Collinear dynamic beyond DGLAP

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arXiv:2210.10061

University of Amsterdam

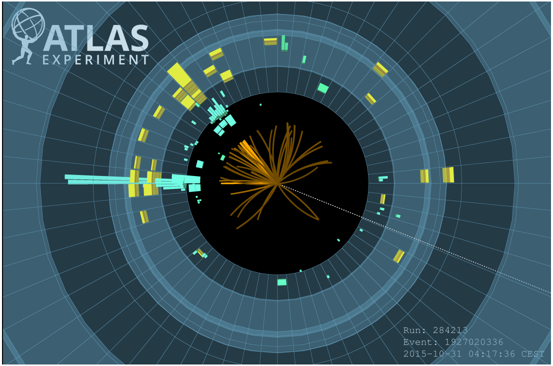
SCET 2023

What is there to love about tracks?

Precision

Track-based measurements can reach higher precision than calorimeter based measurements

- Superior angular resulution of tracking systems
- Pile-up removal



Not so fast

Track-based observables are not infrared-collinear safe

- KLN theorem: Not including all final states $\Rightarrow$ IRC unsafe

Track function formalism

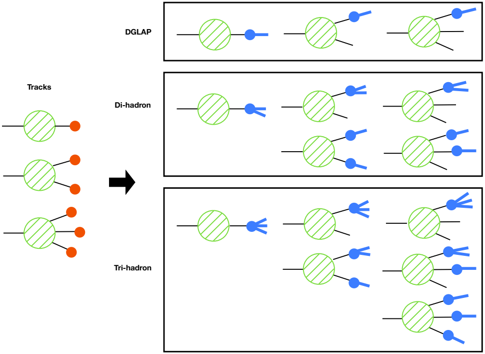
IR divergences are absorbed into non-perturbative track functions

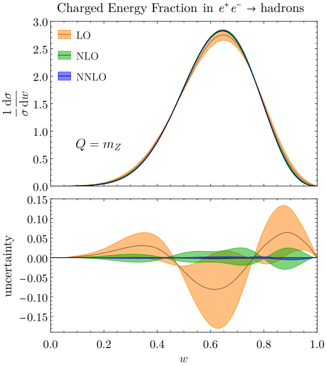
Chang, Procura, Thaler, Waalewijn (2013)

What can you expect today?

$$\frac{\mathrm{d}}{\mathrm{d}\log\mu^2} T_q = a_s \Big[K_{q\to q} \otimes T_q + K_{q\to qg} \otimes T_q T_g + K_{q\to qgg} \otimes T_q T_g T_g$$

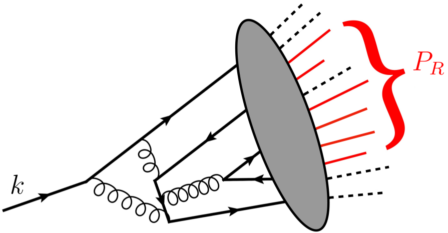
$$+ K_{q\to qq\bar{q}} \otimes T_q T_q T_{\bar{q}} + K_{q\to qQ\bar{Q}} \otimes T_q T_Q T_{\bar{Q}} + \dots \Big]$$

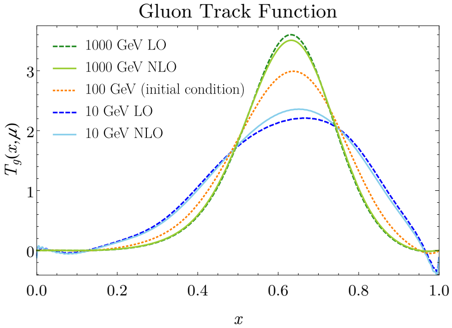




Outline

- Introduction
  - Definition and properties
- Evolution beyond DGLAP
  - Structure of the evolution
  - Calculation
- Universality
  - Reduction to DGLAP
  - N-hadron fragmentation
- Application
  - $e^+e^- \to$ hadrons
- Conclusion





Introduction

Implementation

For a generic cross section with some observable e

$$\frac{\mathrm{d}\sigma}{\mathrm{d}e} = \sum_N \int \mathrm{d}\Pi_N \, \frac{\mathrm{d}\sigma_N}{\mathrm{d}\Pi_N} \, \delta\big[e - \hat{e}(\{p_i\})\big]$$

