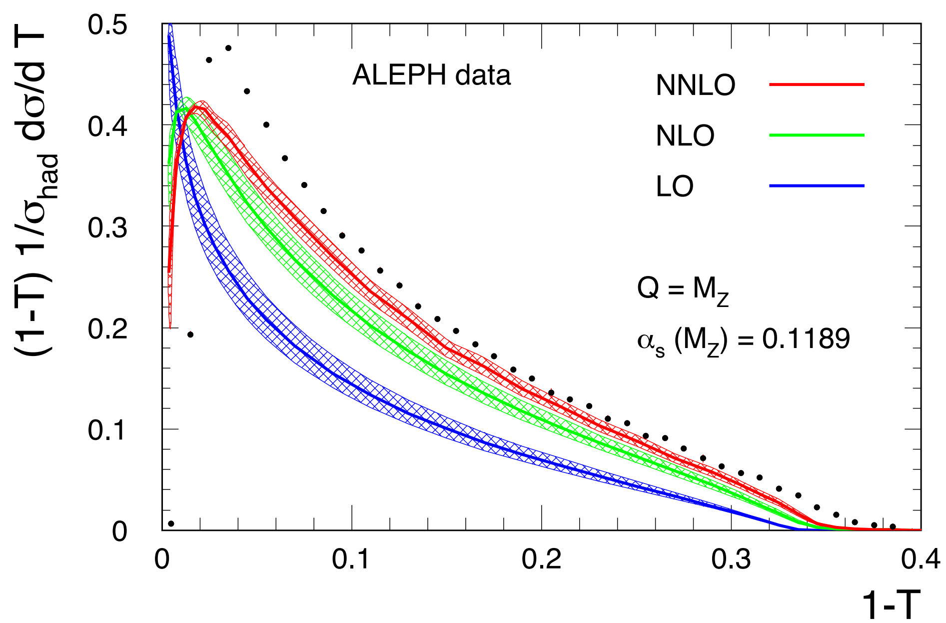
- We present the NNLL resummation of Sudakov shoulder logarithms in heavy jet mass.
  - Differentiating twice and scale setting in the Fourier space removes all spurious poles in the momentum space.
  - ρ_L subtraction scheme gives us control on the ambiguous slope in both shoulders
- We also provide the joint resummation of both dijet and shoulder logs, which is an essential part in the α_s measurement.
- Future works:
  - The complete uncertainty estimation
  - Renormalon and power corrections in the trijet limit
  - Extract the value of α_s from heavy jet mass data

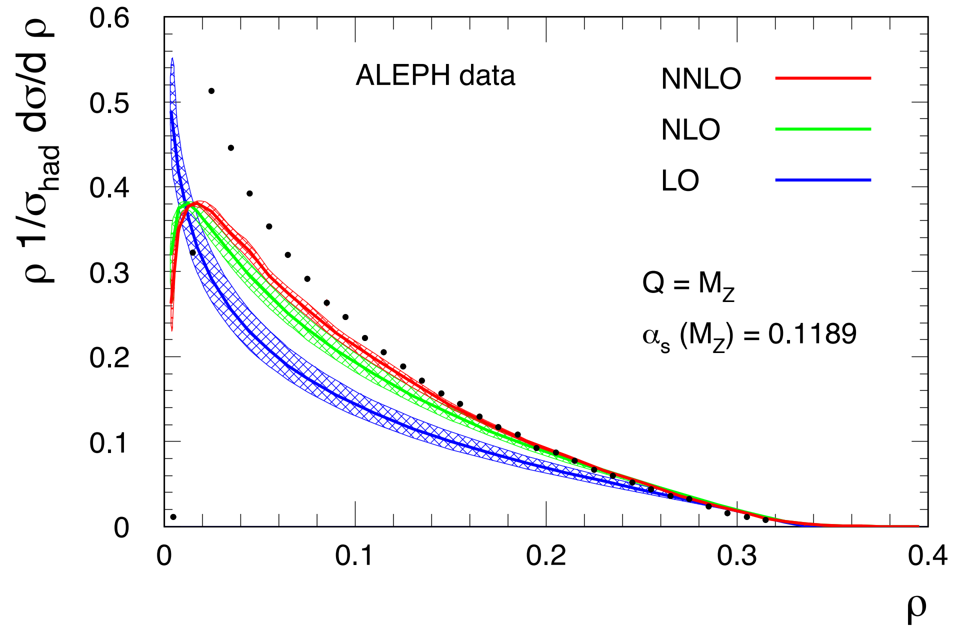
Thank you for your attention!

