LaMET3.0:
Precision parton calculations on lattice

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Outline

- Introduction to Large momentum expansion (LaMET) for parton physics
- Precision controls (LaMET3.0)
    1) Continuum limit
    2) RG resummation
    3) Leading power accuracy
    4) Threshold resummation (SCET)
- Outlook

Large momentum expansion (LaMET) for parton physics

High-energy processes & soft-collinear physics

- In high-energy QCD processes, physical observables can be factorized in terms of soft-collinear physics + perturbative calculable matching functions
- Soft-collinear physics has interesting renormalization properties that can be used to sum perturbative large logarithms (running)
- On the other hand, the soft-collinear matrix elements at $\Lambda_{\rm QCD}$ scale are non-perturbative quantities (PDFs, Soft functions, DA, TMDPDFs, etc) which can only be calculated in low-energy QCD.

LaMET: Connecting non-perturbative soft-collinear physics with lattice QCD

- Much of the soft-collinear physics is tied with light-cone/ray (correlations), and appears impossible to access through Euclidean lattice QCD.
- However, some of these light-cone physics can be formulated as

  Infinite Momentum Limit of Euclidean QCD

- Therefore, non-perturbative soft-collinear physics can be obtained from large momentum expansion of lattice matrix elements (through standard EFT matching and running.)

Example: PDF

- PDFs have their origin in mom. dis. in a moving hadron $n\left(\vec{k}, P^z\right)$

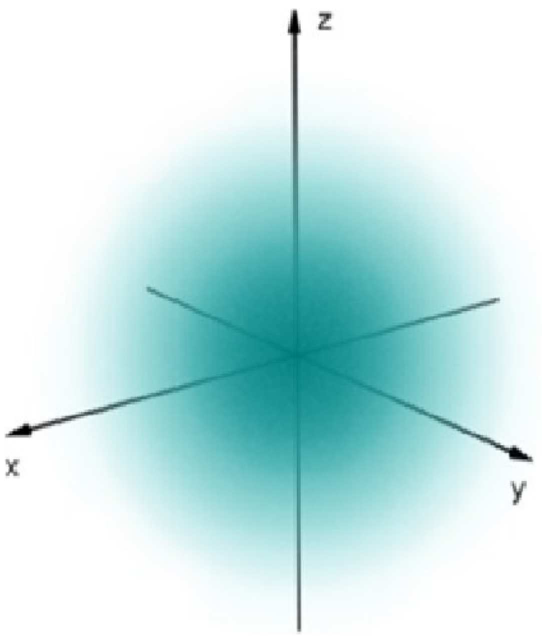
  fundamental property of a quantum (many-body) system,

$$n\left(\vec{k}\right) = \left|\psi\left(\vec{k}\right)\right|^2 \sim \int \psi^*(\vec{r})\psi(0)e^{i\vec{k}\vec{r}}d^3\vec{r}$$

- Static correlation functions can be calculated on lattice QCD

  with lattice spacing a
  as a UV cutoff $\Lambda_{UV} \sim 1/a$

  $n\left(\vec{k}, P^z, a\right)$



Connecting with light-cone distribution

- Longitudinal mom. dis. is

$$n(k^z, P^z, a) = \int d^2\vec{k}_\perp \, n(k^z, \vec{k}_\perp, P^z, a)$$

- When P^z is large, one must first take the continuum limit with

$$P^z \ll \Lambda_{UV} \sim 1/a, \quad \rightarrow \quad n\left(\vec{k}, P^z\right)$$

- Followed infinite momentum limit to get PDF

$$n(k^z, P^z) \rightarrow_{P^z \rightarrow \infty} f(x)\,? \quad \text{with } x = \frac{k^z}{P^z},$$

Does the limit exist?

Large momentum expansion

- When it does, $n(k^z, P^z)$ has a Taylor expansion around $P^z = \infty$,

$$n(k^z, P^z) = f(x) + f_4(x)(M/P^z)^2 + \ldots$$

A precise statement about large-P symmetry!

- One can get the PDFs from Mom. Dis. at large but finite P^z so long as M/P^z is small.

t' Hooft model

- 1+1D QCD with $N_c = \infty$

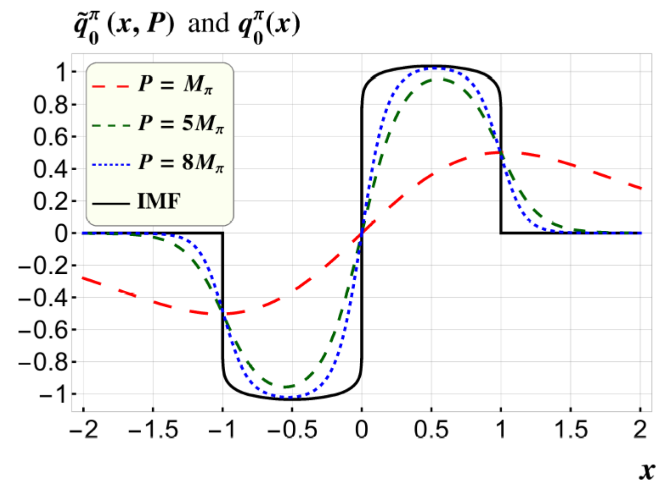
  Can be solved exactly at any finite Pz.

