Bottom mass corrections to boosted-top cross sections

Alejandro Bris[1]

in colaboration with

Vicent Mateu, Fernando Gil

[1]Instituto de Física Teórica -
Universidad Autónoma de Madrid

SCET 2023
Berkeley, March 29 2023

Outline

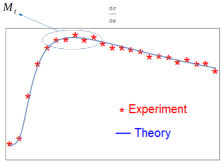
Outline

Motivation

- Precise determinations of the top quark mass need to account for bottom mass efects:
  - Observable sensible to the quark mass. Hemisphere Jet mass:

$$s \equiv \left(\sum_{i\in h} p_i\right)^2 \qquad\qquad s_{\rm min} = M_t^2$$

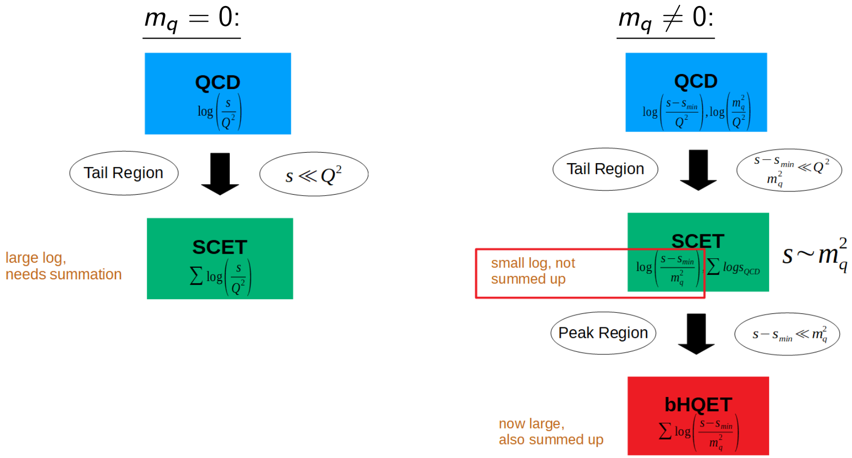
  - Extract the primary quark mass from the peak position of the distribution



  - Close to the peak, fluctuations around the primary quark mass are very small: $\frac{s-M_t^2}{M_t} \ll M_t$
  - A non-vanishing secondary quark mass is relevant for a precise determination of the peak position
- Secondary charm mass corrections might play an important role for bottom mass determinations as well
- These corrections are the missing piece of $\mathcal{O}(\alpha_s^2)$ computations for $e^+e^- \to t\,\bar{t}\,X$

Dijets' primary quarks production

(s = jet invariant mass)



* Q center-of-mass energy

Factorization theorems

$e^+e^- \to$ Hadrons. 2-jettiness. $\left(\tau \approx \frac{s_{h^+}+s_{h^-}}{Q^2}\right.$ in the peak region$\left.\right)$

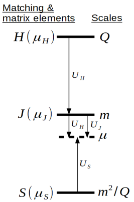
SCET: [0801.4569], [hp-ph/0703207], [0711.2079]

($\hat{\sigma} \to$ partonic cross section)

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}_{\mathrm{SCET}}}{\mathrm{d}\tau} = Q^2 \overbrace{H(Q,\mu)}^{\text{matching coefficient}} \int_0^{Q(\tau-\tau_{\min})} \mathrm{d}\ell\, J_\tau(Q^2\tau - Q\ell, \mu)\, \overbrace{S_\tau(\ell,\mu)}^{\text{soft radiation}} + \text{p.c.}$$

$$J_\tau(s,\mu) \equiv \int_{s_{\min}}^{s-s_{\min}} \mathrm{d}s'\, J_n(s-s',\mu) J_{\bar{n}}(s',\mu) \quad \text{collinear radiation}$$

each element evaluated at its natural scale. Running to a common scale sums up large logs



bHQET (Boosted Heavy Quark): [0801.4569], [hp-ph/0703207], [0711.2079]

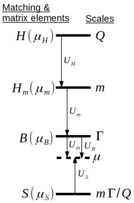
1) Integrate out heavy quark mass

2) Boost back to c.o.m. frame

3) Match onto SCET in order to account for global soft radiation.