Partonic cross section with N final state partons

Track function formalism (leading order)

Observe on tracks only ⇒ attach track function to each parton

For the same observable on tracks $\bar{e}$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\bar{e}} = \sum_N \int \mathrm{d}\Pi_N \, \frac{\mathrm{d}\bar{\sigma}_N}{\mathrm{d}\Pi_N} \int \left(\prod_{i=1}^{N} \mathrm{d}x_i \, T_i(x_i) \right) \, \delta\big[\bar{e} - \hat{e}(\{x_i p_i\})\big]$$

Chang, Procura, Thaler, Waalewijn (2013)

Let's get a bit more formal

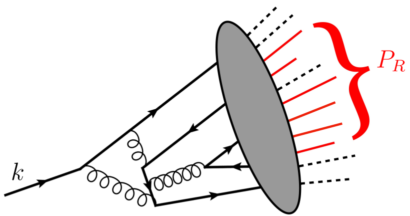
Track function definition: English

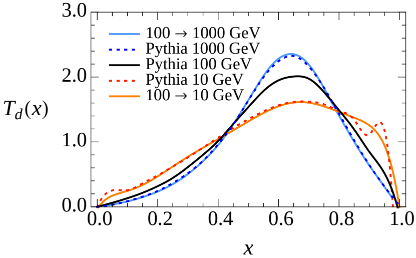
$$T_i(x) \overset{\text{LO}}{=} \begin{pmatrix} \text{Probability density of finding that the charged} \\ \text{particles fragmenting from a parton } i \text{ carry} \\ \text{a certain fraction } x \text{ of the total momentum} \end{pmatrix}$$

Track function definition: QFT

$$T_q(x) = \frac{1}{2N_c} \sum_X \delta\left(x - \frac{P_c^-}{P^-}\right) \int \mathrm{d}y^+ \, \mathrm{d}^{d-2}\mathbf{y} \, e^{iy^+ P^-/2} \times \mathrm{tr}\left[\bar{n}\!\!\!/ \, \langle 0| \chi_n(y^+, 0^-, \mathbf{y}) |X\rangle \langle X| \bar{\chi}_n(0) |0\rangle\right]$$

- Like a FF, but measuring the momentum of a group of particles





Probability density concerning a subset of fragments

- Only has support for $x \in [0, 1]$
- Normalised to 1

$$\int_0^1 \mathrm{d}x \, T_i(x) = 1$$

- Calculable scale dependence

Evolution

Motivation

Why should we care?

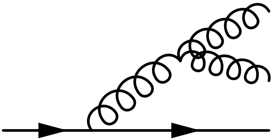
To achieve precise predictions for track-based observables the evolution of track functions must be known beyond leading order

But wait, there is more!

- Predicts structure of IR divergences in perturbative calculations
  For fragmentation at NNLO see: Gehrmann, Schürmann (2022), Gehrmann, Stagnitto (2022)

- Higher-order collinear corrections to next-gen parton showers
  Dasgupta et al. (2020), Li, Skands (2017), Höche, Prestel (2017), . . .

Changing the scales



There goes linearity

All parton branches contribute $\Rightarrow$ Non-linear evolution equations

For the quark track function

$$\frac{\mathrm{d}}{\mathrm{d}\log\mu^2} T_q = a_s \Big[K_{q\to q} \otimes T_q + K_{q\to qg} \otimes T_q T_g + K_{q\to qgg} \otimes T_q T_g T_g + K_{q\to qq\bar{q}} \otimes T_q T_q T_{\bar{q}} + K_{q\to qQ\bar{Q}} \otimes T_q T_Q T_{\bar{Q}} + \dots \Big]$$

Beyond LO

At leading order track evolution kernels are just the DGLAP kernels

$$K^{(0)}_{q\to qg}(z, 1-z) = P^{(0)}_{qq}(z)$$

Goal

Calculate the evolution kernels beyond LO

Problem: Track functions are scaleless objects

$$T_q(x) = \delta(1-x) + 2a_s C_F \left(\frac{1}{\epsilon_{\text{uv}}} - \frac{1}{\epsilon_{\text{ir}}}\right)\left[\frac{1+x^2}{1-x}\right]_+ + \mathcal{O}(a_s^2)$$