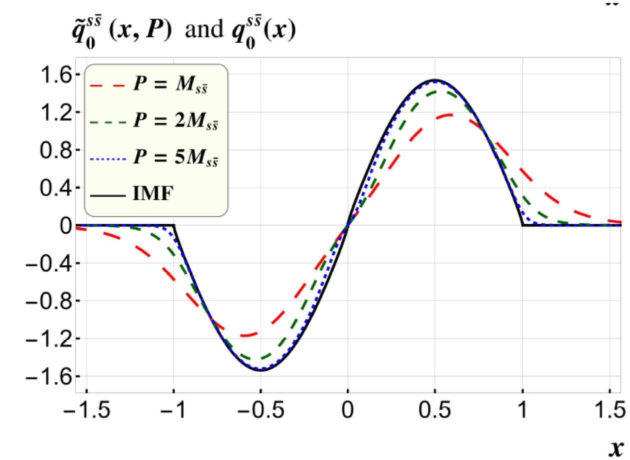
- Mom dis. Calculated at various mom:

  $$p_\pi^z = m_\pi, 5m_\pi, 8m_\pi \ldots$$

  $$p_\phi^z = m_\phi, 2m_\phi, 5m_\phi \ldots$$

- PDF obtained from the smooth limit of $p^z \to \infty$





Infinite momentum limit in 3+1 QCD

- A simple Feynman integral

$$\int^{\Lambda_{UV}} d^4k \frac{(P^z)^2}{(P\cdot k)^2 k^2}$$

- Integral is UV divergent, Λ_{UV} shall be larger than any physics scales. The result depends on lnP.
- **Naïve infinite momentum limit does not exist!**
- However, parton physics is obtained by taking a different limit $P^z \to \infty$ first under the integral sign, followed by $\Lambda_{UV} \to \infty$ (Weinberg 1966, Dirac's LFQ 1949)

Matching relation between two limits

Instead of the simple Taylor expansion,

$$n(k^z, P^z) = f(x) + f_4(x)(M/P^z)^2 + \ldots$$

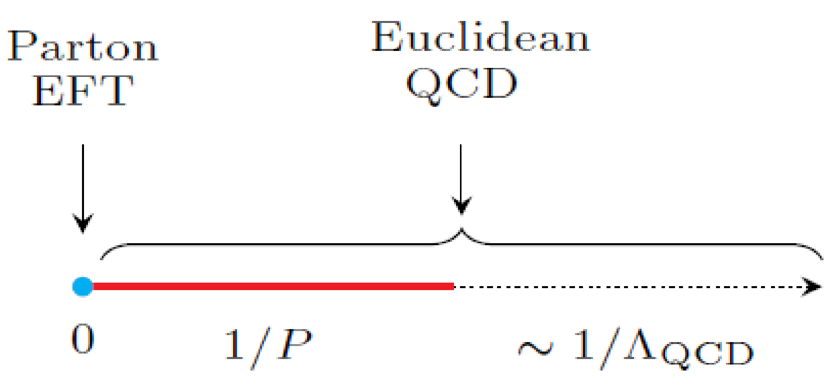
We have the relation between mom dis. in full QCD and PDFs in IMF disregarding UV div (Ji, 2013)

$$n(y, P^z) = \int Z(y/x, xP^z/\mu) f(x, \mu) dx + \mathcal{O}\left(\frac{\Lambda^2_{\text{QCD}}}{y^2(P^z)^2}, \frac{\Lambda^2_{\text{QCD}}}{(1-y)^2(P^z)^2}\right),$$

All order in pert. QCD Ma and Qiu (2018), Izubuchi et al. (2018)

Generalization 1: Universality

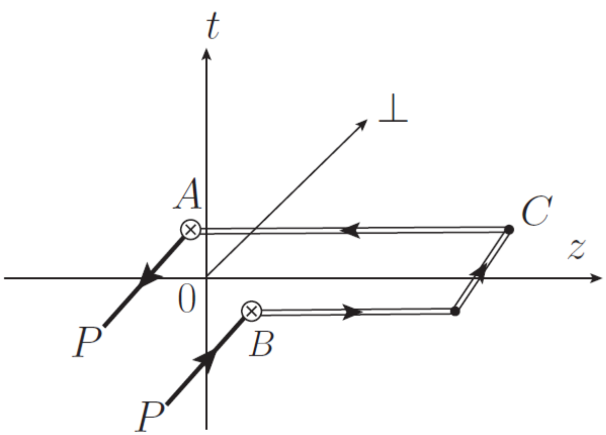
- The most natural quantities as starting point of large momentum expansion are the corresponding finite P physical quantities (quasi-PDFs).
- One can use infinite number of Euclidean observables to achieve the same parton physics, such as current correlators, etc.

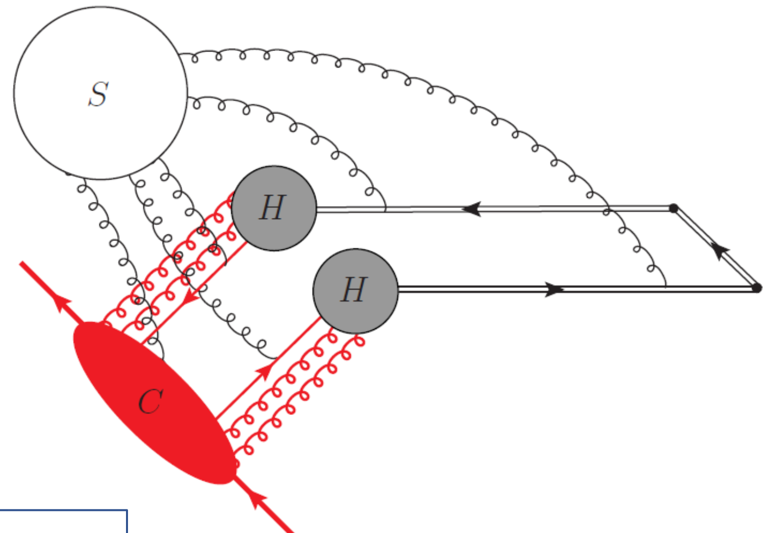


Generalization 2: TMDs, high twists, etc

- Large-momentum expansion can be naturally applied to TMDs.
  - TMD PDFs
  - TMD Wave Functions
- Soft functions
- Higher twists for parton correlations
- Other light-ray observables?

