Glauber Quarks in Backward Scattering

Arindam Bhattacharya (Harvard University)

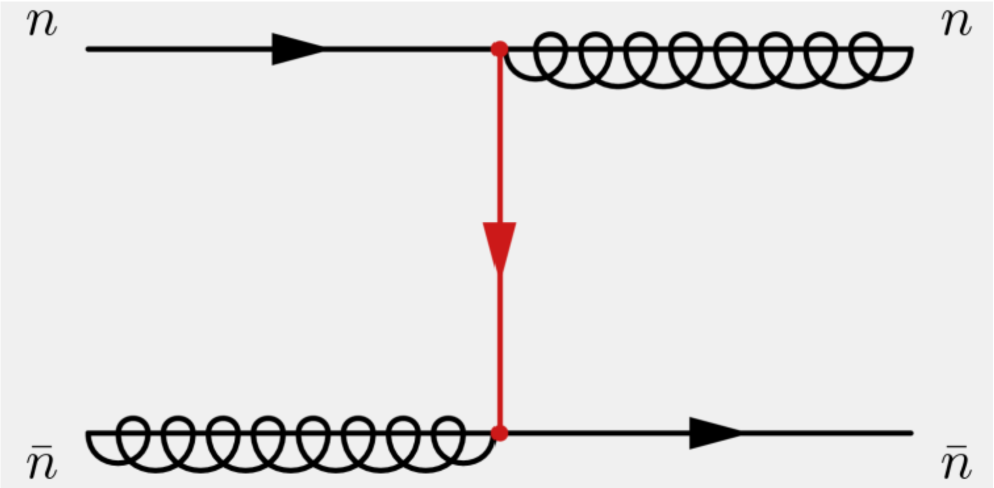
SCET 2023, LBNL

Based on work JHEP 02 (2022) 091 with M.D. Schwartz (Harvard University) & A.V. Manohar (UCSD)

And ongoing work with X.Y. Zhang (Harvard University)

Outline

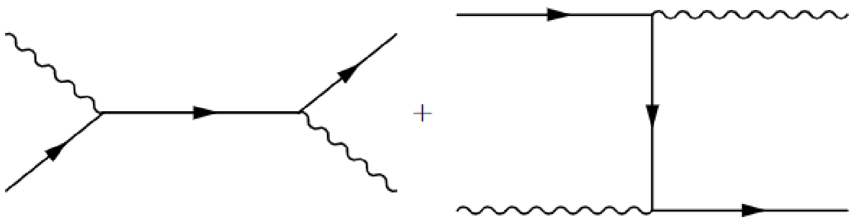
- Motivation
- Backscattering and Glauber Quarks
- SCET and Glauber Modes
- Application to Back Scattering Resummation
- Conclusion



Motivation : Compton Scattering Cross Section

- $e^- \gamma \to e^- \gamma$ (Compton) scattering has a weird feature. Large Log in a tree level process!

$$\frac{1}{s}\int d\Pi_2 \left|\ \cdots\ \right|^2$$



$$= \frac{2\pi\alpha^2}{s}\left[\ln\frac{s}{m_e^2} + \cdots\right]$$

- No well defined lower order process that radiatively generates this log.

- What's going on? Why this large log? Can we resum this large log in the *total inclusive cross section*? $\sigma(e^-\gamma \to e^-(+n\gamma))$

- Let's see what higher order FO computations tell us.

Motivation : Total Cross Sections in QED

- *Total* cross sections of **all** $2 \to 2(+1)$ processes in QED show this novel enhancement

$$\sigma(e^-\gamma \to e^-\gamma(+\gamma)) = \sigma_{\rm LO}\left[1 + \frac{1}{3}\frac{\alpha L^2}{2\pi} - \frac{1}{2}\frac{\alpha L}{2\pi} + \frac{17}{4}\frac{\alpha}{2\pi}\cdots\right]$$

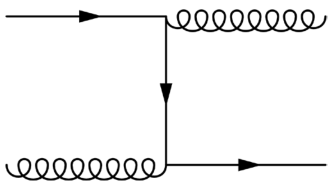
Phenomenologically relevant!
Correction $\sim 8.5\%$@ 1 GeV
$\sim 30\%$ @ 1 TeV

R.N. Lee, M.D. Schwartz, X.Y. Zhang (2021)

$$\sigma_{\rm LO} = \frac{2\pi\alpha^2}{s}L\ , \qquad L \equiv \ln\left(\frac{s}{m_e^2}\right) \gg 1$$

- Log series $\sim (\alpha L^2)^n$. Sudakov log of soft/collinear origin? Since this is an inclusive total cross section, unclear
- Interesting to understand this theoretically as well. How fast do total cross sections grow in the high energy limit? (Froissart bound)
- What region in phase space is responsible for this large log?

Backward Scattering



$$\left| \cdots \right|^2 \sim \frac{1}{-t} \sim \frac{1}{1+\cos\theta_e}$$

- Backward scattering gives large contribution

- Electron mass cutsoff singularity and generates the large log

$$\int_{-m_e^2}^{s} \frac{dt}{-t} \sim L$$

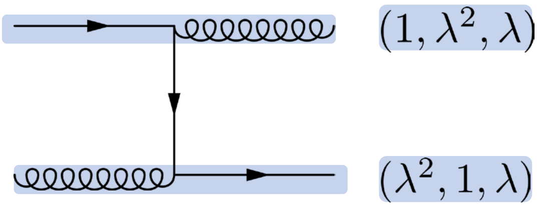
- Previous work has explored this limit at amplitude level (virtual cross section) (Ashoke Sen (1985))

- Can we work in $m_e = 0$ limit? Differential cross section for large angle scattering (Arbuzov, et al. 2011) is IR finite in this limit. Only LL correct, so needs improvement

- Setup EFT to explore this $m = 0$ backscaterring limit. What are the modes?