The burden of being scaleless

Calculate evolution kernels $\Rightarrow$ single out UV divergence

Beyond LO

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The burden of being scaleless

Calculate evolution kernels $\Rightarrow$ single out UV divergence

Plan B

Indirect calculation

Calculate evolution via another object defined on tracks

Recipe for calculating evolution kernels to any order

1. Calculate bare track jet function
2. Treat overlapping divergences
3. Renormalize the track jet function
4. Extract evolution kernels from single poles

Step 1

1. Calculate bare track jet function
2. Treat overlapping divergences
3. Renormalize the track jet function
4. Extract evolution kernels from single poles

Step 1: Calculate bare track jet function

Indirect calculation

Track evolution by matching track jet function onto track functions

Consider the standard invariant mass jet function

$$J_i^{\text{bare}}(s) = \sum_N \sum_{\{i_f\}} \int \mathrm{d}\Phi_N^c \, \frac{1}{S_{\{i_f\}}} \sigma_{i\to\{i_f\}}\big(\{z_f\}, \{s_{ff'}\}, s\big)$$

Now consider the same object, but on tracks

$$J_{\text{track},i}^{\text{bare}}(s, x) = \sum_N \sum_{\{i_f\}} \int \mathrm{d}\Phi_N^c \, \frac{1}{S_{\{i_f\}}} \sigma_{i\to\{i_f\}}\big(\{z_f\}, \{s_{ff'}\}, s\big)$$
$$\times \int \mathrm{d}x_1 \cdots \mathrm{d}x_N \; T_{i_1}^{(0)}(x_1) \cdots T_{i_N}^{(0)}(x_N) \; \delta\big(x - z_1 x_1 \ldots - z_N x_N\big)$$

Ritzmann, Waalewijn (2014)
Li, Moult, Schrijnder van Velzen, Waalewijn, Zhu (2022)

Step 1: Calculate bare track jet function

Indirect calculation

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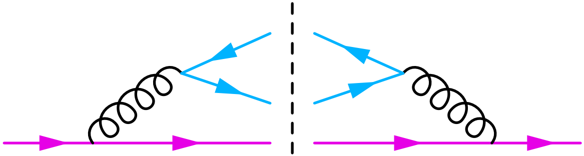


- For the standard jet function

$$J_i^{\text{bare}}(s) = \delta(s)\, J^{(0)}_{q\to qQ\bar{Q}} \;+\; \mathcal{L}_n(s)\ \text{terms} + \ldots$$

- For the track jet function

$$J^{\text{bare}}_{\text{track},i}(s,x) = \delta(s)\, \mathcal{J}^{(0)}_{q\to qQ\bar{Q}} \otimes T_q^{(0)} T_Q^{(0)} T_{\bar{Q}}^{(0)}(x) + \ldots$$

Kosower, Uwer (2003)
Campbell, Glover (1998)
Catani, Grazzini (1999)

Step 2

1. Calculate bare track jet function
2. Treat overlapping divergences
3. Renormalize the track jet function
4. Extract evolution kernels from single poles

Step 2: Treat overlapping divergences

Single momentum fraction $\Rightarrow$ soft divergences at $z \to 0, 1$

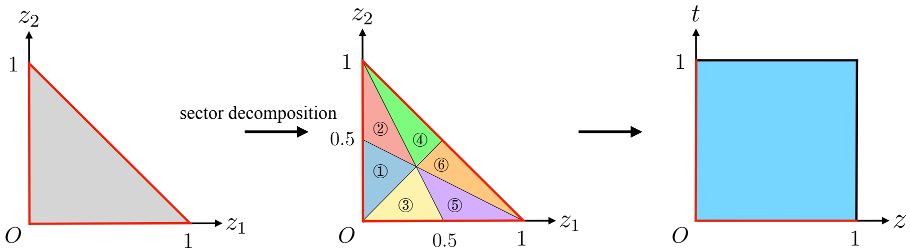
$$\frac{1}{z^{1+\epsilon}} = -\frac{1}{\epsilon}\delta(z) + \left[\frac{1}{z}\right]_+ + \ldots \qquad \int_0^1 \mathrm{d}z \left[\frac{1}{z}\right]_+ f(z) = \int_0^1 \mathrm{d}z \, \frac{f(z) - f(0)}{z}$$