Backup: Thrust vs HJM (NP)

- NNLO HJM seems to have different shape from data

  [Ridder, Gehrmann, Glover, Heinrich, 0711.4711]

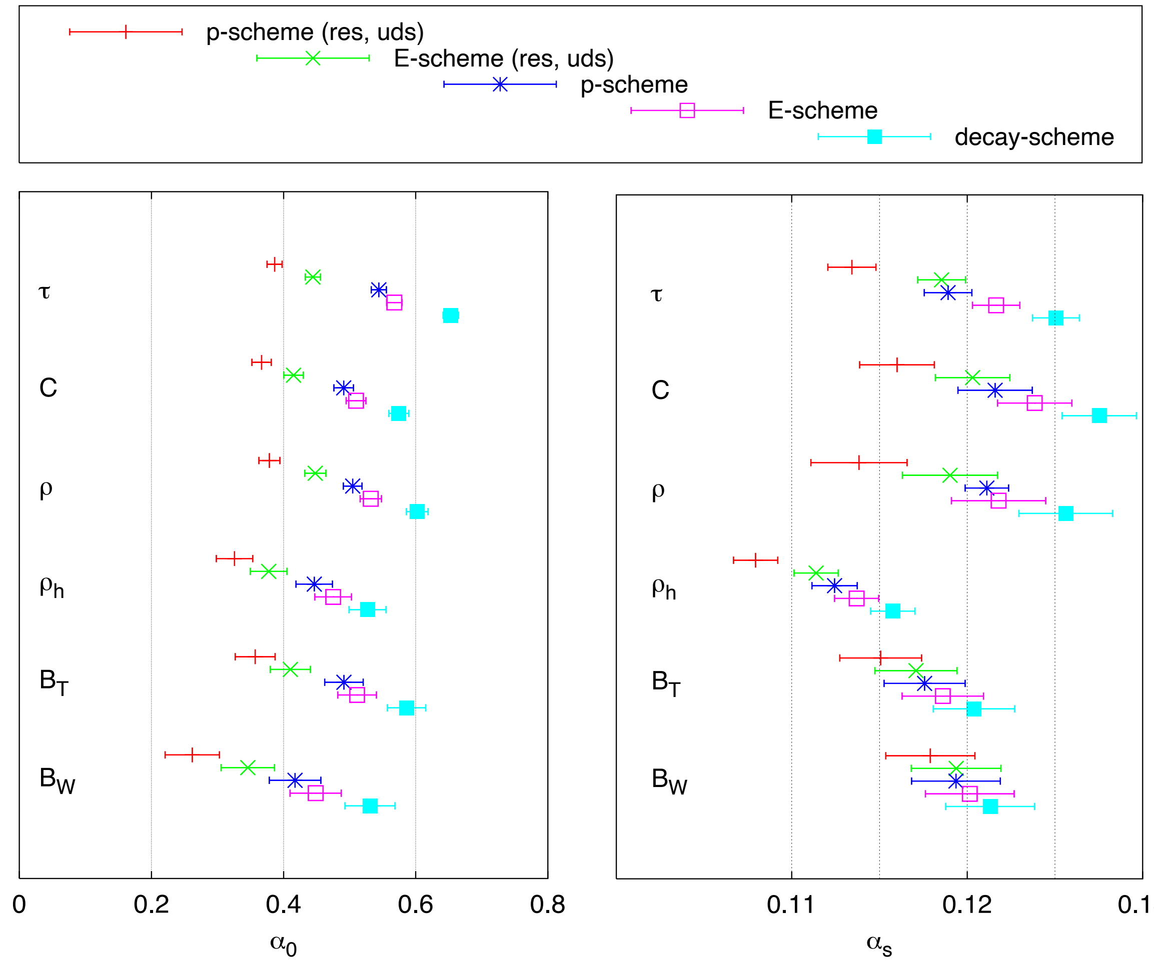




- Scheme dependence:
  - p-scheme: in terms of 3-momenta
  - E-scheme: in terms of energy and angles

  [Salam, Wicke, hep-ph/0102343]