Backward Scattering : Glauber Quarks

- Outgoing and incoming particles have collinear scaling



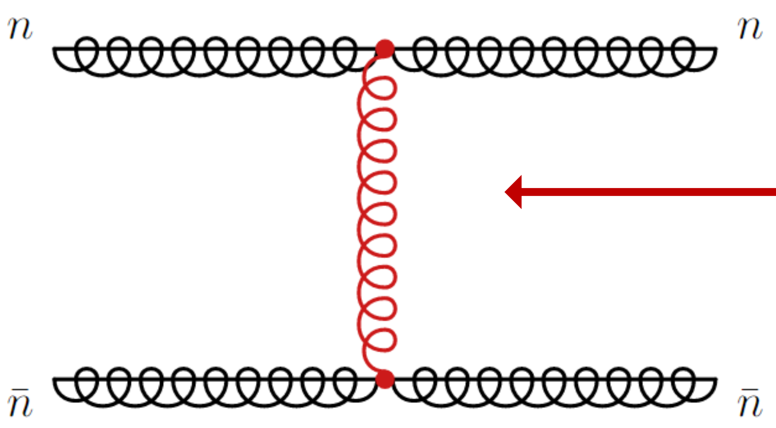
- t channel electron is offshell and subdominant scaling to collinear.

$$p_e \lesssim Q(1, \lambda^2, \lambda) \qquad\qquad p_e \lesssim Q(\lambda^2, 1, \lambda)$$

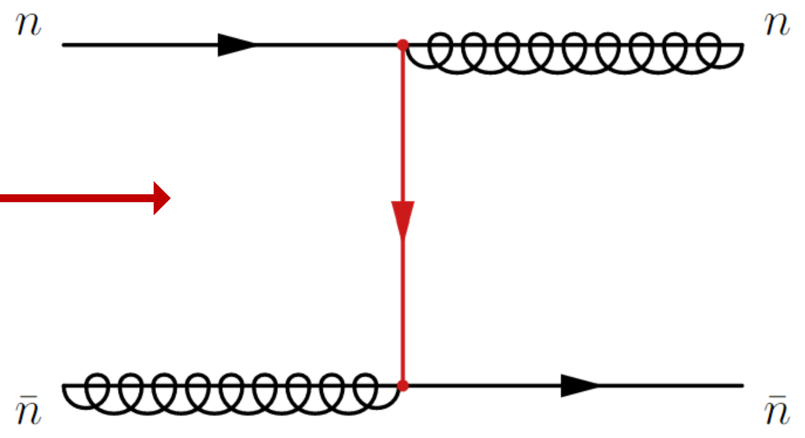
$$\text{Glauber} \implies p_e \sim Q(\lambda^2, \lambda^2, \lambda)$$

- EFT description of backscattering region $\implies$ SCET + Glauber modes

- Regge Limit of QCD
- Small x Resummation
- BFKL Evolution

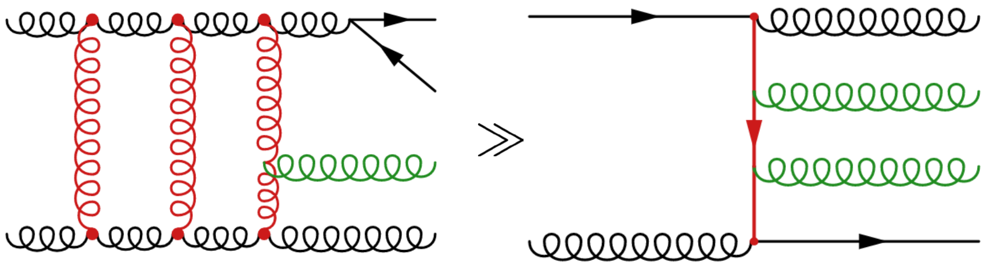


$$(\lambda^2, \lambda^2, \lambda)$$



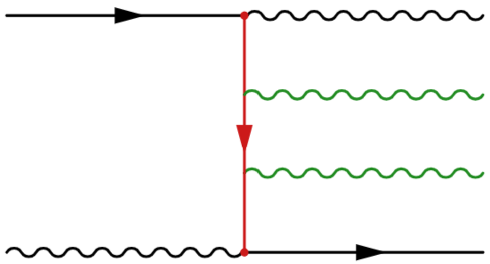
$$\lambda^2 \sim \frac{-t}{s} \sim \frac{1+\cos\theta}{2} \ , \ \theta \approx \pi$$

Glauber Physics: QED vs QCD



$$\sigma \sim \frac{1}{m^2}\left(\alpha_s \ln^2 \frac{s}{m^2}\right)^n \quad + \quad \frac{1}{s}\left(\alpha_s \ln^2 \frac{s}{m^2}\right)^n$$

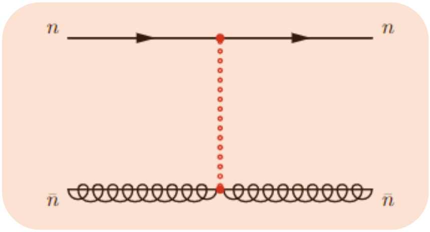
(Catani, Ciafaloni, Hautmann (1991))
(Ball, Ellis (2000))

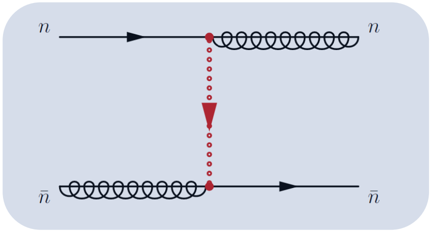


$$\sigma \sim \frac{1}{s}\left(\alpha \ln^2 \frac{s}{m_e^2}\right)^n$$

- Glauber Gluons are power enchanced compared to Glauber Quarks
- Treatment of forward/backward scattering in QCD needs careful treatment of PDFs and non-perturbative contributions