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}_{\mathrm{bHQET}}}{\mathrm{d}\tau} = Q^2 H(Q,\mu_m)\, \overbrace{H_m\!\left(m, \frac{Q}{m}, \mu_m, \mu\right)}^{\text{new matching coefficient}} \int \mathrm{d}\ell\, \overbrace{B_\tau\!\left(\frac{Q^2(\tau-\tau_{\min}) - Q\ell}{m}, \mu\right)}^{\text{new jet function}} S_\tau(\ell,\mu) + \text{p.c.}$$

$$B_\tau(\hat{s},\mu) = m\int_0^{\hat{s}} \mathrm{d}\hat{s}'\, B_n(\hat{s}-\hat{s}',\mu) B_n(\hat{s}',\mu)$$



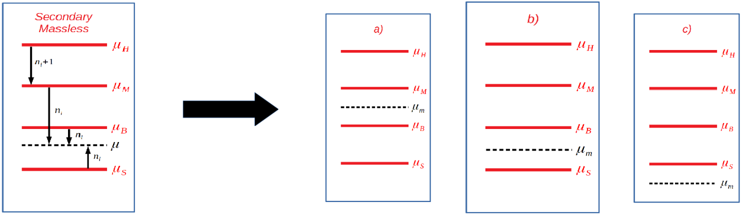
* $S_\tau(\ell,\mu)$ massive secondary quark corrections already known: [S. Gritschacher, A.Hoang, I.Jemos, P. Pietrulewicz, 2013]

bHQET

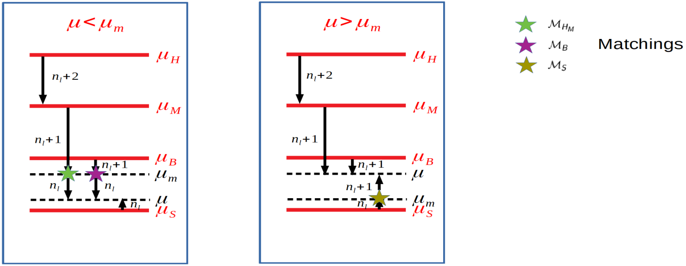
Massive secondary quark corrections

New scale (secondary quark mass) brings a richer structure of EFTs: (Based on [1405.4860] ↔ SCET)

→ Different scenarios: (b relevant in peak region)



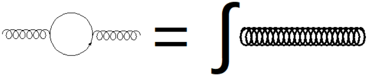
→ Consistency conditions: freedom to choose scale at which everyone runs to.



Outline

1. Motivation & Introduction
2. Computations
   - Dispersive Integral Method
   - Matrix Elements
   - Flavour Matchings
3. Preliminary Numerical Results

Dispersive Integral Method



Method:

1) Write massive bubble diagram as an integral of an effective gluon propagator
2) Perform computations at previous loop order with the modified gluon propagator
3) Carry out dispersive integral

- MASSIVE GLUON:

$$\text{MASSIVE BUBBLE} \longrightarrow \frac{-i}{p^2}\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)\Pi(p^2, m^2)$$

$$\frac{\Pi(p^2, m^2) - \Pi(0, m^2)}{p^2} = T_F \frac{\alpha_s}{4\pi}\int_{4m^2}^{\infty} \mathrm{d}\tilde{m}^2 \, \frac{\mathcal{V}^{(d)}(\tilde{m}, \mu)}{p^2 - \tilde{m}^2 + i\epsilon}$$

$$\mathcal{V}^{(d)}(\tilde{m}, \mu) = \frac{8\Gamma(2-\varepsilon)}{\Gamma(4-2\varepsilon)}\frac{\beta_{\tilde{m}}}{\tilde{m}^2}\left(\frac{4\pi\tilde{\mu}^2}{\beta_{\tilde{m}}^2 \tilde{m}^2}\right)^{\varepsilon}\left(1 - \varepsilon + \frac{2m^2}{\tilde{m}^2}\right)$$

* m bubble quark mass, * $\tilde{m}$ gluon effective mass, * $\beta_{\tilde{m}} \equiv \sqrt{1 - 4m^2/\tilde{m}^2}$

Dispersive Integral Method

Using Mellin-Barnes representation for the modified gluon propagator reduces complexity to massless 1-loop with modified exponents. Same computation as in renormalon calculus

- MELLIN PLANE:
  - I Closed expression for $\Pi(p^2, m^2)$ obtained from massive gluon integration:

$$\Pi(p^2, m^2) = \frac{T_F\,\Gamma(\varepsilon)}{2\varepsilon - 3}\,\frac{\alpha_s}{\pi}\left(\frac{4\pi\tilde{\mu}^2}{m^2}\right)^{\varepsilon}\left[\left(1 - \varepsilon + \frac{2m^2}{p^2}\right)\,{}_2F_1\left(1, \varepsilon; \frac{3}{2}; \frac{p^2}{4m^2}\right) - \frac{2m^2}{p^2}\right]$$

  - II Integral representation of ${}_2F_1$:

$${}_2F_1\left(1, \varepsilon; \frac{3}{2}; \frac{p^2}{4m^2}\right) = \frac{\Gamma(3/2)}{\Gamma(\varepsilon)\,\Gamma(3/2-\varepsilon)}\int_0^1 \mathrm{d}x\,\frac{x^{-1+\varepsilon}(1-x)^{1/2-\varepsilon}}{1 - \frac{p^2}{4m^2}\,x}$$

