TMD fragmentation functions for polarized J/ψ production

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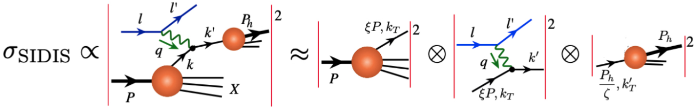
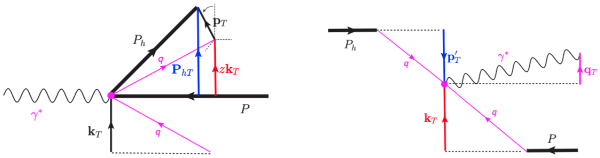
Motivation

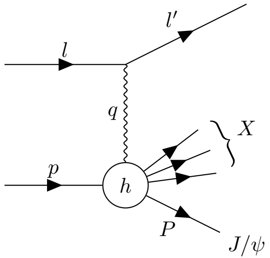




(a) photon-hadron frame (b) hadron-hadron frame

- TMD fragmentation functions (FFs) for polarized parton and polarized hadron have yet to be computed in the literature
- studying quarkonium production is a way to learn about gluon TMDs ; EIC will be able to study it via DIS

EIC Yellow Report, Nucl. Phys. A 1026 (2022) 122447 ; figures from TMD Handbook



$$n^\mu = \frac{1}{\sqrt{2}}(1,0,0,1)\ ,$$

$$\bar{n}^\mu = \frac{1}{\sqrt{2}}(1,0,0,-1)\ .$$

$$P^\mu = (\bar{n}\cdot P)n^\mu + \frac{M^2}{2\bar{n}\cdot P}\bar{n}^\mu\ ,$$

$$Q^2 = \ -q^2 = -(l-l')^2,$$

$$x = \frac{Q^2}{2p\cdot q}, \quad y = \frac{p\cdot q}{p\cdot l}, \quad z = \frac{p\cdot P}{p\cdot q}.$$

Cross section

$$\frac{d\sigma}{d\phi^l \, dx \, dz \, dy \, d\mathbf{q}_T} = 2M \frac{\alpha^2}{4Q^4} \frac{y}{z} L_{\mu\nu} W^{\mu\nu} \ ,$$

- leptonic tensor:

$$L_{\mu\nu} = e^{-2} \langle l' | \, J_\mu(0) \, | l \rangle \, \langle l | \, J_\nu^\dagger(0) \, | l' \rangle \ ,$$

- hadronic tensor:

$$2MW^{\mu\nu} = 2z \int d^2\mathbf{p}_T \, d^2\mathbf{k}_T \, \delta^2(\mathbf{p}_T + \mathbf{q}_T - \mathbf{k}_T) \, \mathrm{tr} \left[2\Phi(x, \mathbf{p}_T) \gamma^\mu 2\Delta(z, \mathbf{k}_T) \gamma^\nu \right] .$$

- Φ is TMD PDF, Δ is TMD FF

TMDFFs

$$\Delta_{q\to J/\psi}(z,\mathbf{b}_T,P^+/z) = \frac{1}{2zN_c}\mathrm{Tr}\int\frac{db^-}{2\pi}e^{ib^-P^+/z}\sum_X \Gamma_{\alpha\alpha'}$$
$$\times \langle 0|\, W_{\bar n}\psi_i^{\alpha}(b)\,|J/\psi(P),X\rangle\,\langle J/\psi(P),X|\,\bar\psi_i^{\alpha'}(b)W_{\bar n}^{\dagger}\,|0\rangle\ ,$$

$$\Delta^{\alpha\alpha'}_{g\to J/\psi}(z,\mathbf{b}_T,P^+) = \frac{1}{2z^2P^+}\int\frac{db^-}{2\pi}e^{ib^-(P^+z)}\,\langle 0|\, W_{\bar n}G^{+\alpha}\,|J/\psi(P),X\rangle$$
$$\times \langle J/\psi(P),X|\,G^{+\alpha'}W_{\bar n}^{\dagger}\,|0\rangle\ .$$

NRQCD factorization

$$d\sigma(a+b\to \mathcal{Q}+X)=\sum_n d\sigma(a+b\to Q\bar{Q}(n)+X)\left\langle \mathcal{O}_n^{\mathcal{Q}}\right\rangle$$

