# Quantum entanglement and PDFs in 1+1D

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# What problem do we want to solve ?

- Can we compute structure functions such as the PDF by appealing to some simple principle that emerges from strong interactions among partons?

$$f_q(x) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{-itx(n\cdot P)} \left\langle \mathscr{H} \middle| \mathcal{O}_2 \middle| \mathscr{H} \right\rangle,$$

- A description of the hadron state.
- Renormalization of the operator .

- At a more basic level, understand how collective long distance d.o.f. emerge from the microscopic ones connecting different Effective descriptions of gauge theories.
- For QCD, no systematic expansion parameter available, neither is there a dual theory.
- Several simple models in lower dimensions that exhibit QCD like behavior and are solvable in various limits.
- Can we extract some universal information that does not depend on the special symmetries or specifics of these theories and apply them to higher dimensions?

# Gauge Theories in 1+1 D

Abelian : Schwinger model 1+1 D QED

Non-Abelian : ’t Hooft model 1+1 D QCD

- Gauge coupling is dimensionful $[\mathrm{g}] = M^1 \rightarrow \Lambda_{QCD}$
- The strength of the interaction is controlled by $\bar{g} = g/m_q$
- Running effects of the PDF operator suppressed by g/Q
- No spin
- Gauge field is not a propagating degree of freedom but gives rise to a confining potential.
- There is no renormalization

Problem reduces to describing the hadron bound state

# Gauge theories in 1+1D

**Hamiltonian in light cone gauge $A^+ = 0$, $x^+$ is the time variable.**

**Schwinger model : 1+1 QED**

$$P^- = \frac{1}{2}\int dx^- \bar{\psi}_+ \gamma^+ \frac{m^2}{i\partial^+}\psi_+ + \frac{e^2}{2}\int dx^- J^+ \frac{1}{(i\partial^+)^2} J^+$$

Kinetic energy

Instantaneous Linear potential in position space

In the regime of strong coupling $e/m \to \infty$, the spectrum can be solved by looking at the dual theory → A theory of free bosons with mass $e/\sqrt{\pi}$

confinement even at small coupling

**'t hooft model : 1+1 D QCD Hamiltonian**

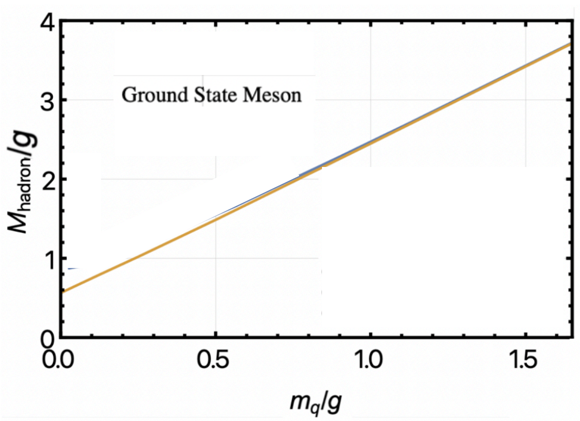
$$P^- = \frac{1}{2}\int dx^- \bar{\psi}_+ \gamma^+ \frac{m^2}{i\partial^+}\psi_+ + \frac{g^2}{2}\int dx^- J^{+,a} \frac{1}{(i\partial^+)^2} J^{+,a}$$

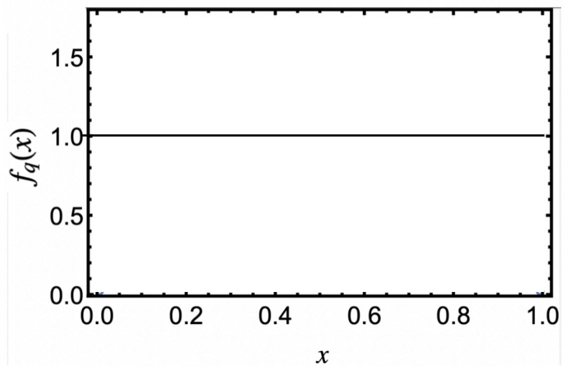
- Large N limit meson spectrum expressed as solution of Bethe Salpeter eqn → 2 body ($q\bar{q}$ ) state schrodinger equation in a confining potential
- Finite N meson, Baryon spectrum solved by numerical methods

- Ground state meson is a $q\bar{q}$ state; contributions from higher fock states is suppressed
- So there is no contribution from sea partons -> Same is true for ground state Baryon