  - III Mellin-Barnes representation:

$$\frac{1}{1 - \frac{p^2}{4m^2}\,x} = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \mathrm{d}h\left(-\frac{p^2}{4m^2}\,x\right)^{-h}\Gamma(h)\,\Gamma(1-h)$$

  - IV Integration of x variable can be carried out right away

Dispersive Integral Method

After integrating x one finds:

Mellin Plane Result

$$\frac{\Pi(p^2, m^2) - \Pi(0, m^2)}{p^2} = \frac{T_F\, \alpha_s}{2\pi^2 i p^2} \left(\frac{4\pi\tilde{\mu}^2}{m^2}\right)^{\varepsilon} \int_{c-i\infty}^{c+i\infty} \mathrm{d}h \left(-\frac{m^2}{p^2}\right)^{-h} \frac{1+h}{3+2h} \frac{h\Gamma^2(h)\Gamma(1-h)\Gamma(h+\varepsilon)}{\Gamma(2h+2)}$$

$\Rightarrow$ Modified gluon propagator: same as renormalon calculus "analytic regulator"

$$\frac{-ig^{\mu\nu}}{p^2} \longrightarrow \frac{ig^{\mu\nu}}{(-p^2)^{1-h}}$$

- Does not introduce an additional energy scale in previous order computations ✓
- Integration of h by residues yields directly expansions for large or small mass (converse mapping theorem) ✓
- It also may give rise to closed forms in terms of Meijer functions ✓
- Dealing with distributions needs extra effort ✗

Jet Function

One loop results

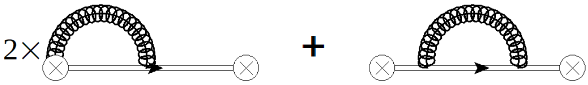
Definition:

$$\mathcal{B}_n(\hat{s},\mu) = \frac{i}{4\pi N_C M}\ \mathrm{Tr}\left[\int \mathrm{d}^d x\, e^{ikx} \left\langle 0\right| T\left\{ W_n^{+}(x)\, h_v(x)\ \bar{h}_v(0)\, W_n(0)\right\} \left|0\right\rangle\right]$$

$$B_n(\hat{s},\mu) = \mathrm{Im}\left[\mathcal{B}_n(\hat{s},\mu)\right]$$

- W_n Wilson lines with u-collinear gluons
- h_v Heavy quark field

Contributing diagrams:



Jet Function

One loop results. Massive gluon

Calculation comments:

- → Rapidity divergence appears in vertex diagram due to non-zero gluon mass $\Rightarrow$ Introduce δ regulator that changes de exponent.
- → $\mathcal{B}_n(\hat{s},\mu,\tilde{m})$ involves hypergeometric functions with $\hat{s}$ dependence which will lead to "hidden" distributions when taking the imaginary part $\Rightarrow$ Easiest way:

$$\mathrm{Im}\Big[\mathcal{B}_n(\hat{s},\mu,\tilde{m})\Big] = \mathrm{Im}\Big[\mathcal{B}_n(\hat{s},\mu,\tilde{m}\to\infty)\Big] + \mathrm{Im}\Big[\mathcal{B}_n(\hat{s},\mu,\tilde{m}) - \mathcal{B}_n(\hat{s},\mu,\tilde{m}\to\infty)\Big]_{\varepsilon\to 0} + \mathcal{O}(\varepsilon)$$

- → regulator dependence disappears when taking imaginary part

$B_n(\hat{s},\mu,\tilde{m})$

$$\frac{\alpha_s C_F}{4\pi M}\left\{2\,\Gamma(\varepsilon)\left(\frac{\mu^2}{\tilde{m}^2}e^{\gamma}\right)^{\varepsilon}\left[\left(-H_{\varepsilon-1}+2\log(\tilde{m})+1\right)\delta(\hat{s})-2\left[\frac{\theta(\hat{s})}{\hat{s}}\right]_+ - \frac{2\tilde{m}\pi^{1/2}\,\Gamma(1/2+\varepsilon)}{(2\varepsilon-1)\,\Gamma(\varepsilon)}\,\delta'(\hat{s})\right]\right.$$
$$\left. +\,\theta\left(\hat{s}^2-4\tilde{m}^2\right)\left[\frac{8}{\hat{s}}\log\left(\frac{\hat{s}+\sqrt{\hat{s}^2-4\tilde{m}^2}}{2\tilde{m}}\right)-\frac{4\sqrt{\hat{s}^2-4\tilde{m}^2}}{\hat{s}^2}\right]+\mathcal{O}(\varepsilon)\right\}$$