- $d\sigma(a+b\to Q\bar{Q}(n)+X)$ is the perturbative short-distance cross section
  - expansion in α_s
- $\left\langle \mathcal{O}_n^{\mathcal{Q}}\right\rangle$ is the NRQCD long-distance matrix element (LDME)
  - terms scale in powers of v

NRQCD LDMEs

$$\mathcal{O}_n^H = \chi^\dagger \mathcal{K}_n \psi \left(\sum_X \sum_{m_J} |H + X\rangle\langle H + X| \right) \psi^\dagger \mathcal{K}_n' \chi$$
$$= \chi^\dagger \mathcal{K}_n \psi \left(a_H^\dagger a_H \right) \psi^\dagger \mathcal{K}_n' \chi,$$

$$\mathcal{O}_8^H(^3S_1) = \chi^\dagger \sigma^i T^a \psi \left(a_H^\dagger a_H \right) \psi^\dagger \sigma^i T^a \chi.$$
$$\mathcal{O}_1^H(^3P_0) = \frac{1}{3}\, \chi^\dagger (-\tfrac{i}{2}\overleftrightarrow{\mathbf{D}} \cdot \boldsymbol{\sigma}) \psi \left(a_H^\dagger a_H \right) \psi^\dagger (-\tfrac{i}{2}\overleftrightarrow{\mathbf{D}} \cdot \boldsymbol{\sigma}) \chi;$$
$$\mathcal{O}_1^H(^3S_1) = \chi^\dagger \sigma^i \psi \left(a_H^\dagger a_H \right) \psi^\dagger \sigma^i \chi,$$
$$\mathcal{O}_8^H(^1S_0) = \chi^\dagger T^a \psi \left(a_H^\dagger a_H \right) \psi^\dagger T^a \chi,$$

- at lowest order in v we have $\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]}) \rangle \sim v^3$
  - color singlet does not contribute at LO
- lowest order non-vanishing LDME for our purposes is $\langle \mathcal{O}_n^{\mathcal{Q}} \rangle = \langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle \approx 4m_c^2 \xi^\dagger \sigma^i T^a \eta \eta^\dagger \sigma^i T^a \xi \sim v^7$.
  - $^1S_0^{[8]}$ and $^3P_J^{[8]}$ also scale as v^7 but do not contribute at LO in our current calculation

Uncertainty surrounding LDMEs

- polarization puzzle: NRQCD global fits (Butenschoen & Kniehl PRD 84 (2011) 051501) predict J/ψ to be transversely polarized at high p_T but this disagrees with data which says it is unpolarized
- some work done to resolve the puzzle by removing small p_T values from fits
  - Chao et al. PRL 108 (2012) 242004
  - Bodwin et al. PRL 113 (2014) 022001
  - Faccioli et al. PLB 736 (2014) 98
  - Lourenço et al. NPA 932 (2014) 466-471
- these different approaches lead to disagreeing values for the LDMEs
  - we will be able to compute new observables to extract the LDMEs

	$\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]})\rangle$ $\times$ GeV^3	$\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]})\rangle$ $\times 10^{-2}$ GeV^3	$\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]})\rangle$ $\times 10^{-2}$GeV^3	$\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]})\rangle/m_c^2$ $\times 10^{-2}$GeV^3
B & K [5, 6]	1.32 ± 0.20	0.224 ± 0.59	4.97 ± 0.44	-0.72 ± 0.88
Chao, et al. [12]	1.16 ± 0.20	0.30 ± 0.12	8.9 ± 0.98	0.56 ± 0.21
Bodwin et al. [13]	1.32 ± 0.20	1.1 ± 1.0	9.9 ± 2.2	0.49 ± 0.44

Table from Bain et al. PRL 119 (2017) 3, 032002

Projection operators

Quark :

- unpolarized: $\Gamma = \not{n}$ ← this has been done by Echevarria et al. JHEP 10 (2020) 164
- longitudinally polarized: $\Gamma = \not{n}\gamma_5$
- transversely polarized: $\Gamma_{+\alpha} = \sigma_{+\alpha}\gamma_5$

Gluon :

- unpolarized: contract with $g_T^{\alpha\alpha'}$
- longitudinally polarized: contract with $\epsilon_T^{\alpha\alpha'}$
- transversely polarized: contract with symmetric traceless tensor combinations of k_T