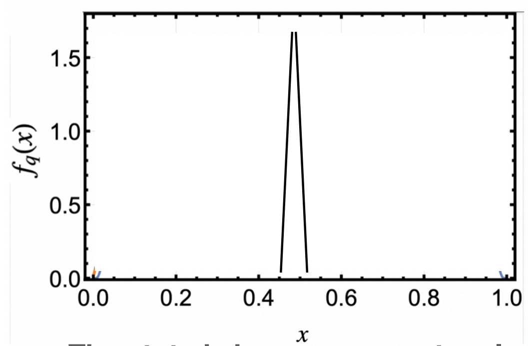
# Spectrum and PDF in 1+1 D

## Schwinger model



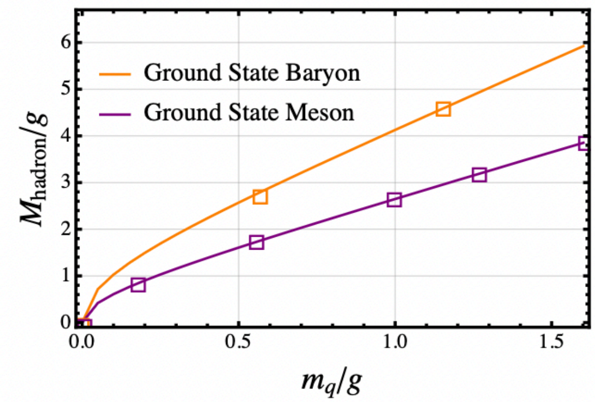




The state is maximally entangled in momentum space at $g/m \to \infty$




The state is becomes unentangled in momentum space at $g/m \to 0$

## ’t hooft model




Conclusion: We have universality of behavior for the PDF in the two limits across theories .

**Perhaps in both cases the system is organizing itself based on some coupling strength dependent measure of Quantum entanglement.**

# A guess at the answer

- The simplest guess is the Von Neumann entropy S as a measure of entanglement.
- For $g/m \to \infty$ , we will extremize S → Maximum entropy principle.
- To describe the system as it moves away from $g/m \to \infty$ , we can draw an analogy with thermodynamics,
- Conjecture a free energy F= E- TS

Expectation value of the **Kinetic Energy**

$$E = \langle P^- \rangle_0 \propto m^2$$

# The Meson wave function

- $q\bar{q}$ state

$$|\psi\rangle = \sum_{i,\bar{j}} p_{i,\bar{j}} \delta_{i+\bar{j},1} |i, \bar{j}\rangle,$$

i ($\bar{j}$) → fraction of the longitudinal momentum $P^+$ carried by quark (antiquark)

- Reduced density matrix is diagonal

$$\rho_1 = \sum_i |p_{i,1-i}|^2 |i\rangle\langle i|.$$

- Entanglement entropy

$$S = -\sum_i |p_{i,1-i}|^2 \ln |p_{i,1-i}|^2,$$

- Kinetic energy

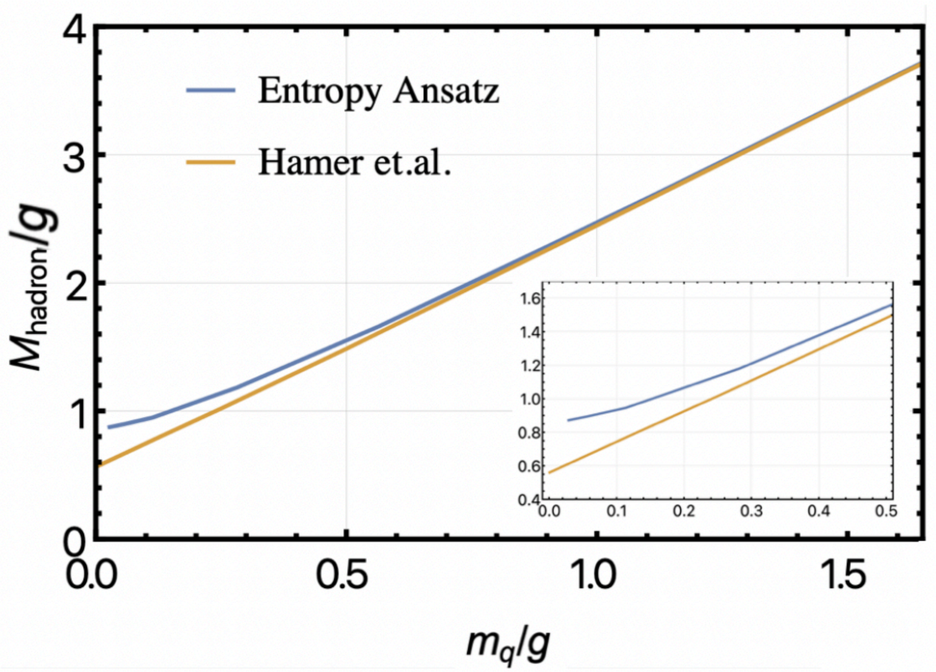
$$\langle E \rangle \equiv \langle P^- \rangle_{\text{kinetic}} = \frac{m^2}{P^+} \sum_i |p_i|^2 \left( \frac{1}{i} + \frac{1}{1-i} \right)$$