TMDPDF Matching (Ji, Liu, Liu, 2020, Ebert et al 2022)



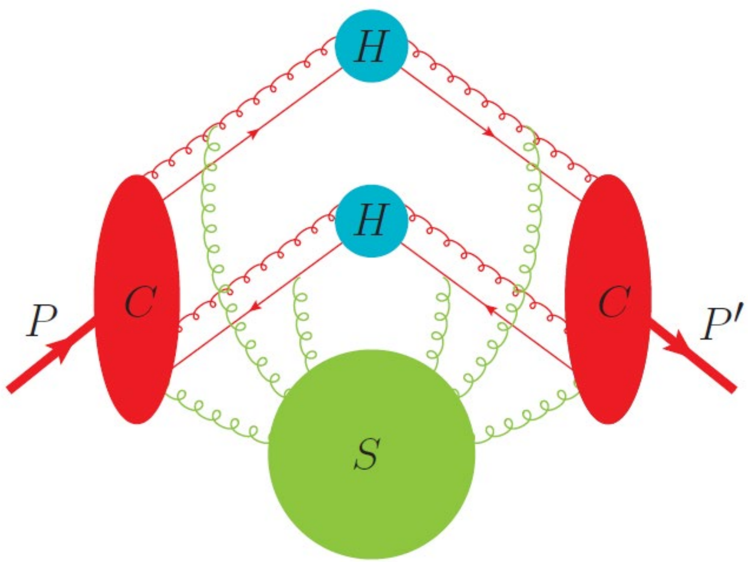


$$\tilde{f}(x, b_\perp, \mu, \zeta_z)\sqrt{S_r(b_\perp, \mu)}$$
$$= H\left(\frac{\zeta_z}{\mu^2}\right) e^{K(b_\perp,\mu)\ln(\frac{\zeta_z}{\zeta})} f^{\mathrm{TMD}}(x, b_\perp, \mu, \zeta) + \ldots$$

$$\mu\frac{d}{d\mu}\ln H\left(\frac{\zeta_z}{\mu^2}\right) = \Gamma_{\mathrm{cusp}}\ln\frac{\zeta_z}{\mu^2} + \gamma_C$$

Soft function

- Soft function can be extracted from “Form Factor” of meson in Breit frame.



Generalization 3: Light-Front Quantization (LFQ)

- Light-front quantized theory is formal (undefined!) and cannot be solved without regularizing light-cone singularities.
- If the regularization breaks Lorentz symmetry (almost all regulators in the LFQ literature do), theory ends up non-renormalizable.
- LFQ can be defined through large-momentum effective theories, including wave functions.

  (X. Ji & Y. Liu, 2022 & to be published)

Precision Control

1) Continuum limit
2) RG resummation
3) leading power accuracy
4) Threshold resummation

...

Power counting

- In the large-momentum expansion, small parameters are

$$\epsilon_i = \left(\frac{\Lambda_{QCD}}{k_i}\right)$$

  where k is ANY physical momentum scale.

- In PDF calculation, k can be
  - Active quark/gluon, $k^z = xP^z$
  - Spectator, $k^z = (1-x)P^z$
- Thus, LaMET approach cannot calculate small and large-x partons unless P^z is very large, such that

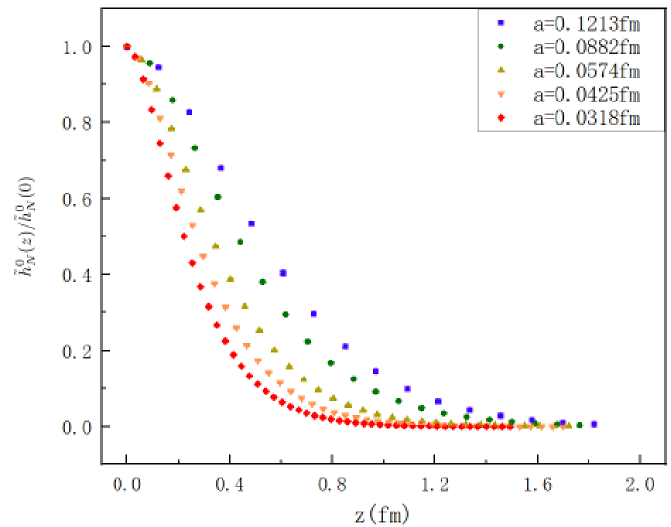
$$xP^z, (1-x)P^z \gg \Lambda_{QCD}$$

Linear divergence and continuum limit

- The quasi-PDF operator has linear Wilson line, which generate power law divergence (mass renorm.)
- These divergences must be subtracted carefully to take the continuum limit.
- Hybrid renormalization scheme (LPC, Ji et al 2021)
- Renormalon ambiguity in mass subtraction (twist-3)