☐ reproduces massless result

Jet Function

One loop results. Mellin plane

Calculation comments:

- → Simpler integrals than massive gluon case because there is only one scale: $\hat{s}$
- → Rapidity regulator is not needed
- → Imaginary part of non-integer powers:

$$(-\hat{s} - i\epsilon)^{-a} = \hat{s}^{-a} e^{ai\pi} \xrightarrow[i\mathrm{Im}]{} \hat{s}^{-a} \frac{i\pi}{\Gamma(a)\Gamma(1-a)},$$

$B_n^h(\hat{s}, \mu)$ ($\equiv$ One loop jet function with gluon propagator $\frac{ig^{\mu\nu}}{(-p^2)^{1-h}}$)

$$-C_F \frac{\alpha_s}{\pi M} \frac{\Gamma(2+h-\varepsilon)\,\hat{s}^{-1+2h} e^{\varepsilon\gamma}}{(\varepsilon - h)\Gamma(1-h)\Gamma(2+2h-2\varepsilon)} \left(\frac{\mu}{\hat{s}}\right)^{2\varepsilon}$$

- no need to expand in epsilon or h
- agreement with [N. G. Gracia, V. Mateu, 2021]

Jet Function

Two loop massive secondary quark corrections. Massive gluon Computation

All terms in dispersive integral analytically computed easily except:

$$I[\hat{s}, m] \equiv \frac{1}{4\hat{s}} \int_{4m^2}^{\hat{s}^2/4} \mathrm{d}\tilde{m}^2 \; \mathcal{V}^{(4)}(\tilde{m}, \mu) \log\left(\frac{\hat{s} + \sqrt{\hat{s}^2 - 4\tilde{m}^2}}{2\tilde{m}}\right)$$

Integrating by parts and defining $a \equiv \sqrt{1 - \frac{16m^2}{\hat{s}^2}}$ we managed to obtain an expression in terms of hypergeometric functions and elliptic integrals E and K:

$$I[\hat{s}, m] = \frac{1}{3\hat{s}}\left[\frac{1}{9}\left(16 - 2a^2\right) E\left(a^2\right) - \frac{1}{9}\left(16 - a^2\right) K\left(a^2\right) + f(a)\right]$$

$$\begin{aligned} f(a) = &- \frac{1}{8}\left(1 - a^2\right) {}_5F_4\left(1, 1, 1, \frac{3}{2}, \frac{3}{2}; 2, 2, 2, 2; 1 - a^2\right) \\ &+ \left[\frac{1 - a^2}{4} + \frac{1 - a^2}{8} \log\left(\frac{1 - a^2}{16}\right)\right] {}_4F_3\left(1, 1, \frac{3}{2}, \frac{3}{2}; 2, 2, 2; 1 - a^2\right) \\ &+ \frac{1 - a^2}{4} \frac{d}{d\epsilon}\left[{}_4F_3\left(1, 1, \frac{3}{2}, \frac{3}{2} + \epsilon; 2, 2, 2 + \epsilon; 1 - a^2\right)\right]_{\epsilon = 0} + \frac{1}{4}\log^2\left(\frac{1 - a^2}{16}\right) - \frac{\pi^2}{6} \end{aligned}$$

Jet Function

Two loop massive secondary quark corrections. Mellin plane Computation

$$B_n^{m-\text{bubble},OS}(\hat{s},\mu,m) = \frac{T_f\,\alpha_s}{2\pi^2 i\,M}\left(\frac{4\pi\tilde{\mu}^2}{m^2}\right)^{\varepsilon}\int_{c-i\infty}^{c+i\infty}\mathrm{d}h\,\left(m^2\right)^{-h}\frac{1+h}{3+2h}\,\frac{h\,\Gamma^2(h)\Gamma(1-h)\Gamma(h+\varepsilon)}{\Gamma(2h+2)}\,B_n^h(\hat{s},\mu)$$

$$= -\frac{\alpha_s^2\,T_f\,C_F}{2^{4-2\varepsilon}\pi^2 i\,M}\left(\frac{\mu^2 e^{\gamma}}{m}\right)^{2\varepsilon}\hat{s}^{-1-2\varepsilon}\int_{c-i\infty}^{c+i\infty}\mathrm{d}h\left(\frac{16m^2}{\hat{s}^2}\right)^{-h}\frac{(1-\varepsilon+h)(1+h)\,\Gamma(h)\Gamma(h+\varepsilon)}{(\varepsilon-h)\,\Gamma(\frac{5}{2}+h)\Gamma(\frac{3}{2}-\varepsilon+h)}$$

$$= -\frac{\alpha_s^2\,T_f\,C_F}{2^{3-2\varepsilon}\,\pi\,M}\left(\frac{\mu^2 e^{\gamma}}{m}\right)^{2\varepsilon}\hat{s}^{-1-2\varepsilon}\,G_{5,5}^{4,1}\left(\frac{16m^2}{\hat{s}^2}\Big|\begin{array}{c}1-\varepsilon,1,1-\varepsilon,\frac{5}{2},\frac{3}{2}-\varepsilon\\ 0,\varepsilon,2,2-\varepsilon,-\varepsilon\end{array}\right)$$