- Glauber Photons give rise to Coulomb phase, simple exponentiation
- Glauber Electrons give rise to large logs in the cross section at *leading power* in QED
- Theory fully perturbative in backward/forward scattering region





In particular $qg \to qg$ is dominated by forward scattering mediated by Glauber gluons

while

$e\gamma \to e\gamma$ is dominated by backward scattering mediated by Glauber electron (quark)

$\Longrightarrow$ QED is a better testbed to flesh out Glauber quark physics

Resumming Logs in Inclusive Cross section using Glaubers

Find large logs in QED Total cross sections

Relevant modes, find correct EFT description

Collinear $\sim (1, \lambda^2, \lambda)$

Soft $\sim (\lambda, \lambda, \lambda)$

Glauber Quarks $\sim (\lambda^2, \lambda^2, \lambda)$

$\lambda^2 \sim \frac{-t}{s} \sim \frac{1+\cos\theta}{2}$, $\theta \approx \pi$

Matching

2110.02978 - Work by **AB**, A.V. Manohar, M.D. Schwartz

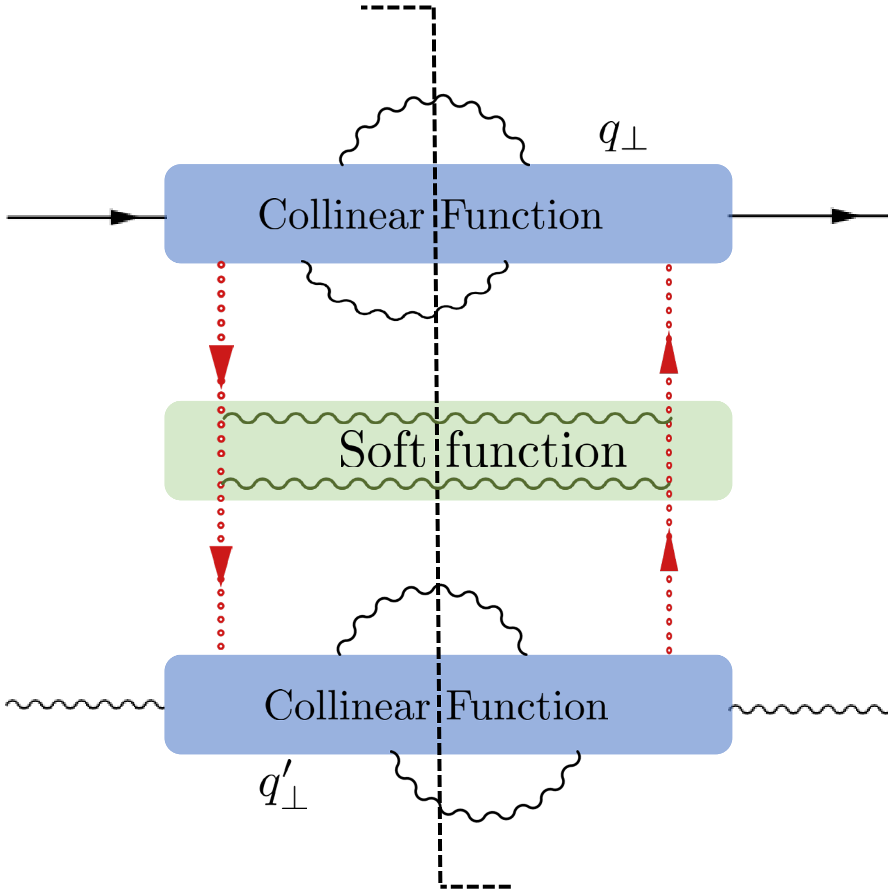
SCET_{II} + Glauber Quark Operators

Ongoing work by **AB**, A.V. Manohar, M.D. Schwartz, X.Y. Zhang

Construct Factorization theorem for inclusive cross section

Resum large logs using RGE in SCET

Prelude: Factorization for Inclusive Cross Section



- Expect a factorization theorem as follows

Rothstein and Stewart (2016)

$$\frac{d^4\sigma}{d^2q_\perp d^2q'_\perp} = C_n(q_\perp, p_e\cdot\bar{n}) \times S(q_\perp, q'_\perp) \times C_n(q'_\perp, p_\gamma \cdot n)$$

$$C_n(q_\perp, p_e\cdot\bar{n}) \sim \frac{1}{q_\perp^2}\sum_{X_n} \langle e^-| \mathcal{O}_n |\gamma, X_n\rangle \langle \gamma, X_n| \mathcal{O}_n |e^-\rangle$$

$$S(q_\perp, q'_\perp) \sim \sum_{X_s} \langle 0| \mathcal{O}_s |X_s\rangle \langle X_s| \mathcal{O}_s |0\rangle$$

$$C_{\bar{n}}(q_\perp, p_e\cdot\bar{n}) \sim \frac{1}{{q'_\perp}^2}\sum_{X_{\bar{n}}} \langle \gamma| \mathcal{O}_{\bar{n}} |e^-, X_{\bar{n}}\rangle \langle e^-, X_{\bar{n}}| \mathcal{O}_{\bar{n}} |\gamma\rangle$$

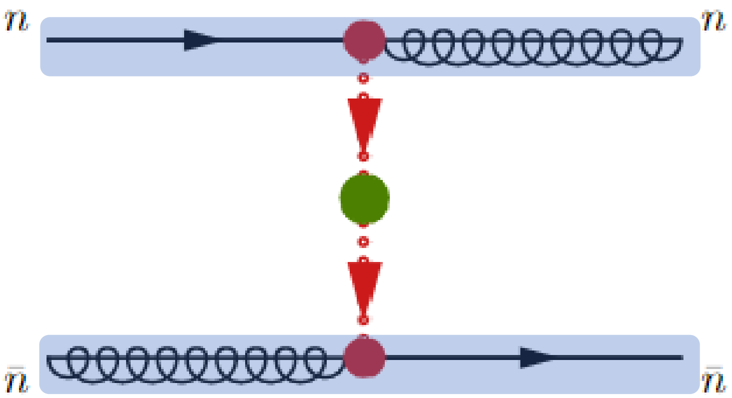
- No hard function? Clear from the matching computation (next)