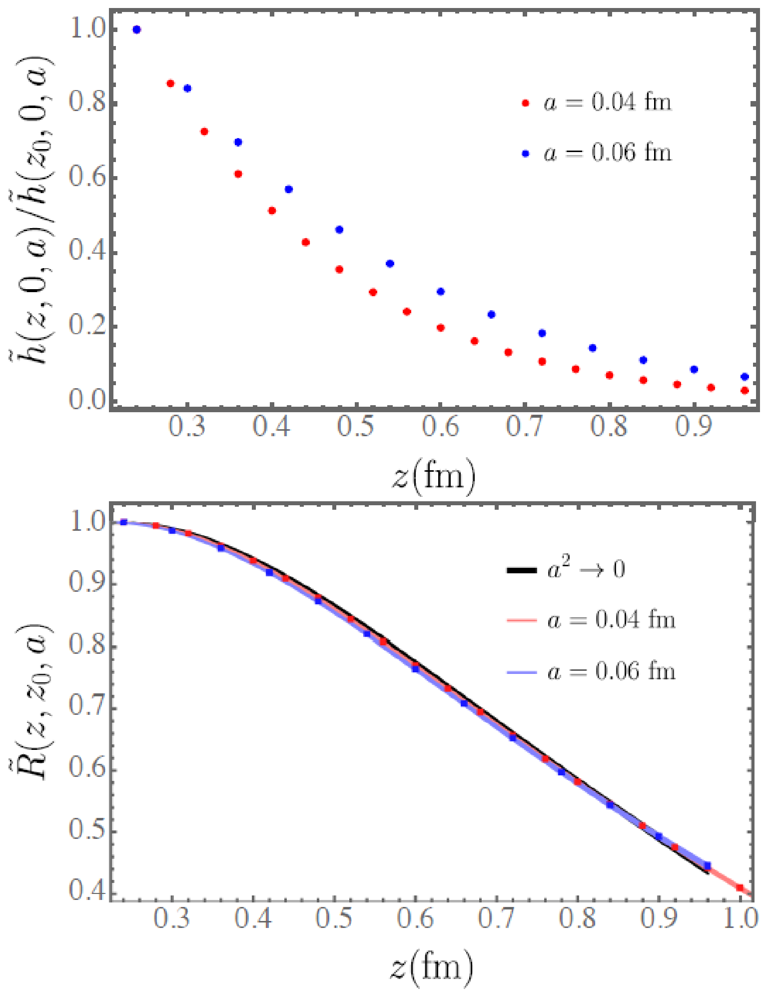
$$O_\Gamma(z)_R = Z_O^{-1} e^{\delta \bar{m} z} O_\Gamma(z),$$

$$\delta \bar{m} = m_{-1}(a)/a - m_0 \ ,$$



Pion qPDF correlation

Gao et al, Phys. Rev. Lett. 128, 142003 (2022)



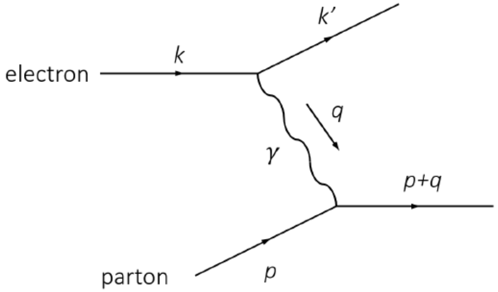
One-loop matching

$$\tilde{f}(x, P_z) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{|x|P_z}\right) f(y,\mu) + \mathcal{O}\left[\frac{\Lambda_{\text{QCD}}^2}{x^2 P_z^2}, \frac{\Lambda_{\text{QCD}}^2}{(1-x)^2 P_z^2}\right]$$

$$C^{(1)}\left(\xi, \frac{\mu}{|x|P_z}\right) = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left(\frac{1+\xi^2}{1-\xi} \ln \frac{\xi}{\xi-1} + 1 - \frac{3}{2(1-\xi)}\right)_{+(1)}^{[1,\infty]} & \xi > 1 \\ \left(\frac{1+\xi^2}{1-\xi} \left[-\ln \frac{\mu^2}{4x^2 P_z^2} + \ln\left(\frac{1-\xi}{\xi}\right) - 1\right] + 1 + \frac{3}{2(1-\xi)}\right)_{+(1)}^{[0,1]} & 0 < \xi < 1 \\ \left(-\frac{1+\xi^2}{1-\xi} \ln \frac{-\xi}{1-\xi} - 1 + \frac{3}{2(1-\xi)}\right)_{+(1)}^{[-\infty,0]} & \xi < 0 \end{cases}$$