$*$ Trick: one can postpone MB inversion after resummation

- Converse mapping theorem:

  Fundamental strip: $h\in(0,\varepsilon)$. Expansion series: $\begin{cases}\text{l.h.s poles: } \frac{16m^2}{\hat{s}^2}\to 0\\ \text{r.h.s poles: } \frac{16m^2}{\hat{s}^2}\to\infty\end{cases}$

- Distributional structure:

$$B_n^{m-\text{bubble},OS}(\hat{s},\mu,m) \equiv f_\delta\,\delta(\hat{s}) + \sum_{i=0} f_{\mathcal{L}_i}\left[\frac{\theta(\hat{s})\log^i(\hat{s})}{\hat{s}}\right]_+ + \sum_{j=1} f_{\delta^{(j)}}\,\delta^{(j)}(\hat{s}) + \theta\Big(g(\hat{s},m)\Big)\,f_{\text{nd}}$$

$^*\,\delta^{(j)}(x) \equiv \frac{\mathrm{d}^j}{\mathrm{d}x^j}\delta(x)$

Recovering distributions:

- l.h.s poles $\Rightarrow \sum_{i=0} f_{\mathcal{L}_i} \frac{\log^i(\hat{s})}{\hat{s}} + f_{\rm nd}$
- r.h.s pole (ONLY $h = \varepsilon$) $\Rightarrow \sum_{i=0} f_{\mathcal{L}_i} \frac{\log^i(\hat{s})}{\hat{s}}$
- Dirac delta and its derivatives:

$$\int_0^{\hat{s}_c} d\hat{s}\ \hat{s}^k\, B_n^{m-\text{bubble},OS}\left(\hat{s},\mu,m\right)\Big|_{\mathcal{O}(\hat{s}_c^0)} = (-1)^k k!\, f_{\delta^{(k)}}$$

Different ways:

- use $\hat{s}^{-1+2(h-\varepsilon)} = \frac{1}{2(h-\varepsilon)}\,\delta(\hat{s}) + \sum_{n=0} \frac{2^n(h-\varepsilon)^n}{n!}\left[\frac{\theta(\hat{s})\log^n(\hat{s})}{\hat{s}}\right]_+$ for dirac delta and plus distributions
- Setting $\varepsilon = 0$ directly in the Mellin plane expression one recovers the non-distributional part as a Meijer G-function:

$$\theta\left(1-\frac{16m^2}{\hat{s}^2}\right) f_{\rm nd} = \frac{\alpha_s^2\, T_f\, C_F}{8\pi\, M}\,\hat{s}^{-1}\, G_{5,5}^{5,0}\left(\frac{16m^2}{\hat{s}^2}\Big|\ \begin{matrix} 1,1,1,\frac{3}{2},\frac{5}{2} \\ 0,0,0,2,2 \end{matrix}\right)$$

- Obtain the Mellin plane two loops expression for $\mathcal{B}_n^{m-\text{bubble},OS}\left(\hat{s},\mu,m\right)$, expand in $\hat{s} = \hat{s} + i\epsilon$ and take then the imaginary part

Jet Function

Final Result

Carrying out renormalization and convolving both hemisphere jet functions we get:

$$
\begin{aligned}
B_\tau^{(n_l+1)}(\hat{s},\mu,m) =& B_\tau^{(n_l+1)}(\hat{s},\mu) + \delta B_m^{dist}(\hat{s},\mu,m) + \delta B_m^{real}(\hat{s},m) \\
\delta B_m^{dist}(\hat{s},\mu,m) =& \left(\frac{\alpha_s^{(n_l+1)}}{4\pi}\right)^2 \frac{C_F}{M}\Bigg[\left(\frac{32}{9}L_m^3 + \frac{128}{9}L_m^2 + \left(\frac{976}{27} - \frac{16\pi^2}{9}\right)L_m + \frac{3568}{81} - \frac{64\pi^2}{27} - \frac{32}{3}\xi_3\right)\delta(\hat{s}) \\
&+ \left(-\frac{32}{3}L_m^2 - \frac{256}{9}L_m - \frac{976}{27} + \frac{16\pi^2}{9}\right)\mathcal{L}^0(\hat{s}) + \left(\frac{64}{3}L_m + \frac{256}{9}\right)\mathcal{L}^1(\hat{s}) - \frac{32}{3}\mathcal{L}^2(\hat{s}) - 8\pi^2 m\,\delta'(\hat{s})\Bigg] \\
\delta B_m^{real}(\hat{s},m) =& \left(\frac{\alpha_s^{(n_l+1)}}{4\pi}\right)^2 \frac{C_F}{M}\frac{\theta\left(\hat{s}^2 - 16m^2\right)}{\hat{s}}\Bigg[\frac{976}{27} - \frac{16\pi^2}{9} + \frac{256}{9}\log\left(\frac{m}{\hat{s}}\right) + \frac{32}{3}\log^2\left(\frac{m}{\hat{s}}\right) \\
&+ \sum_{n=0}^{\infty}\frac{\left(\left(\frac{1}{2}\right)_n\right)^2}{(n!)^2(n+2)^3}\left(\frac{4n-1}{n+2} + 2(1+2n)\Big[\psi(n+1) - \psi(n+1/2) - \log\left(\frac{4m}{\hat{s}}\right)\Big]\right)\left(\frac{16m^2}{\hat{s}^2}\right)^{2+n}\Bigg]
\end{aligned}
$$