Matching to SCET + Glauber Quarks

- Glauber modes talk to all other modes. Additional operators in the Lagrangian

$$\mathcal{L} = \mathcal{L}_n + \mathcal{L}_{\bar{n}} + \mathcal{L}_{\text{soft}} + \mathcal{O}_n \frac{1}{\mathcal{P}_\perp^a} \mathcal{O}_s \frac{1}{\mathcal{P}_\perp^a} \mathcal{O}_{\bar{n}} + \cdots$$

Glauber Gluons $a = 2$,
Glauber Quarks $a = 1$



Collinear $\sim (1, \lambda^2, \lambda)$

Soft $\sim (\lambda, \lambda, \lambda)$

Glauber Quarks $\sim (\lambda^2, \lambda^2, \lambda)$

- Include these operators , compute $qg \to qg$ in backscattering limit of QCD and SCET

- Does SCET + Glauber quark capture entire IR dynamics of QCD(QED) in backscattering limit?

SCET+Glauber Modes : Lagrangian & Operators

- Operator basis for Glauber gluons first worked out by I.Z. Rothstein & I.W.Stewart (2016)

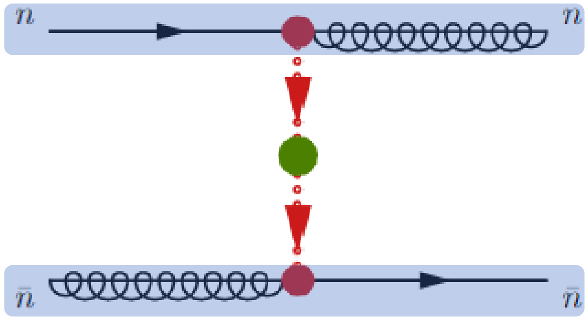
$$\mathcal{O}^{ij}_{ns\bar{n}} = \mathcal{O}^{iB}_{n} \frac{1}{\mathcal{P}^2_\perp} \mathcal{O}^{BC}_{s} \frac{1}{\mathcal{P}^2_\perp} \mathcal{O}^{jC}_{\bar{n}} \qquad \mathcal{O}^{ij}_{ns} = \mathcal{O}^{iB}_{n} \frac{1}{\mathcal{P}^2_\perp} \mathcal{O}^{jB}_{s}$$

Collinear-Soft-Collinear $\quad i, j$ parton forward scattering $\quad$ Collinear-Soft

- Tree level operators needed for Glauber quark exchange worked out by I. Moult, M. Solon, I.W. Stewart, G. Vita (2017)

$$\mathcal{O}_T = \bar{\chi}_n \mathcal{B}_{n\perp} \frac{1}{\mathcal{P}_\perp} \mathcal{O}_s \frac{1}{\mathcal{P}_\perp} \mathcal{B}_{\bar{n}\perp} \chi_{\bar{n}}$$

$$\mathcal{O}_s = -2\pi\alpha_s \left[S^\dagger_{\bar{n}} S_n \mathcal{P}_\perp + \mathcal{P}_\perp S^\dagger_{\bar{n}} S_n - S^\dagger_{\bar{n}} S_n g \mathcal{B}^n_{S\perp} - g \mathcal{B}^{\bar{n}}_{S\perp} S^\dagger_{\bar{n}} S_n \right]$$



- Suffices for tree level matching; More operators needed at one-loop level since collinear operators have non-trivial spin structure

Glauber Quark Operators: Complete Basis

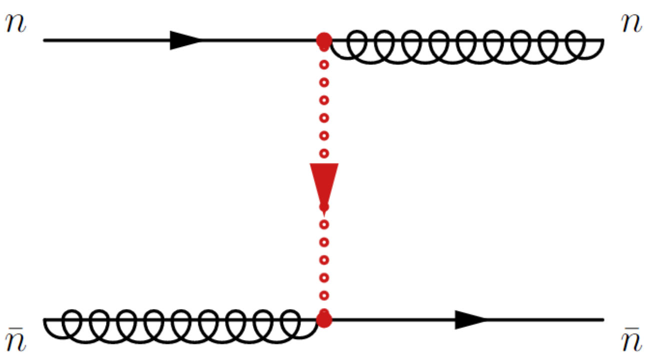
$$\mathcal{O}_{FG} = C_T\mathcal{O}_T + C_{L1}\mathcal{O}_{L1} + C_{L2}\mathcal{O}_{L2} + C_{L3}\mathcal{O}_{L3} + C_{L4}\mathcal{O}_{L4}$$

$$\mathcal{O}_{L1} = \bar{\chi}_n \not{\mathcal{B}}_{n\perp} \frac{1}{\not{\mathcal{P}}_\perp} \mathcal{O}_s \frac{1}{\mathcal{P}_\perp^2} \chi_{\bar{n}} \left(\mathcal{P}_\perp \cdot \mathcal{B}_{\bar{n}\perp}\right) , \tag{1}$$

$$\mathcal{O}_{L2} = \bar{\chi}_n \left(\mathcal{P}_\perp \cdot \mathcal{B}_{n\perp}\right) \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s \frac{1}{\not{\mathcal{P}}_\perp} \not{\mathcal{B}}_{\bar{n}\perp} \chi_{\bar{n}} , \tag{2}$$

$$\mathcal{O}_{L3} = \bar{\chi}_n \left(\mathcal{P}_\perp \cdot \mathcal{B}_{n\perp}\right) \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s \frac{1}{\mathcal{P}_\perp^2} \left(\mathcal{P}_\perp \cdot \mathcal{B}_{\bar{n}\perp}\right) \chi_{\bar{n}} , \tag{3}$$

$$\mathcal{O}_{L4} = \bar{\chi}_n \mathcal{B}_{n\perp}^\mu \frac{1}{\not{\mathcal{P}}_\perp} \mathcal{O}_s \frac{1}{\not{\mathcal{P}}_\perp} \mathcal{B}_{\bar{n}\perp\mu} \chi_{\bar{n}} . \tag{4}$$



- Tree level matching to $qg \to qg$ backscattering in QCD gives

$$C_T = 1 \qquad\qquad C_{L1} = C_{L2} = C_{L3} = C_{L4} = 0$$

- $\mathcal{O}_{L1}, \mathcal{O}_{L2}$ generated at one-loop level. Needed to maintain Ward identity for all external polarization vectors (QED)
- What are the Wilson coefficients at one loop level?