Intrinsic scale of qPDF

- DIS scale Q^2

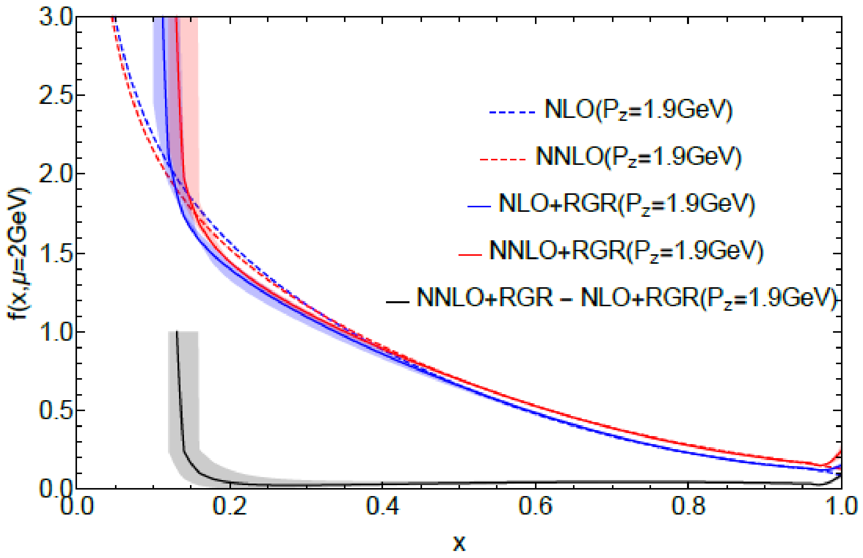


$$
\begin{aligned}
q^\mu &= (0, 0, 0, -2xP_z) \\
p^\mu &= (xP_z, 0, 0, xP_z) \\
(p+q)^\mu &= (xP_z, 0, 0, -xP_z).
\end{aligned}
$$

- Thus the qPDF intrinsic scale is $4x^2P_z^2$

RG resummation (Y. Su et al, e-Print: 2209.01236)

- There is a large scale-gap between μ and 2xP, when x is small, or when P is large.



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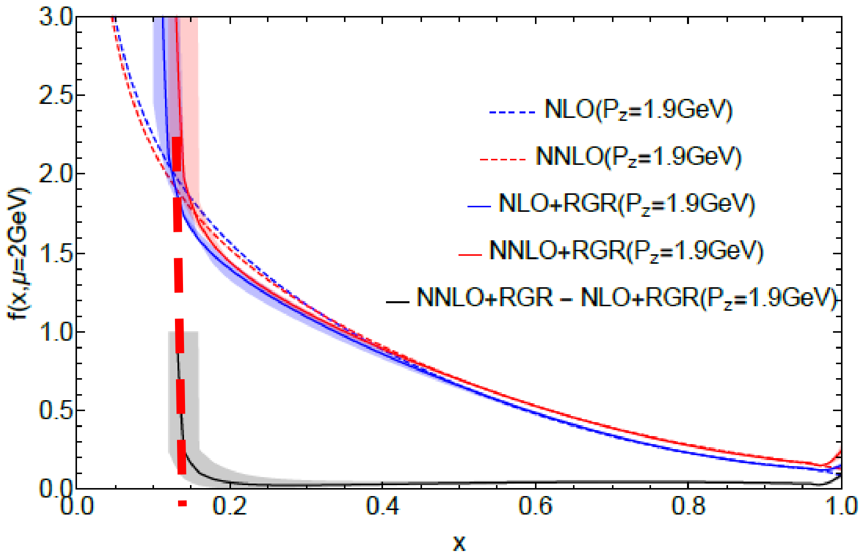


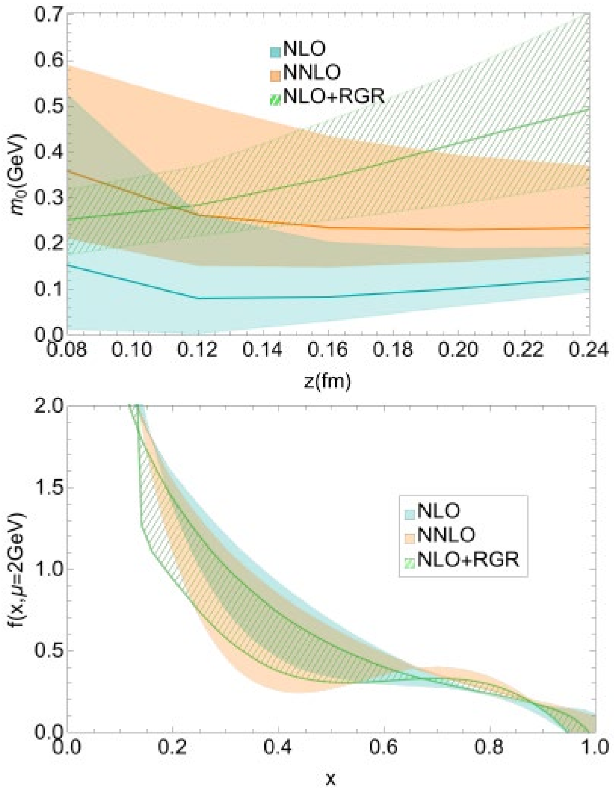
- Consistent with power counting.

Leading power correction (R Zhang et al)

- Euclidean correlations with Wilson line has power div, which introduces extra mass parameter in OPE at twist-3 level,

$$h^R(z, P_z, \mu, \tau) \tag{2}$$
$$= \left(1 - m_0(\tau) z\right) \sum_{k=0}^{\infty} C_k\left(\alpha_s(\mu), \mu^2 z^2\right) \lambda^k a_{k+1}(\mu) + \mathcal{O}(z^2)$$
$$= \sum_{k=0}^{\infty} \left[C_k\left(\alpha_s(\mu), \mu^2 z^2\right) - z m_0(\tau)\right] \lambda^k a_{k+1}(\mu) + \mathcal{O}(z\alpha_s, z^2),$$

- To obtain LaMET matching to the leading power accuracy, one must determine m_0 in consistency