Multiple momentum fractions $\Rightarrow$ divergent regions overlap

- Partons can become soft either individually or simultaneous

$$\frac{1}{z_1^{\epsilon}(z_1 + z_2)^{1+\epsilon}}$$

Sector decomposition



Binoth, Heinrich (2000)
Binoth, Heinrich (2004)
Anastasiou, Melnikov (2004)

1. Divide the phase space up into 6 six sectors
   - sector 1: $z_1 < z_2 < z_3$, ...

2. Change variables in each sector

Sector decomposition

Sector decomposition disentangles overlapping divergences

Step 3

1. Calculate bare track jet function
2. Treat overlapping divergences
3. Renormalize the track jet function
4. Extract evolution kernels from single poles

Step 3: Renormalize the track jet function

A useful property

Renormalization of J is not sensitive to the momentum fractions

Renormalization kernels for the two jet functions are the same

$$J_i(s) = \int_0^s \mathrm{d}s' \; Z_{J_i}(s') \; J_i^{\text{bare}}(s-s')$$

$$J_{\text{track},i}(s,x) = \int_0^s \mathrm{d}s' \; Z_{J_i}(s') \; J_{\text{track},i}^{\text{bare}}(s-s',x)$$

Ritzmann, Waalewijn (2014)
Becher, Neubert (2006)
Becher, Bell (2010)

Step 4

1. Calculate bare track jet function
2. Treat overlapping divergences
3. Renormalize the track jet function
4. Extract the evolution kernels from the single poles

Step 4: Extract evolution from the single poles

After renormalization J_{track} will still contain (IR) poles in ϵ

$$J_q^{(2)}(s) = \text{finite}$$

$$J_{\text{track},i}^{(2)}(s,x) = \text{finite} - \frac{1}{2\epsilon}\delta(s)\, K_{q\to qgg}^{(1)} \otimes T_q^{(0)} T_g^{(0)} T_g^{(0)}(x) + \dots$$

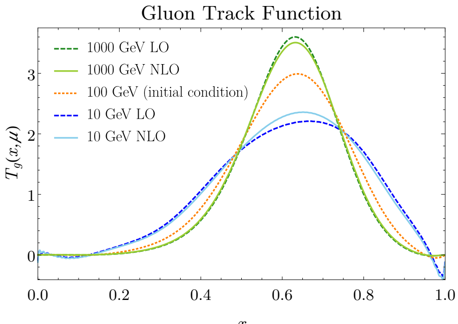
Evolution kernels

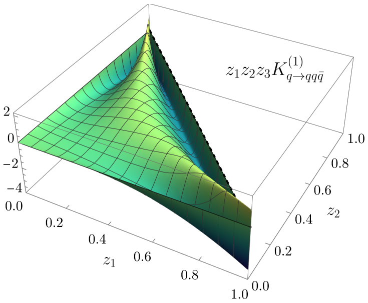
Obtain evolution kernels from poles of J_{track} that remain after renormalization*

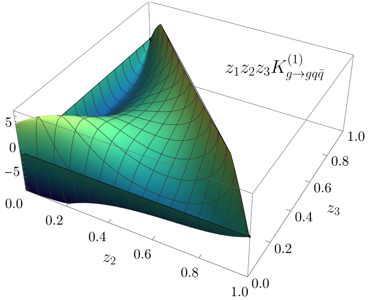
*Some one-loop cross terms need to be subtracted first

Results

- NLO kernels for all channels
- Correlations in fragmentation
- Agrees with moment analysis
- Public code available!
  - **https://github.com/HaoChern14/Track-Evolution**







Universality

More than just tracks

Claim

The track function evolution kernels are universal

What do we mean with this claim?