- $B_\tau^{(n_l+1)}(\hat{s},\mu)$ can be found in [A. Jain, I. Scimemi and I.W. Stewart, 2008]
- n_l number of massless flavours
- $L_m \equiv \log\left(\frac{m}{\mu}\right)$
- $\mathcal{L}^i(\hat{s}) \equiv \frac{1}{\mu}\left[\frac{\theta(\hat{s})\log^i(\hat{s}/\mu)}{\hat{s}/\mu}\right]_+$

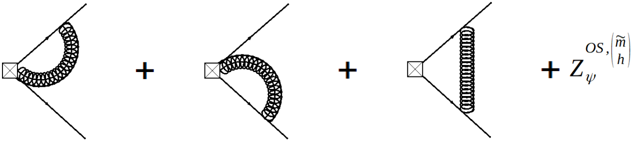
Definition:

$$\mathcal{J}_{\text{SCET}} = C_M \, \mathcal{J}_{\text{bHQET}}$$
$$H_M = |C_M|^2$$

Computation:

$$C_M = \frac{\langle q, \bar{q} | \mathcal{J}_{\text{SCET}} | 0 \rangle}{\langle q, \bar{q} | \mathcal{J}_{\text{bHQET}} | 0 \rangle} = \frac{F_{\text{SCET}}}{F_{\text{bHQET}}}$$

Contributing diagrams:



bHQET Hard Function

One loop results

- Massive gluon

$F^{\tilde{m}}_{\text{SCET}} \to$ [A. H. Hoang, A. Pathak, P. Pietrulewicz, I. W. Stewart, 2015]

$$F^{\tilde{m}}_{\text{bHQET}} = \frac{\alpha_s C_F}{2\pi} \Gamma(\varepsilon) \left(\frac{\mu^2}{\tilde{m}^2} e^{\gamma} \right)^{\varepsilon} \left[1 + \log\left(\frac{M^2}{Q^2} \right) + i\pi \right]$$

- Mellin plane

$$F^{h}_{\text{SCET}} = \frac{\alpha_s C_F}{\pi} \left(\mu^2 e^{\gamma} \right)^{\varepsilon} (h-1) \left(1 + \frac{(3-2\varepsilon)(\varepsilon-h-1)(\varepsilon-h)}{2+h-2\varepsilon} \right) \frac{\Gamma(\varepsilon-h)\Gamma(2h-2\varepsilon)}{\Gamma(2+h-2\varepsilon)} \left(M^2 \right)^{h-\varepsilon}$$

$$F^{h}_{\text{bHQET}} \text{ Scaleless} \to C^{h}_{M} = F^{h}_{\text{SCET}}$$

bHQET Hard Function

Final result for the expansion

$$C_M^{(n_l+1)}\left(M, \frac{Q}{M}, \mu, m\right) = C_M^{(n_l+1)}\left(M, \frac{Q}{M}, \mu\right) + \delta C_M\left(\frac{m}{M}\right)$$

$$\delta C_M\left(\frac{m}{M}\right) = \left(\frac{\alpha_s^{(n_l+1)}}{4\pi}\right)^2 C_F T_f\left[6\pi^2\frac{m}{M} + \frac{4m^2}{M^2}\left(6+8\hat{L}_m\right) - \frac{110}{9}\pi^2\frac{m^3}{M^3}\right.$$
$$\left. + \frac{m^4}{3M^4}\left(145+12\pi^2-72\left(2-\hat{L}_m\right)\hat{L}_m\right) + \frac{m^6}{M^6}\sum_{n=0}^{\infty} a_n(m/M)\left(\frac{m^2}{M^2}\right)^n\right]$$

$$a_n(m/M) = \frac{8}{(n+1)(n+2)(n+3)^3(2n+3)(2n+5)}\left[506+750n+413n^2+100n^3+9n^4\right.$$
$$+\left(408+778n+589n^2+221n^3+41n^4+3n^5\right)\log(4) + (n+2)(n+3)(n+4)\left(17+14n+3n^2\right)\times$$
$$\left.\times\left[2\hat{L}_m + \psi(-1/2-n) + \psi(1+n) - 2\psi(7+2n)\right]\right]$$