- Minimizing F = $\langle E \rangle - TS$ yields the thermal ansatz.

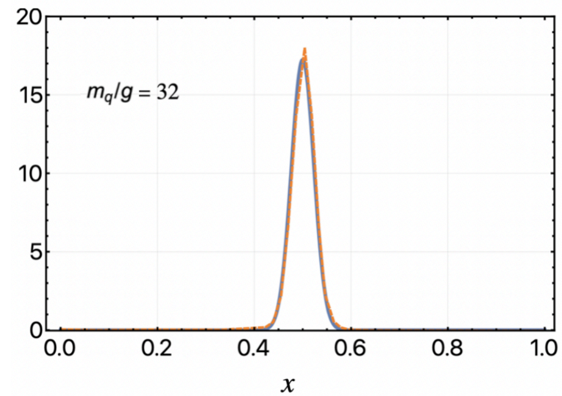
$$|p_{i,1-i}|^2 = \mathcal{N} e^{-\frac{m^2}{TP^+}\left(\frac{1}{i} + \frac{1}{1-i}\right)} \equiv e^{-\frac{\langle E \rangle_0}{T}}$$

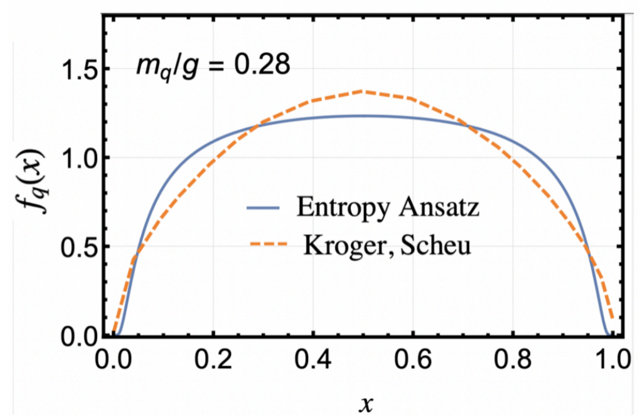
**Minimizing $P^+\langle P^- \rangle = M^2$ allows us to calculate the invariant bound state mass and T.**