- Reduction to DGLAP kernels
- Reduction to the evolution of di-hadron fragmentation functions
- Generalization to N-hadron fragmentation functions

Reduction to DGLAP

Track functions: All branches $\Rightarrow$ non-linear evolution

$$\frac{\mathrm{d}}{\mathrm{d}\log\mu^2}T_i = K_{i\to i}\otimes T_i + K_{i\to i_1 i_2}\otimes T_{i_1}T_{i_2} + K_{i\to i_1 i_2 i_3}\otimes T_{i_1}T_{i_2}T_{i_3} + \dots$$

Fragmentation functions: One branch $\Rightarrow$ linear evolution

$$\frac{\mathrm{d}}{\mathrm{d}\log\mu^2}D_{i\to h} = P_{ij}\otimes D_{j\to h}$$

Gribov, Lipatov (1972)
Dokshitzer (1977)
Altarelli, Parisi (1977)

Universality

The DGLAP kernels can be obtained from the track evolution kernels by integrating over all unobserved branches

Reduction to DGLAP

$$P_{qq}(z) = K_{q\to q}\delta(1-z) \quad + \quad K_{q\to qg}(z, 1-z)$$

$$+ \int \mathrm{d}z_1\, \mathrm{d}z_2\, \mathrm{d}z_3\, \delta(1-z_1-z_2-z_3)\, K_{q\to qgg}(z_1, z_2, z_3)\, \delta(z-z_1)$$

$$+ \int \mathrm{d}z_1\, \mathrm{d}z_2\, \mathrm{d}z_3\, \delta(1-z_1-z_2-z_3)\, K_{q\to qq\bar{q}}(z_1, z_2, z_3)\, \delta(z-z_1)$$

$$+ \int \mathrm{d}z_1\, \mathrm{d}z_2\, \mathrm{d}z_3\, \delta(1-z_1-z_2-z_3)\, K_{q\to qq\bar{q}}(z_1, z_2, z_3)\, \delta(z-z_2)$$

$$+ \ldots$$

Reduction to DGLAP

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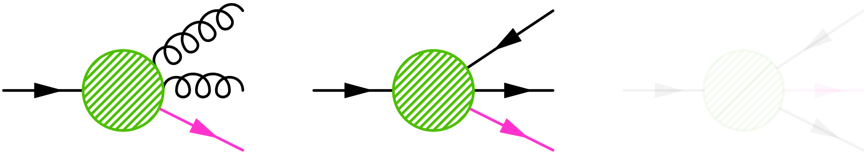
$$+ \int \mathrm{d}z_1\,\mathrm{d}z_2\,\mathrm{d}z_3\,\delta(1-z_1-z_2-z_3)\,K_{q\to qq\bar{q}}(z_1,z_2,z_3)\,\delta(z-z_2)$$
$$+ \ldots$$



$$
\begin{aligned}
P_{qq}(z) = {} & K_{q\to q}\,\delta(1-z) \quad + \quad K_{q\to qg}(z, 1-z) \\
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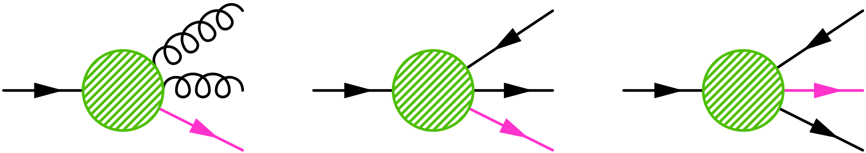
& + \int \mathrm{d}z_1\,\mathrm{d}z_2\,\mathrm{d}z_3\,\delta(1-z_1-z_2-z_3)\,K_{q\to qq\bar{q}}(z_1,z_2,z_3)\,\delta(z-z_2) \\
& + \ldots
\end{aligned}
$$

Reduction to DGLAP



$$P_{qq}(z) = K_{q\to q}\delta(1-z) \quad + \quad K_{q\to qg}(z, 1-z)$$
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$$+ \int dz_1\, dz_2\, dz_3\, \delta(1-z_1-z_2-z_3)\, K_{q\to qq\bar{q}}(z_1, z_2, z_3)\, \delta(z-z_1)$$
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$$+ \dots$$

Evolution of track functions

$$\frac{\mathrm{d}}{\mathrm{d}\log\mu^2} T_i = K_{i\to i} \otimes T_i + K_{i\to i_1 i_2} \otimes T_{i_1} T_{i_2} + K_{i\to i_1 i_2 i_3} \otimes T_{i_1} T_{i_2} T_{i_3} + \ldots$$