One Loop Matching : Ingredients

Regulator choices a bit involved at one loop

Rapidity Divergences → η regulator

$$\int_0^{p^-} \frac{dk^-}{k^-}$$

$$\left(\frac{\nu}{k^-}\right)^\eta, \left(\frac{\nu}{|2k_z|}\right)^\eta .$$

Chiu, et al. (2012)

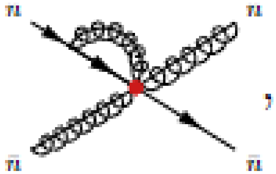
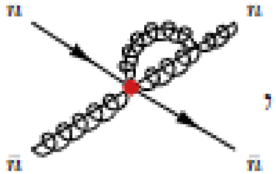
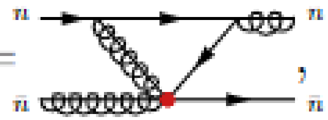
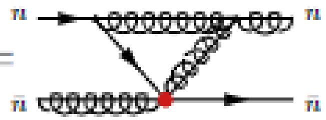
UV divergences → Dim reg

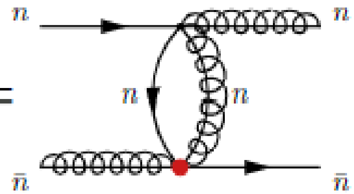
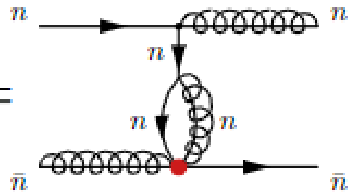
IR Divergences → Dim reg

Previous work used gluon mass regulator (Rothstein and Stewart (2016) Moult, et al. (2017))

Does not regulate all IR divergences when $m_q = 0$

Collinear Loops

$i\mathcal{I}_{WQ,n} =$  , $i\mathcal{I}_{WG,n} =$  , $i\mathcal{I}_{VQ,n} =$  , $i\mathcal{I}_{VG,n} =$ 

$i\mathcal{I}_{T1,n} =$  , $i\mathcal{I}_{T2,n} =$  .

- T graphs vanish as in gluonic case

Matches collinear rapidity anomalous dimension computed by Moult, et al. (2017)

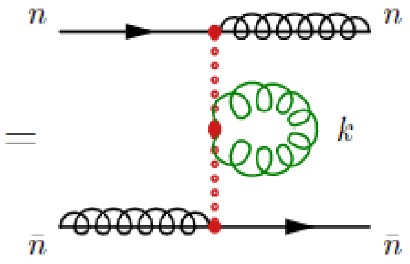
$$\begin{aligned}\mathcal{I}_{n,\mathrm{coll}} &= -\frac{g^4}{16\pi^2}T^aT^bC_F\left[\frac{2}{\eta}\left\{\frac{1}{\varepsilon_{\mathrm{IR}}}+\ln\frac{\mu^2}{-t}\right\}+\frac{1}{\varepsilon_{\mathrm{UV}}}+\frac{2+2\ln\frac{\nu}{Q}}{\varepsilon_{\mathrm{IR}}}+2\ln\frac{\mu^2}{-t}\ln\frac{\nu}{Q}+3\ln\frac{\mu^2}{-t}-\frac{2\pi^2}{3}+8\right]\mathcal{M}_T\\ &-\frac{g^4}{16\pi^2}T^aT^bC_A\left[-\frac{1}{\varepsilon_{\mathrm{UV}}}+\frac{1}{\varepsilon_{\mathrm{IR}}^2}+\frac{1+\ln\frac{\mu^2}{-t}}{\varepsilon_{\mathrm{IR}}}+\frac{1}{2}\ln^2\frac{\mu^2}{-t}-\frac{\pi^2}{12}-1\right]\mathcal{M}_T-\frac{g^4}{16\pi^2}T^aT^b2(C_A-C_F)\mathcal{M}_{L1}\\ &+\frac{g^4}{16\pi^2}T^aT^bC_A\left[\frac{1}{4\varepsilon_{\mathrm{UV}}}+\frac{1}{4}\ln\frac{\mu^2}{-t}+1\right]\left(\mathcal{M}_{1\bar{n}}+\frac{1}{2}\mathcal{M}_{q12}\right).\end{aligned}$$

New operators generated beyond tree level

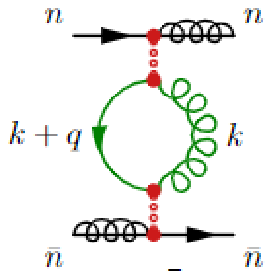
Non-gauge invariant piece; Critically removed by Collinear-Soft Zero Bin