Summing over the leading renormalon

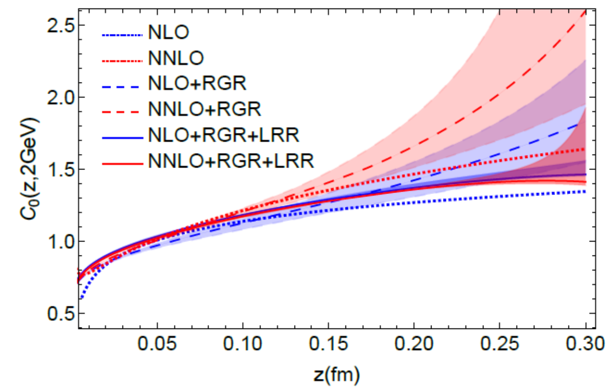
- Leading renormalon in the matching coefficient

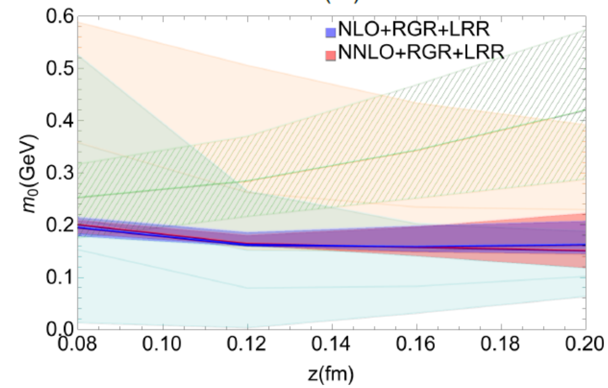
$$C_k(\alpha_s(1/z),1)_{\rm PV} = 1 + N_m \frac{4\pi}{\beta_0}\int_{0,{\rm PV}}^{\infty} du$$
$$\times\, e^{-\frac{4\pi u}{\alpha_s(1/z)\beta_0}} \frac{1}{(1-2u)^{1+b}}\left(1 + c_1(1-2u) + \ldots\right),$$

its uncertainty is also linear in z.

N_m can be obtained from high-order lattice pert (Bali et al, 2014)