Evolution for di-hadron fragmentation functions

$$\frac{\mathrm{d}}{\mathrm{d}\log\mu^2} D_{i\to h_1 h_2} = \ldots \otimes D_{j\to h_1 h_2} + \otimes \ldots D_{i_1\to h_1} D_{i_2\to h_2} + \ldots$$

- Linear term: $i \to j \ldots$ followed by $j \to h_1 h_2 \ldots$
- Quadratic term: $i \to i_1 i_2 \ldots$ followed by $i_1 \to h_1 \ldots$ and $i_2 \to h_2 \ldots$

Konishi, Ukawa (1979)
Sukhatme, Lassila (1980)
Sukhatme, Lassila, Orava (1982)
de Florian, Vanni (2004)
Vendramin (1981)
Majumder, Wang (2004)

Evolution of track functions

$$\frac{\mathrm{d}}{\mathrm{d}\log\mu^2} T_i = K_{i\to i} \otimes T_i + K_{i\to i_1 i_2} \otimes T_{i_1} T_{i_2} + K_{i\to i_1 i_2 i_3} \otimes T_{i_1} T_{i_2} T_{i_3} + \ldots$$

Evolution for di-hadron fragmentation functions

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Konishi, Ukawa (1979)
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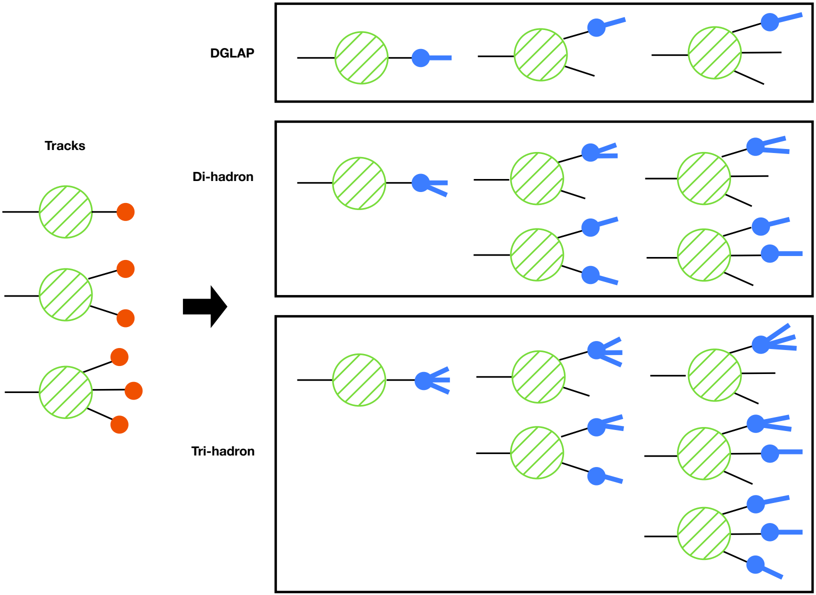
Reduction



Application

$e^+e^- \to$ hadrons

- w: Charged energy fraction

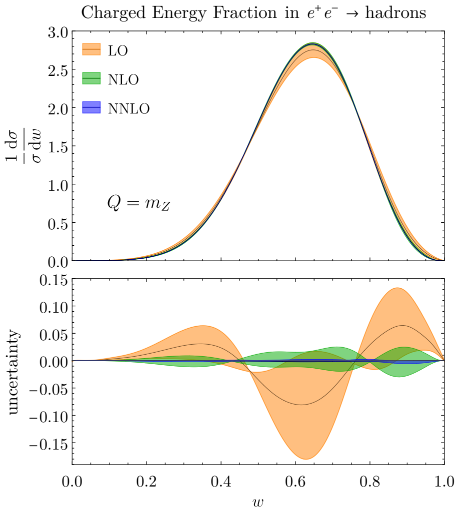
$$\frac{\mathrm{d}\sigma_{e^+e^-\to\text{hadrons}}}{\mathrm{d}w}$$

- Convolution of track functions

$$\frac{\mathrm{d}\sigma^{\text{LO}}}{\mathrm{d}w} \propto \int \mathrm{d}x\, T_q(x) T_{\bar{q}}(w-x)$$

- Requires evolution of $T_i(x)$

- Significant error reduction



Energy correlators (work in progress)

What is an energy correlator?