- $C_M^{(n_l+1)}\left(M, \frac{Q}{M}, \mu\right)$ can be found in [A. H. Hoang, A. Pathak, P. Pietrulewicz, I. W. Stewart, 2015]
- n_l number of massless flavours
- $\hat{L}_m \equiv \log\left(\frac{m}{M}\right)$

bHQET Hard Function

Final result unexpanded

$$\delta C_M\left(\frac{m}{M}\right) = \left(\frac{\alpha_s^{(n_l+1)}}{4\pi}\right)^2 C_F T_f \left\{ \frac{1747}{81} + \frac{52\pi^2}{27} + \left(\frac{532}{27} + \frac{16}{9}\pi^2\right)\hat{L}_m + \frac{104}{9}\hat{L}_m^2 + \frac{32}{9}\hat{L}_m^3 \right.$$
$$+ \frac{1}{2}\int_{4m^2}^{\infty} \mathrm{d}\tilde{m}^2\, \mathcal{V}^{(4)}(\tilde{m},\mu)\left[\frac{\left(-b^3-5b+6\right)}{b(b+1)^2}\log\left(\frac{1-b}{2}\right) - \frac{\left(b^3+5b+6\right)}{(1-b)^2 b}\log\left(\frac{b+1}{2}\right)\right.$$
$$\left.\left. + \frac{b^2-25}{2\left(1-b^2\right)} + 4\log\left(\frac{1-b}{2}\right)\log\left(\frac{b+1}{2}\right)\right]\right\}$$

- $b \equiv \sqrt{1 - \frac{4M^2}{\tilde{m}^2}}$
- n_l number of massless flavours
- $\hat{L}_m \equiv \log\left(\frac{m}{M}\right)$

Based on [P. Pietrulewicz, S. Gritschacher, A. H. Hoang, I. Jemos, V. Mateu, 2014]

$$\mathcal{M}_B = M \int \mathrm{d}\hat{s}' \, B_\tau^{(n_l)}(\hat{s}-\hat{s}', \mu_m, m) \left[B_\tau^{(n_l+1)}(\hat{s}', \mu_m, m) \right]^{-1}$$

$$B_\tau^{(n_l)}(\hat{s}, \mu, m) = B_\tau^{(n_l)}(\hat{s}, \mu) + B_\tau^{m-\text{bubble},OS}(\hat{s}, \mu, m) - Z_B^{OS}\left(\hat{s}, \mu, m\right)$$

$Z_B^{OS}\left(\hat{s}, \mu, m\right)$ is obtained from the decoupling condition:

$$\lim_{m\to\infty} \left[B_\tau^{(n_l)}(\hat{s}, \mu, m) \right] = B_\tau^{(n_l)}(\hat{s}, \mu) \underbrace{- \left(\frac{\alpha_s}{4\pi}\right)^2 \frac{C_F}{M} 8\pi^2 m \, \delta'(\hat{s})}_{\Diamond}$$

$\Diamond$ comes from the Lagrangian of bHQET$^{(n_l)}$ theory

$$\mathcal{L}_{\mathrm{bHQET}^{(n_l+1)}} = \bar{h}_v \left(i\, v \cdot \mathrm{D} - \delta M \right) h_v$$

$$\delta M = M_{pole} - M$$

$$\Downarrow$$

$$\boxed{\mathcal{L}_{\mathrm{bHQET}^{(n_l)}} = \bar{h}_v \left[i\, v \cdot \mathrm{D} - \delta M - \left(\frac{\alpha_s}{4\pi} \right)^2 C_F\, 2\pi^2 m \right] h_v}$$

- Massive secondary quark bubbles remain as a contribution to the primary quark self-energy

$$\mathcal{M}_B(\hat{s}, \mu_m, m) = \left(\frac{\alpha_s^{(n_l+1)}}{4\pi} \right)^2 \frac{C_F}{M} \Bigg\{ \Bigg[-\frac{32}{9} L_m^3 - \frac{128}{9} L_m^2 + \left(-\frac{688}{27} + \frac{4\pi^2}{9} \right) L_m - \frac{440}{27} + \frac{5\pi^2}{27} + \frac{28}{9} \xi_3 \Bigg] \delta(\hat{s}) + \left(\frac{32}{3} L_m^2 + \frac{160}{9} L_m + \frac{224}{27} \right) \mathcal{L}^0(\hat{s}) \Bigg\}$$