- After including LRR, we have a stable twist-3 mass parameter

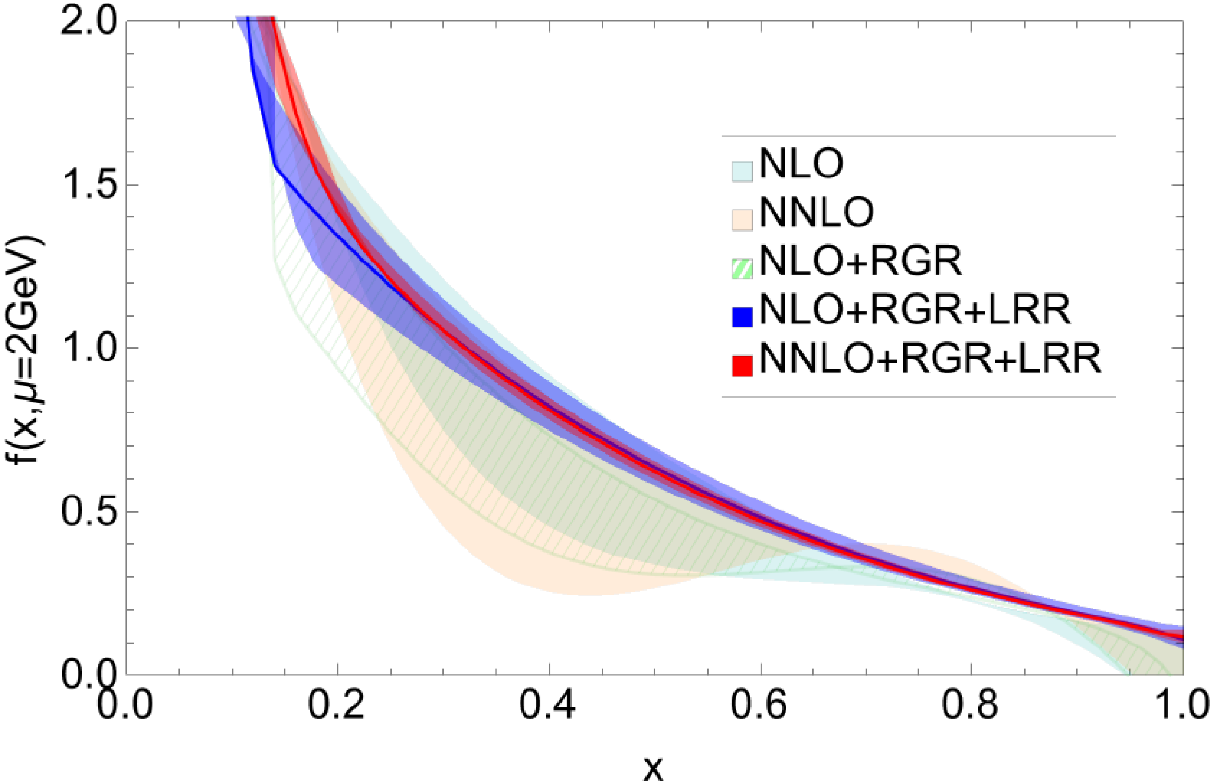




Improving accuracy of matching

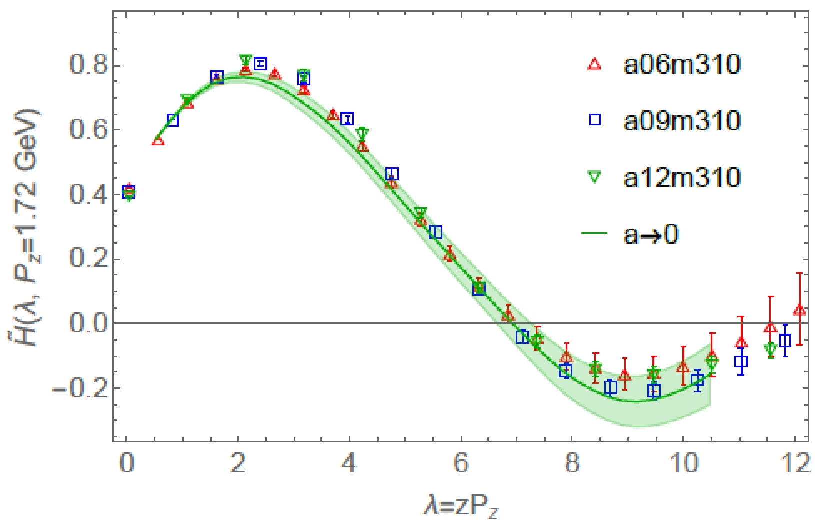
- Example: pion PDF

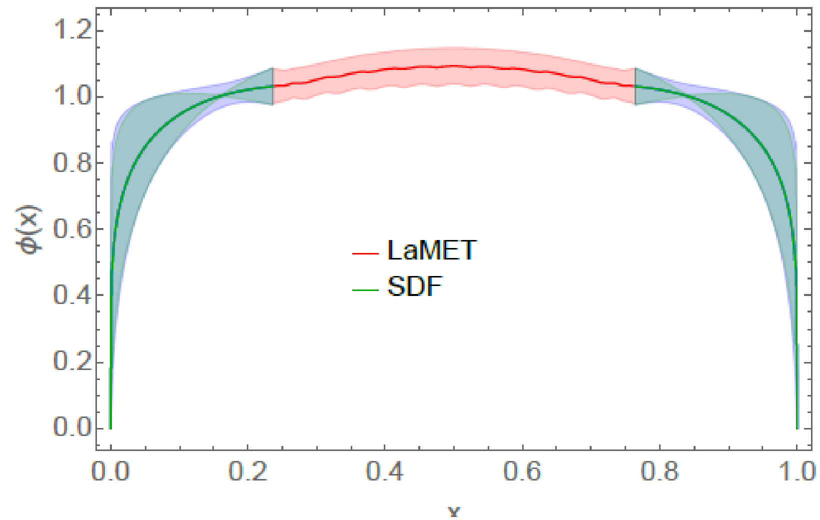
Data: BNL-ANL Collaboration (PRL128,2022)

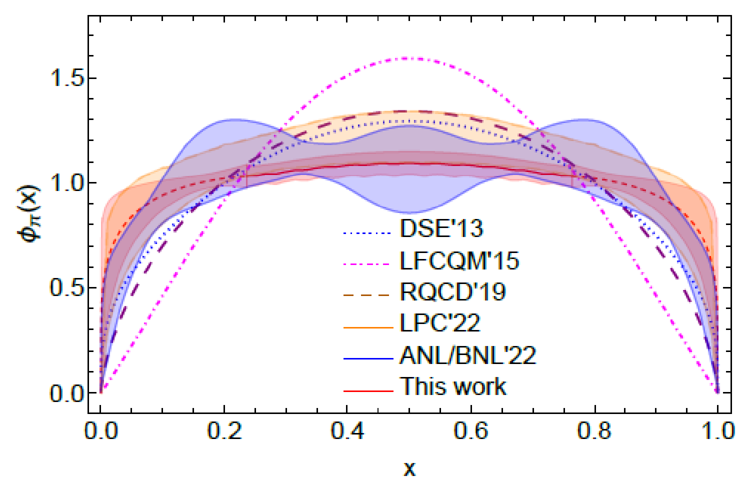


Pion distribution amplitude

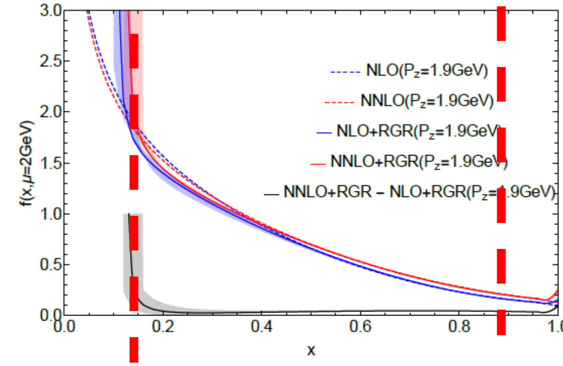
e-Print: 2301.10372







Threshold resummation



- When $x \to 1$, the hadron remnant moment $(1-x)P^z$ becomes soft.
- This is now an incomplete cancellation of IR divergences between real and virtual contributions.

$$C^{(1)}\left(\xi, \frac{\mu}{|x|P_z}\right) = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left(\frac{1+\xi^2}{1-\xi}\ln\frac{\xi}{\xi-1} + 1 - \frac{3}{2(1-\xi)}\right)_{+(1)}^{[1,\infty]} & \xi > 1 \\ \left(\frac{1+\xi^2}{1-\xi}\left[-\ln\frac{\mu^2}{4x^2P_z^2} + \ln\left(\frac{1-\xi}{\xi}\right) - 1\right] + 1 + \frac{3}{2(1-\xi)}\right)_{+(1)}^{[0,1]} & 0 < \xi < 1 \\ \left(-\frac{1+\xi^2}{1-\xi}\ln\frac{-\xi}{1-\xi} - 1 + \frac{3}{2(1-\xi)}\right)_{+(1)}^{[-\infty,0]} & \xi < 0 \end{cases}$$

- Large logs of type $\left[\frac{\ln(1-x)}{1-x}\right]_+$ shall be resumed, can be done in momentum space as in DIS

Threshold resummation using SCET (Y. Liu et al)

- Cannot be done in moment space!