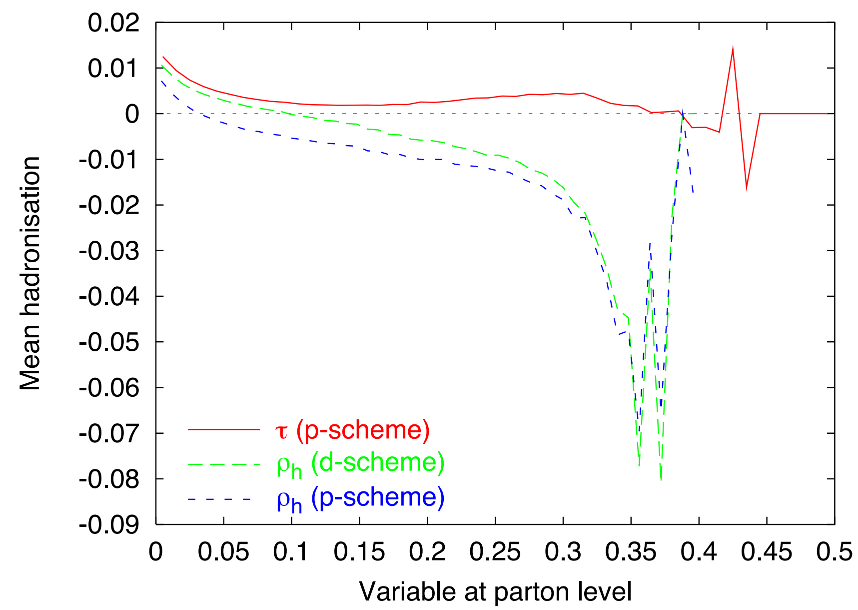
  [Mateu, Stewart, Thaler, 1209.3781]



Backup: Thrust vs HJM (NP)

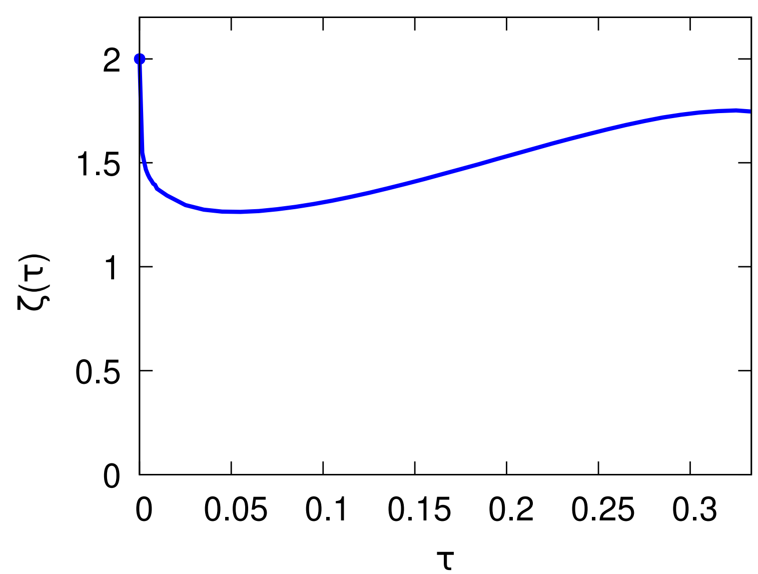
- Hadronization effects

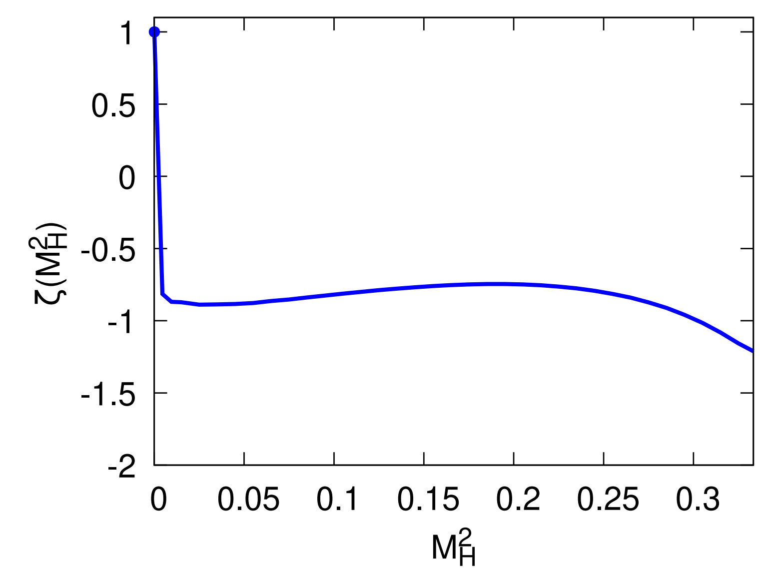
[Salam, Wicke, hep-ph/0102343]