Flavour Matching

bHQET Hard Function

$$\mathcal{M}_{C_M} = \frac{C_M^{(n_l)}\left(M, \frac{Q}{M}, \mu_m, m\right)}{C_M^{(n_l+1)}\left(M, \frac{Q}{M}, \mu_m, m\right)} \quad ; \qquad \mathcal{M}_{H_M} = \left|\mathcal{M}_{C_M}\right|^2$$

$$C_M^{(n_l)}\left(M, \frac{Q}{M}, \mu, m\right) = C_M^{(n_l)}\left(M, \frac{Q}{M}, \mu\right) + C_M^{m-\text{bubble},OS}\left(M, \mu, m\right) - Z_{C_M}^{OS}\left(M, \mu, m\right)$$

- Decoupling limit $\Rightarrow$ Remove the quark also from the running of SCET hard function:

$$\lim_{m\to\infty}\left[H^{(n_l+2)}(Q,\mu,m)\, H_M^{(n_l)}\left(M, \frac{Q}{M}, \mu, m\right)\right] = H^{(n_l+1)}(Q,\mu)\, H_M^{(n_l)}\left(M, \frac{Q}{M}, \mu\right)$$

$$H^{(n_l+1)}(Q,\mu) = \lim_{m\to\infty}\left[|\mathcal{M}_C(Q,\mu,m)|^2\; H^{(n_l+2)}(Q,\mu,m)\right]$$

$$\Downarrow$$

$$2\mathrm{Re}\left[C_M^{m-\text{bubble},OS}\Big(M,\mu,m\Big) - Z_{C_M}^{(2),OS}\Big(M,\mu,m\Big) - \mathcal{M}_C^{(2)}(Q,\mu,m)\right] \underset{m\to\infty}{\longrightarrow} 0$$

$\mathcal{M}_C(Q,\mu,m) \rightarrow$[P. Pietrulewicz, S. Gritschacher, A. H. Hoang, I. Jemos, V. Mateu, 2014]

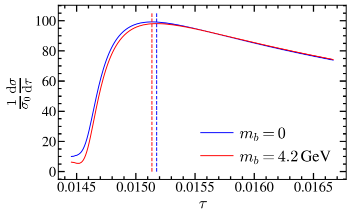
$$\mathcal{M}_{H_M}^{(2)}\left(M,\frac{Q}{M},\mu_m,m\right) = \left(\frac{\alpha_s^{(n_l+1)}}{4\pi}\right)^2 C_F\,\frac{16}{27}\left[9\log^2\left(\frac{m}{\mu_m}\right) + 15\log\left(\frac{m}{\mu_m}\right) + 7\right]\left[1 - 2\log\left(\frac{Q}{M}\right)\right]$$

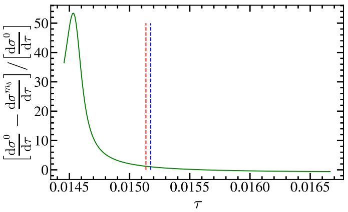
Outline

Results. Pole-scheme cross section

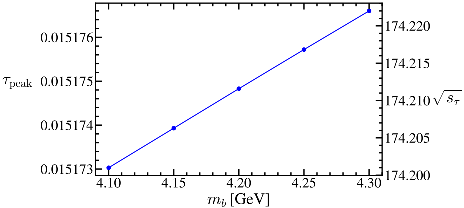
work in progress

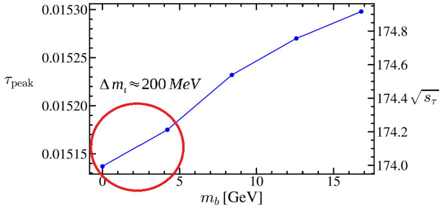
Based on [B. Bachu, A. H. Hoang, V. Mateu, A. Pathak, I. W. Stewart]





$$M_t^{\text{pole}} = 170.034\ \text{GeV}$$
$$Q = 2000\ \text{GeV}$$
$$\Gamma_t = 1.32\ \text{GeV}$$





Conclusions

- $\rightarrow$ For a precise determination of the peak position, massive secondary quark corrections need to be taken into account.
- $\rightarrow$ Missing pieces of thrust and hemisphere mass distribution in the bHQET factorization theorem were computed.
- $\rightarrow$ We proposed a dispersive integration method in Mellin plane to compute massive bubble corrections.
- $\rightarrow$ The results were implemented in a numerical code for thrust bHQET cross section at $\mathrm{N^3LL} + \mathcal{O}(\alpha_s^2)$ accuracy.