Soft Loops and Glauber Boxes

$i\mathcal{I}_{\rm flower} =$



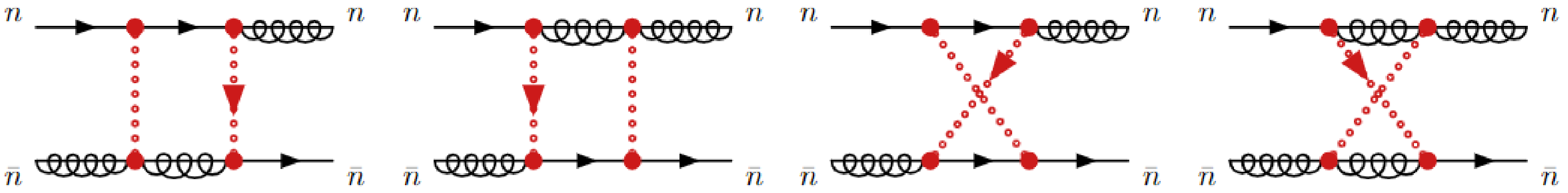
$i\mathcal{I}_{\rm eye} =$



Soft sector gauge invariant, analogous to gluon case

$$\mathcal{I}_{\rm soft} = \mathcal{I}_{\rm eye} + \mathcal{I}_{\rm flower}$$
$$= \frac{2g^2 C_F}{16\pi^2} T^a T^b \left[\frac{1}{\eta}\left(\frac{2}{\varepsilon_{\rm IR}} + 2\ln\frac{\mu^2}{-t}\right) - \frac{1}{\varepsilon_{\rm IR}^2} - \frac{\ln\frac{\mu^2}{\nu^2}}{\varepsilon_{\rm IR}} + \frac{3}{2\varepsilon_{\rm UV}} - \frac{1}{2}\ln^2\frac{\mu^2}{-t} + \ln\frac{\mu^2}{-t}\ln\frac{\nu^2}{-t} + \frac{3}{2}\ln\frac{\mu^2}{-t} - \frac{\pi^2}{12} + \frac{7}{2}\right]\mathcal{M}_T\,.$$

Matches soft rapidity anomalous dimension computed by Moult, et al. (2017)



- Time ordered product with Glauber Gluon operators generates pure phases, computed previously

Glauber/ Coulomb Phase

$$\mathcal{I}_{\rm boxes} = \frac{g^4}{16\pi^2}\delta^{ab}\delta_{ij}\left[\frac{1}{\varepsilon_{\rm IR}} + \ln\frac{\mu^2}{-t}\right](i\pi)\,.$$

Full One Loop Matching

- Collinear + Soft + Glauber − Zero Bins

$$= \frac{g^4}{16\pi^2} C_F \left[\frac{1}{\varepsilon_{\rm UV}} - \frac{2}{\varepsilon_{\rm IR}^2} - \frac{4 + 2\ln\frac{\mu^2}{Q^2}}{\varepsilon_{\rm IR}} - 2\ln\frac{\mu^2}{Q^2}\ln\frac{\mu^2}{-t} + \ln^2\frac{\mu^2}{-t} - 3\ln\frac{\mu^2}{-t} + \frac{7\pi^2}{6} - 9 \right] \mathcal{M}_T (T^a T^b)_{ij}$$

$$+ \frac{g^4}{16\pi^2} C_A \left[\frac{2}{\varepsilon_{\rm UV}} - \frac{2}{\varepsilon_{\rm IR}^2} - \frac{2 + 2\ln\frac{\mu^2}{-t}}{\varepsilon_{\rm IR}} - \ln^2\frac{\mu^2}{-t} + \frac{\pi^2}{6} + 2 \right] \mathcal{M}_T (T^a T^b)_{ij}$$

$$- \frac{g^4}{16\pi^2}(C_A - C_F)(2\mathcal{M}_{L1} + 2\mathcal{M}_{L2})(T^a T^b)_{ij} + \frac{g^4}{16\pi^2}\delta^{ab}\delta_{ij}\left[\frac{1}{\varepsilon_{\rm IR}} + \ln\frac{\mu^2}{-t}\right](i\pi).$$

= QCD Backscattering limit for all polarizations

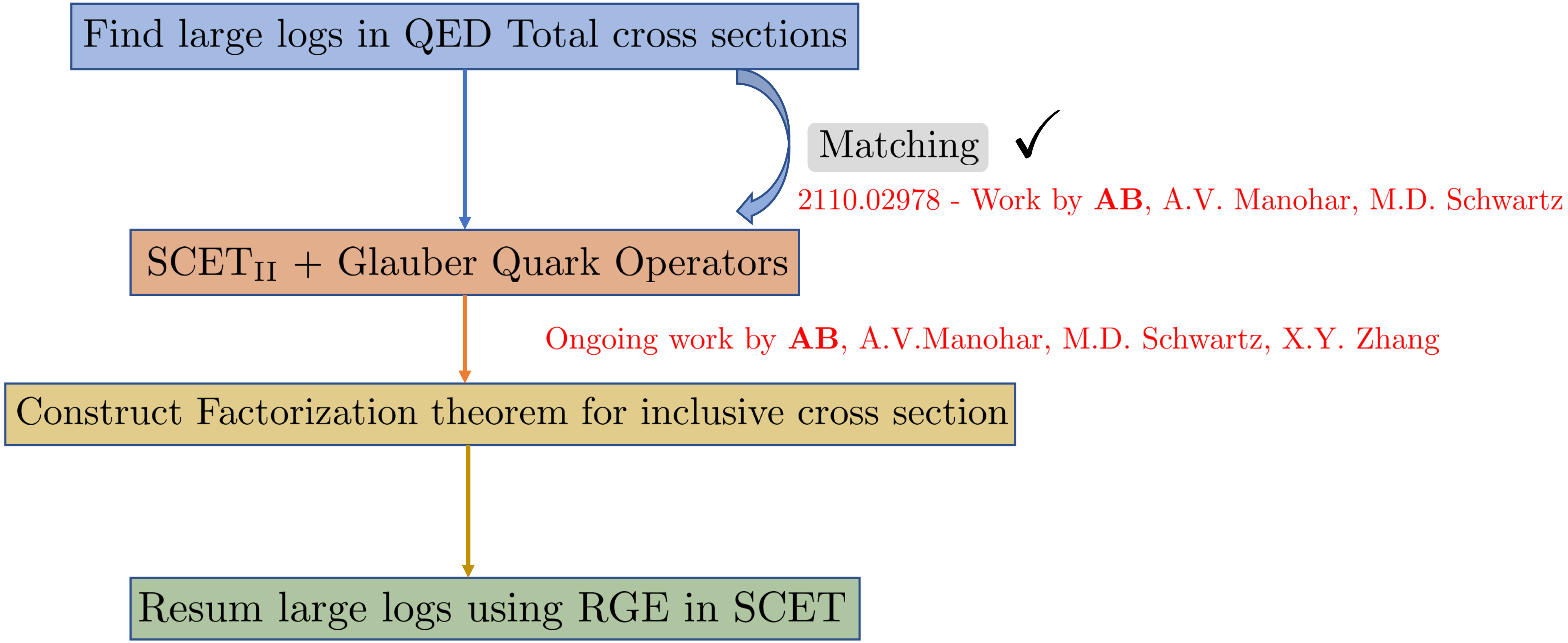
- One loop matching is same as tree level (i.e.)