[Nason, Zanderighi, 2301.03607]

$$\zeta(v) = -\left(\frac{d\sigma_{\mathrm{B}}}{dv}\right)^{-1} \frac{(\Sigma_{\mathrm{B+NP}}(v) - \Sigma_{\mathrm{B}}(v))}{H_{\mathrm{NP}}}$$





- Something fishy about the power corrections for HJM that we need to understand

Backup: fixed-order matching

- We first introduce the matched second derivative

$$\sigma^{\rm match}(r,\mu^{\rm res}) = \sigma^{\rm resum}(r,\mu^{\rm res}) + \sigma^{\rm FO}(r) - \sigma^{\rm FO,S}(r)$$

- Then require the integrated spectrum agrees with fixed order at two points $\rho_{L,R}$

$$\frac{d\sigma^{\rm match}}{d\rho} = \begin{cases} \frac{d\sigma^{\rm FO}}{d\rho} & \text{if } \rho = \rho_L \text{ or } \rho_R \\ \left.\frac{d\sigma^{\rm FO}}{d\rho}\right|_{\rho=\rho_L} + C_\rho(\rho-\rho_L) + \sigma_{\rm LO}\int_{r_L}^{r} dr' \int_{r_L}^{r'} dr'' \sigma^{\rm match}(r'',\mu^{res}) \end{cases}$$

The boundary $\left.\frac{d\sigma^{\rm match}}{d\rho}\right|_{\rho=\rho_R} = \left.\frac{d\sigma^{\rm FO}}{d\rho}\right|_{\rho=\rho_R}$ amounts to fix C_ρ

- This is simplified to

$$\begin{aligned} \frac{d\sigma^{\rm match}}{d\rho} &= \frac{d\sigma^{\rm FO}}{d\rho} + \sigma_{\rm LO}\int_{r_L}^{r} dr' \int_{r_L}^{r'} dr'' \left[\sigma^{\rm resum}(r'',\mu^{\rm res}) - \sigma^{\rm resum}(r'',\mu^{\rm FO})\right] \\ &= \frac{d\sigma^{\rm FO}}{d\rho} + \sigma_{\rm LO} 2\Re\left\{\int_0^\infty \frac{dy}{2\pi} K(y,r,r_L)\left[\tilde{\sigma}_i(y,\mu^{\rm res}) - \tilde{\sigma}_i(y,\mu^{\rm FO})\right]\right\} \end{aligned}$$

Backup: dijet profile

[Abbate, Fickinger, Hoang, Mateu, Stewart, 1006.3080]

- We adopt the $\mathrm{N^3LL}$ thrust profile for dijet resummation and rescale the endpoint

$$\mu_h = e_h Q \qquad \mu_j(\rho) = \left[1 + e_j \left(\frac{1}{2} - \rho_1\right)^2\right] \sqrt{\mu_h \mu_s(\rho)}$$

$$\mu_s(\rho) = \begin{cases} \mu_0 + \frac{b}{2t_1}\rho_1^2, & 0 \le \rho_1 \le t_1 \\ b\rho_1 + d, & t_1 \le \rho_1 \le t_2 \\ \mu_h - \frac{b}{1-2t_2}\left(\frac{1}{2} - \rho_1\right)^2, & t_2 \le \rho_1 \le \frac{1}{2} \end{cases}$$

peak

tail

far-tail

$$\rho_1 = \frac{3}{2}\rho$$

$$b = \frac{2(\mu_h - \mu_0)}{t_2 - t_1 + \frac{1}{2}}, d = \frac{\mu_0\left(t_2 + \frac{1}{2}\right) - \mu_h t_1}{t_2 - t_1 + \frac{1}{2}}$$

$$\mu_0 = 2 \pm 0.5\mathrm{GeV}, \quad n_1 = t_1 \frac{Q}{1\mathrm{GeV}} = 5 \pm 3, \quad t_2 = 0.25 \pm 0.05, \quad e_h = 2^{0\pm 1}, \quad e_j = 0 \pm 1$$