# How accurate is a thermal description?

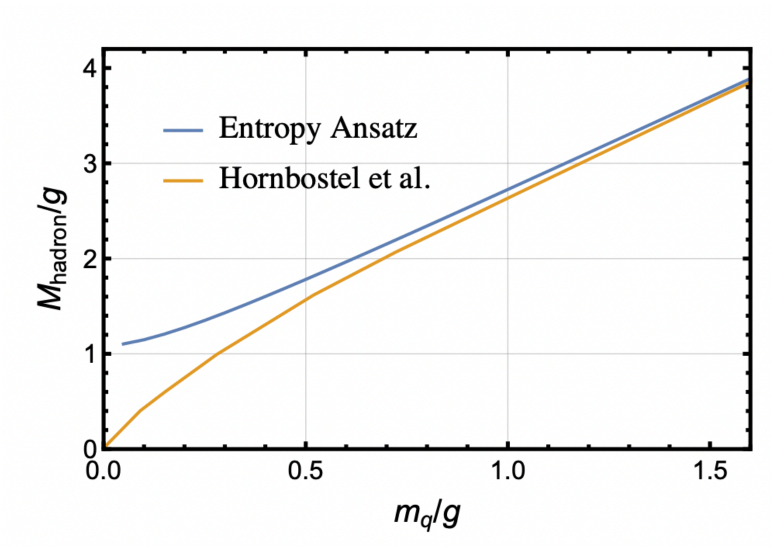


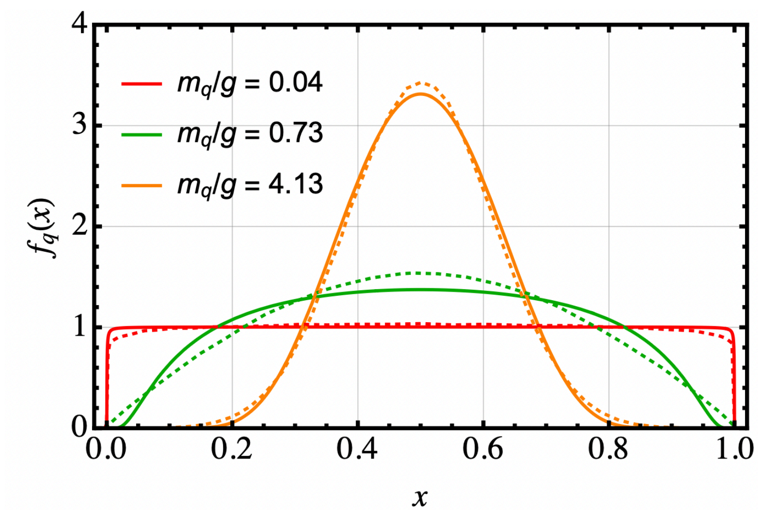

Schwinger model



# How good is a thermal description?




’t hooft model

- A good description for moderate to weak couplings .
- A Thermal description does not work in the non- perturbative regime.

# An Alternative Entanglement measure

- Can we do better than this?
- Perhaps we can use a different measure of entanglement that reduces to the Von Neumann entropy in the weak limit?
- Renyi Entropy. $S_\alpha(\rho) = \frac{1}{1-\alpha} \ln \left( \mathrm{Tr} \left[ \rho^\alpha \right] \right) \xrightarrow{\alpha \to 1} \mathrm{Tr}[\rho \ln \rho]$

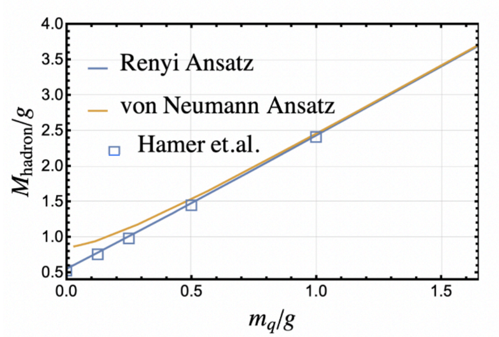
$$F_\alpha = E - TS_\alpha;$$

$$|\psi(x, 1-x)|^2 = \mathcal{N} e^{-\frac{m^2}{TP^+}\left(\frac{1}{x} + \frac{1}{1-x}\right)} \to \mathcal{N}[x(1-x)]^{\frac{1}{1-\alpha}}$$

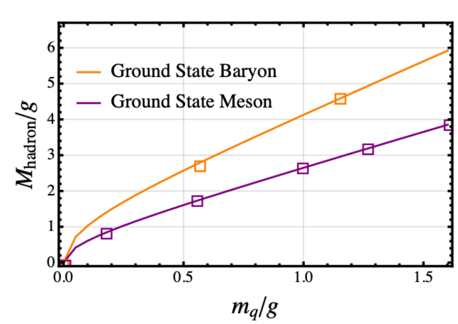
- We still have a single variational parameter $\alpha$, instead of T.