$$C_T = 1 \qquad\qquad C_{L1} = C_{L2} = C_{L3} = C_{L4} = 0$$

- SCET_{II} + Glauber Quark Operators describes backward scattering in QCD

- Tree level Wilson coefficient suffices as there is no hard scale in the matching (same argument as for gluons by Rothstein and Stewart (2016))

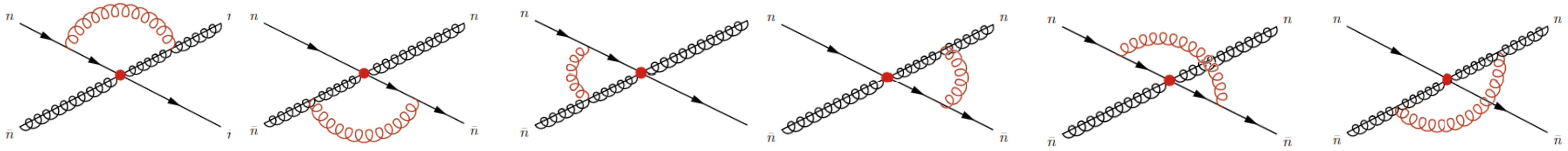
Resumming Logs in Inclusive Cross section using Glaubers



Matching in $\mathbf{SCET_I}$: Ultrasoft modes

- What happens when one includes ultrasoft modes $(\lambda^2, \lambda^2, \lambda^2)$?

New Wilson line graphs involving ultrasoft gluons

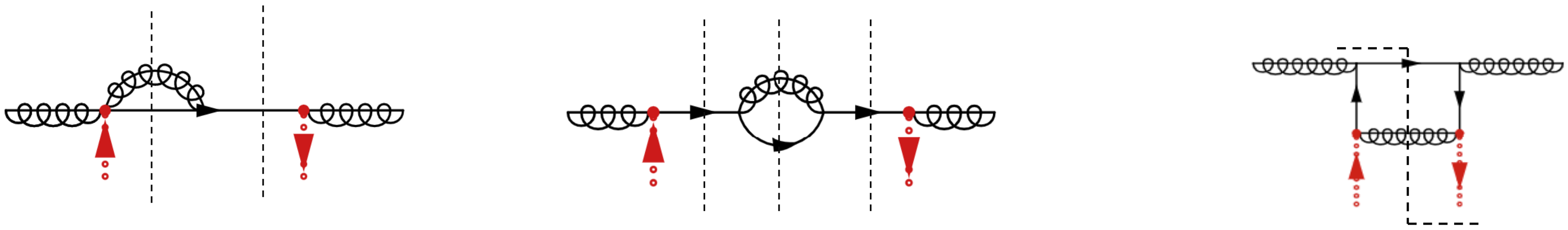


- With offshellness as regulator, US zerobin subtracts parts of collinear, soft and Glauber loops

- Matching doesn't change in $\mathrm{SCET_I}$ (i.e.)

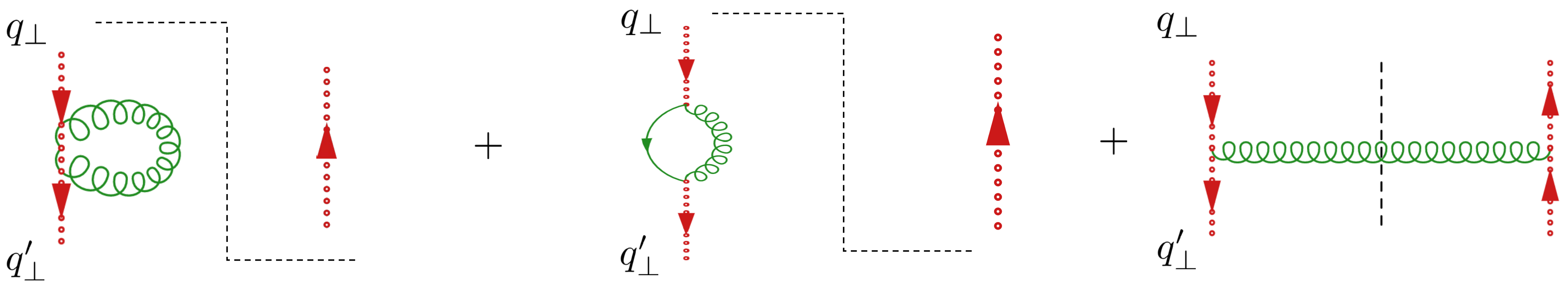
$$C_T = 1 \qquad\qquad C_{L1} = C_{L2} = C_{L3} = C_{L4} = 0$$

Collinear Function



- Computation of real graphs is slightly involved due to injection of Glauber momentum from vertex.
- Differs from inclusive jet function; differential *only* in final state perp momentum
- Get new real graphs which have no virtual counterpart
- All other graphs with virtual counterparts seem to cancel when $m_e = 0$