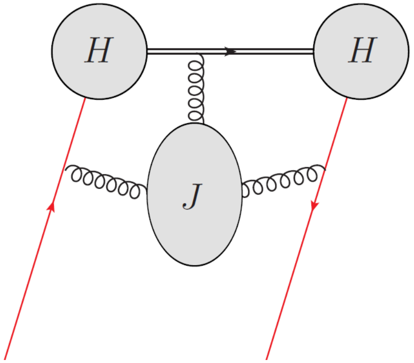
  Sterman, Manohar...

- Direct momentum space re-summation in DIS has been done through SCET

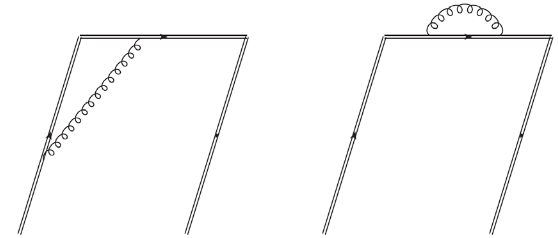
  T. Becher & Neubert, PRL 2006, JHEP, 2007 ...

- Similar approach can be used for LaMET matching.

$$\tilde{f}\left(y, \frac{\zeta_z}{\mu}\right) = \int_0^1 dx H\left(\frac{4x^2 P_z^2}{\mu^2}\right) P^z J_f\left(\frac{(x-y) P^z}{\mu}, \frac{4x^2 P_z^2}{\mu^2}\right) f(x, \mu) \ ,$$



“Heavy quark jet” function



$$\tilde{J}(\mu z, D) = \langle \Omega | \bar{\mathcal{T}} W^{\dagger}_{n,-}(z n_z) W^{\dagger}_{z}(z n_z) \mathcal{T} W_z(0) W_{n,-}(0) | \Omega \rangle \ .$$

- Evolution in coordinate space

$$\tilde{J}\left(\mu^2 z^2, \alpha(\mu)\right) = \exp\left[-2S(\mu_i, \mu) + a_{\mathrm{HL}}(\mu_i, \mu) \right] \left(\frac{4}{z^2 e^{2\gamma_E} \mu_i^2} \right)^{a_\Gamma(\mu_i,\mu)} \tilde{J}(z^2 \mu_i^2, \alpha(\mu_i))$$

- Momentum space

$$J\left(\frac{p}{\mu}, \alpha(\mu)\right) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-ipz} \tilde{J}(z^2\mu^2, \alpha(\mu)) dz$$

$$J\left(\frac{p}{\mu}, \alpha(\mu)\right) = \exp\left[-2S(\mu_i, \mu) + a_{\mathrm{HL}}(\mu_i, \mu) + 2\ln 2 a_\Gamma(\mu_i, \mu) \right] \times \int_{-\infty}^{\infty} dp' \frac{2\sin\frac{\pi\eta}{2}\Gamma(1-\eta)}{(\mu_i e^{\gamma_E})^{\eta} |p - p'|^{1-\eta}} J\left(\frac{p'}{\mu_i}, \mu_i\right) \ .$$

Matching after SCET resummation

$$
\begin{aligned}
&C\left(\xi, \frac{yP^z}{\mu}\right) = yH\left(\frac{4y^2P_z^2}{\mu^2}, \alpha(\mu)\right) P^z J_f\left(\frac{(\xi^{-1}-1)yP^z}{\mu}, \frac{4y^2P_z^2}{\mu^2}, \alpha(\mu)\right) \\
&= H(\alpha(\zeta_z)) \exp\left[2S(\zeta_z,\mu) - a_{\rm C}(\zeta_z,\mu) - 2S(\mu_i,\mu) + a_{\rm HL}(\mu_i,\mu)\right] \\
&\times \tilde{J}\left(\ln\frac{e^{2\gamma_E}z^2\mu_i^2}{4} = -2\partial_\eta, \alpha(\mu_i)\right)\left(\left[\frac{\cos\hat{A}(\zeta_z,\mu)}{|1-\xi|}\left(\frac{2|\xi^{-1}-1|yP^z}{\mu}\right)^\eta\right]_* \sin\left(\frac{\eta\pi}{2}\right)\right. \\
&\left. + \left[\frac{\sin\hat{A}(\zeta_z,\mu)}{1-\xi}\left(\frac{2|\xi^{-1}-1|yP^z}{\mu}\right)^\eta\right]_* \cos\left(\frac{\eta\pi}{2}\right)\right)\frac{\Gamma(1-\eta)\mathrm{e}^{-\eta\gamma_E}}{\pi}\,, \qquad (3
\end{aligned}
$$

- What is missing? Leading renormalon resummation near the threshold, due to linear divergence of the heavy-quark jet function self-energy.

Summary

- LaMET aims to calculate parton physics at intermediate x region without doing global fitting.
- With precision lattice data and perturbative matching, LaMET3.0 can reach 5-10%
- More calculations are need to confirm, which allows further development of high precision calculations.