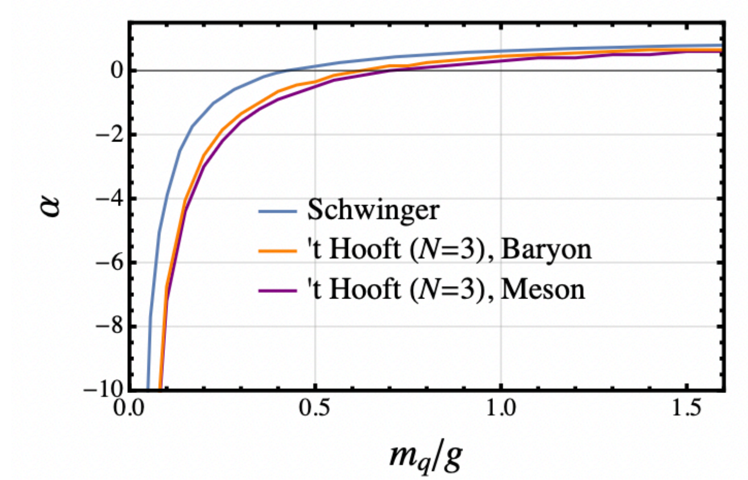
# The Meson mass and PDF

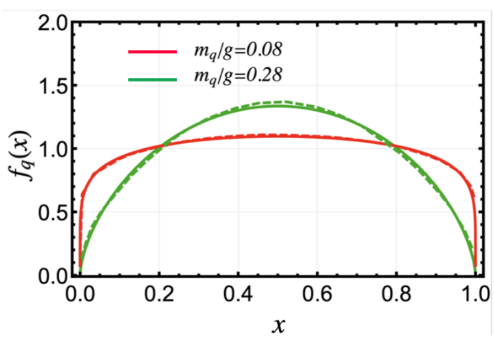
## Schwinger model

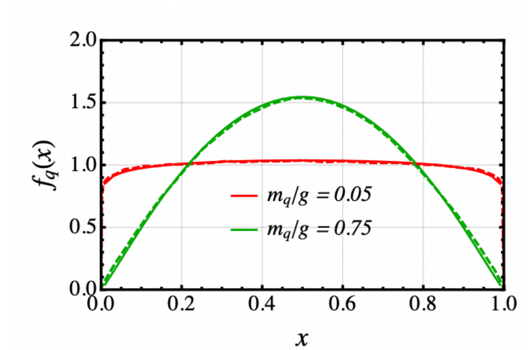



## ’t hooft model




- The order $\alpha$ becomes negative as we enter the non-perturbative regime.
- The system appears to switch from maximizing high probability configurations to minimizing low probability ones.

# The Baryon mass and PDF

- $qqq$ color singlet state .

$$|\psi\rangle = \sum_{ijk=0}^{1} p_{i,j,k}\delta_{i+j+k,1}|ijk\rangle.$$

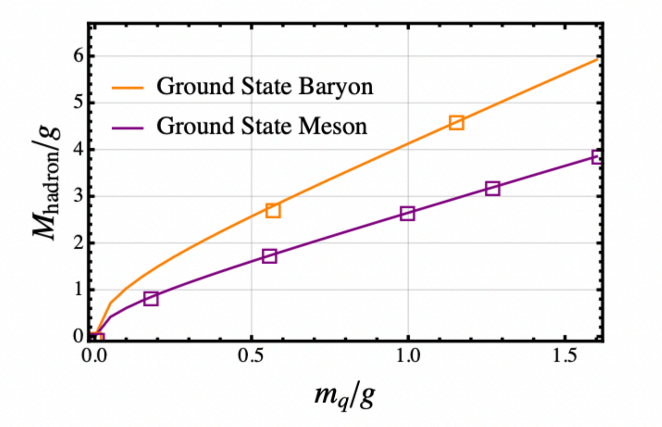
- Reduced density matrix is a weighted sum over two-quark density matrices with a total momentum fraction $1-l$

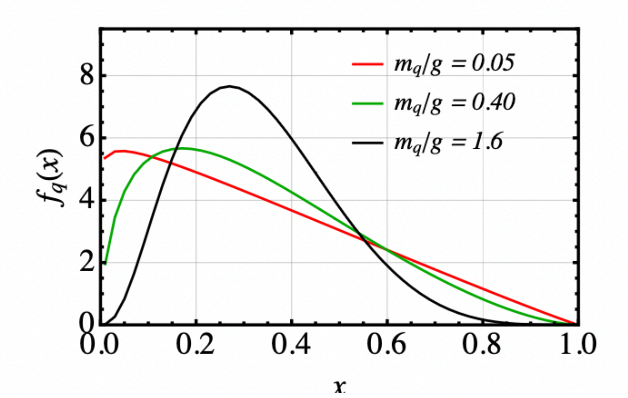
$$\rho_2 = \sum_l \mathcal{N}_l |\psi_l\rangle\langle\psi_l|$$

- Minimize free energy for **each** bi-parton density matrix ,

$$|p_{i,j,1-i-j}|^2 = \mathcal{N}[i \times j \times (1-i-j)]^{\frac{1}{1-\alpha}}$$

- Dictated by the 2-2 interaction in the Hamiltonian



# Outlook

- Does a simple organizing principle persist in higher dimensions?
    - Incorporate renormalization of the theory.
    - Incorporate renormalization of the PDF operator.
    - Contribution from higher occupation Fock states.