Soft Function



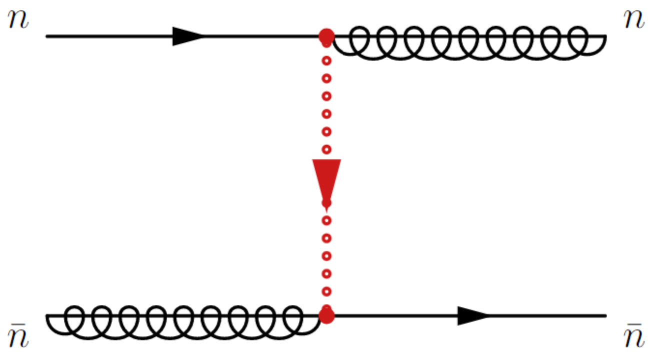
$$S^{(1)}(q_\perp, q'_\perp) = S^{(0)}(q_\perp, q'_\perp)\frac{\alpha}{\pi}\left[-\frac{1}{2}\ln\left(\frac{\mu^2}{q_\perp^2}\right)^2 + \ln\left(\frac{\mu^2}{q_\perp^2}\right)\ln\left(\frac{\nu^2}{q_\perp^2}\right) + \cdots\right] + \frac{64\pi^2\alpha^3}{(q_\perp + q'_\perp)_+^2}\ln\left(\frac{\nu^2}{(q_\perp + q'_\perp)^2}\right) + \cdots$$

- Find an IR finite result. Rapidity divergences renormalized using BFKL (Moult, et al. 2017)
- Integration over $q'_\perp$ to get the full inclusive cross section drives one out of the EFT. Is that justified?
- There is also a non-trivial RG play at work. Rapidity RGE (BFKL) of collinear / soft function gives $(\alpha L)^n$ series, which is at odds with the $(\alpha L^2)^n$ series in cross section. What gives ?

Applications to Backscattering

- Virtual Graphs from SCET

$$\frac{d\sigma}{dt} \sim \frac{\alpha}{2\pi}\left[\cdots - 2\ln\frac{\mu^2}{s}\ln\frac{\mu^2}{-t} + \ln^2\frac{\mu^2}{-t} + \cdots\right]\frac{\tilde{\sigma}_{\rm LO}}{-t}$$



- Natural scale from matching $\mu^2 \sim s$ gives

$$\tilde{\sigma}_{LO} \times \frac{\alpha}{2\pi}\int_{-s}^{-m_e^2}\frac{dt}{-t}\ln^2\frac{s}{-t} = \sigma_{\rm LO}\left[\frac{1}{3}\frac{\alpha L^2}{2\pi}\right]$$

Already hints at LL in total cross sections

$$\sigma(e^-\gamma \to e^-\gamma(+\gamma)) = \sigma_{\rm LO}\left[1 + \frac{1}{3}\frac{\alpha L^2}{2\pi} - \frac{1}{2}\frac{\alpha L}{2\pi} + \frac{17}{4}\frac{\alpha}{2\pi}\cdots\right]$$

$$\sigma(e^+e^- \to \gamma\gamma(+\gamma)) = \sigma_{\rm LO}\left[1 + \frac{1}{3}\frac{\alpha L^2}{2\pi} - \frac{1}{2}\frac{\alpha L}{2\pi} + \left(\frac{2\pi^2}{3} - 2\right)\frac{\alpha}{2\pi}\cdots\right]$$

$$\sigma(\gamma\gamma \to e^+e^-(+\gamma)) = 2\sigma_{\rm LO}\left[1 + \frac{1}{3}\frac{\alpha L^2}{2\pi} - \frac{1}{2}\frac{\alpha L}{2\pi} + \left(6 - \frac{2\pi^2}{3}\right)\frac{\alpha}{2\pi}\cdots\right]$$

- Possibly exponentiates as $\exp\left(\frac{\alpha}{2\pi}\ln^2\frac{s}{-t}\right)$. Need to check this via factorization

Future Work

- Real contibutions do not seem to match virtual ones in collinear function computation. Find sum is zero. How does the resummation work?
- All apparent RGEs (such as rapidity/ BFKL) seem to be summing single logs $(\alpha L)^n$, while we have a double log series $(\alpha L^2)^n$ at hand. What gives?
- Does setting $m = 0$ in the intermediate steps allowed? Is the factorization completely independent of the mass at the end even if we set $m \neq 0$ in the intermediate steps?

Thanks for listening! Questions?

Backup Slides

- **All** $2 \to 2$ inclusive cross sections in QED exhibit the same LL, NLL

$$\sigma(e^-\gamma \to e^-\gamma(+\gamma)) = \sigma_{\rm LO}\left[1 + \frac{1}{3}\frac{\alpha L^2}{2\pi} - \frac{1}{2}\frac{\alpha L}{2\pi} + \frac{17}{4}\frac{\alpha}{2\pi}\cdots\right] \quad (1)$$

$$\sigma(e^+e^- \to \gamma\gamma(+\gamma)) = \sigma_{\rm LO}\left[1 + \frac{1}{3}\frac{\alpha L^2}{2\pi} - \frac{1}{2}\frac{\alpha L}{2\pi} + \left(\frac{2\pi^2}{3} - 2\right)\frac{\alpha}{2\pi}\cdots\right] \quad (2)$$

R.N.Lee (2020)

$$\sigma(\gamma\gamma \to e^+e^-(+\gamma)) = 2\sigma_{\rm LO}\left[1 + \frac{1}{3}\frac{\alpha L^2}{2\pi} - \frac{1}{2}\frac{\alpha L}{2\pi} + \left(6 - \frac{2\pi^2}{3}\right)\frac{\alpha}{2\pi}\cdots\right] \quad (3)$$

Abelian limit of $gg \to t\bar{t}$
M Czakon, A Mitov (2008)