Observable that describes correlations within the energy flow of a jet

Neutral particles are detected via calorimeter cells

- Low angular resolution limits precision

Energy correlators on tracks

Measuring EECs on tracks allows for higher precision measurements

For all charged particles, measure the energies and angles

$$\frac{\mathrm{d}\sigma^{[N]}}{\mathrm{d}z} = \sum_{\{i\}} \int \mathrm{d}\sigma \, \frac{E_{i_1} \cdots E_{i_N}}{Q^N} \delta\left(z - \frac{1 - \cos\theta_{\text{max}}}{2}\right)$$

Lee, Meçaj, Moult (2022)
Dixon, Moult, Zhu (2019)
Chen, Luo, Moult, Yang, Zhang (2020)
Chen, Moult, Zhang, Zhu (2020)

Conclusions

Conclusion

- Track function formalism
  - Calculation of track-based observables

- Evolution
  - Evolution of track functions now known to NLO

- Universality
  - Reduction to DGLAP and N-hadron fragmentation

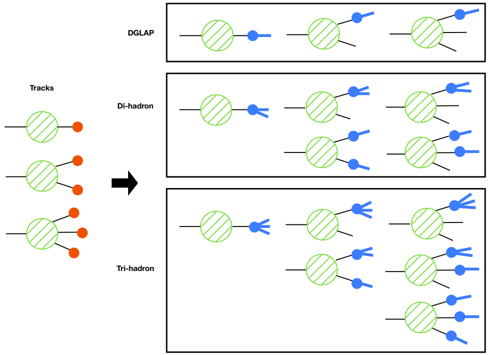
- Application
  - Working towards energy correlators on tracks

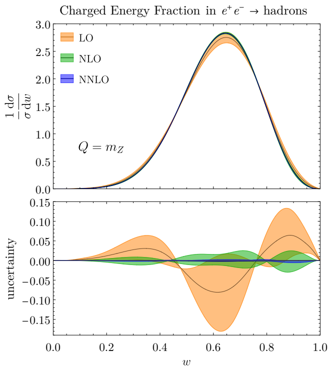
Main achievement

Universal collinear evolution kernels calculated to NLO

Thank you for your attention!

$$\frac{\mathrm{d}}{\mathrm{d}\log\mu^2} T_q = a_s\Big[K_{q\to q}\otimes T_q + K_{q\to qg}\otimes T_q T_g + K_{q\to qgg}\otimes T_q T_g T_g$$
$$+ K_{q\to qq\bar{q}}\otimes T_q T_q T_{\bar{q}} + K_{q\to qQ\bar{Q}}\otimes T_q T_Q T_{\bar{Q}} + \dots\Big]$$





Backup slides

Backup I - Convolution

$$
\begin{aligned}
&K_{i\to i_1 i_2 i_3} \otimes T_{i_1} T_{i_2} T_{i_3}(x) \\
&= \int \mathrm{d}x_1\, \mathrm{d}x_2\, \mathrm{d}x_3\, T_{i_1}(x_1) T_{i_2}(x_2) T_{i_3}(x_3) \int \mathrm{d}z_1\, \mathrm{d}z_2\, \mathrm{d}z_3\, \delta(1 - z_1 - z_2 - z_3) \\
&\qquad \times \delta(x - z_1 x_1 - z_2 x_2 - z_3 x_3)\, K_{i\to i_1 i_2 i_3}(z_1, z_2, z_3)
\end{aligned}
$$

Backup II - Sector decomposition

$$K_{i\to i_1 i_2 i_3} \otimes T_{i_1} T_{i_2} T_{i_3}(x)$$
$$= \sum_{n=1}^{6} \int \mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3\, T_{i_1}(x_1) T_{i_2}(x_2) T_{i_3}(x_3) \int \mathrm{d}z\,\mathrm{d}t$$
$$\times\, \delta\Big(x - {}^n z_1 x_1 - {}^n z_2 x_2 - {}^n z_3 x_3\Big)\, {}^n K_{i\to i_1 i_2 i_3}(z,t)$$

where

$${}^n z_i \in \left\{ \frac{zt}{1+z+zt}, \frac{z}{1+z+zt}, \frac{1}{1+z+zt} \right\}$$