J/ψ spin decomposition

$$[\xi^\dagger \sigma^i T^A \eta][\eta^\dagger \sigma^j T^A \xi] \to \varepsilon^{*i}\varepsilon^j [\xi^\dagger \sigma^l T^A \eta][\eta^\dagger \sigma^l T^A \xi]$$

$$\varepsilon^{*i}\varepsilon^j = \frac{1}{3}\delta^{ij} + \frac{i}{2}\varepsilon^{ijk}S^k - T^{ij},$$

$$\vec{S} = \mathrm{Im}\,(\varepsilon^* \times \varepsilon) = (S_T^x, S_T^y, S_L)$$

$$T^{ij} = \frac{1}{3}\delta^{ij} - \mathrm{Re}\left(\varepsilon^{*i}\varepsilon^j\right) = \frac{1}{2}\begin{pmatrix} -\frac{2}{3}S_{LL} + S_{TT}^{xx} & S_{TT}^{xy} & S_{LT}^x \\ S_{TT}^{yx} & -\frac{2}{3}S_{LL} - S_{TT}^{xx} & S_{LT}^y \\ S_{LT}^x & S_{LT}^y & \frac{4}{3}S_{LL} \end{pmatrix}.$$

Boost to yield

$$\Lambda^\mu{}_i \Lambda^\nu{}_j \varepsilon^{*i}\varepsilon^j = -\frac{1}{3}\left(g^{\mu\nu} - \frac{P^\mu P^\nu}{M^2}\right) + \frac{i}{2M}\epsilon^{\mu\nu\alpha\beta}P_\alpha S_\beta - T^{\mu\nu}$$

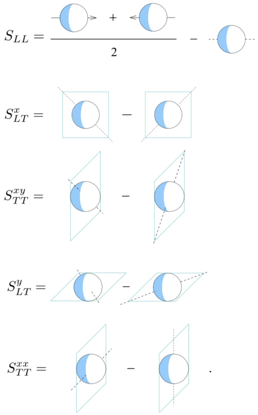
Bacchetta & Mulders, PRD 62(2000) 114004

Components of T^{ij}

$$\boldsymbol{\Sigma}^i \hat{n}_i = \boldsymbol{\Sigma}_x \cos\theta \cos\varphi + \boldsymbol{\Sigma}_y \cos\theta \sin\varphi + \boldsymbol{\Sigma}_z \sin\theta,$$

$$P\left(m_{(\theta,\varphi)}\right) = \mathrm{Tr}\ \left\{\boldsymbol{\rho}\, |m_{(\theta,\varphi)}\rangle\langle m_{(\theta,\varphi)}|\right\}.$$

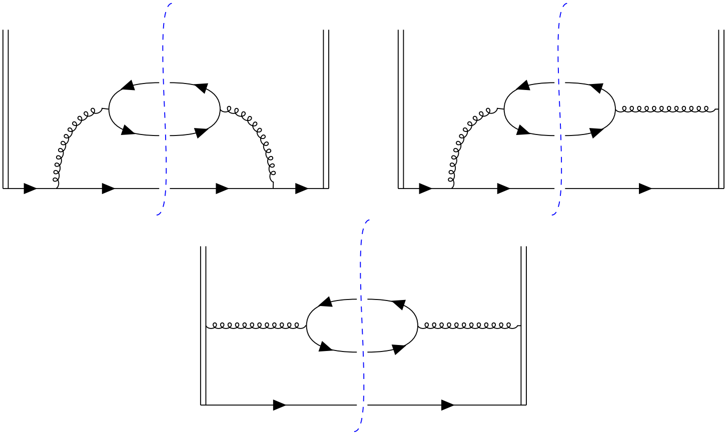
$$\begin{aligned}
S_{LL} &= \frac{1}{2} P\left(1_{(0,0)}\right) + \frac{1}{2} P\left(-1_{(0,0)}\right) - P\left(0_{(0,0)}\right), \\
S_{LT}^{x} &= P\left(0_{(-\frac{\pi}{4},0)}\right) - P\left(0_{(\frac{\pi}{4},0)}\right), \\
S_{LT}^{y} &= P\left(0_{(-\frac{\pi}{4},\frac{\pi}{2})}\right) - P\left(0_{(\frac{\pi}{4},\frac{\pi}{2})}\right), \\
S_{TT}^{xx} &= P\left(0_{(\frac{\pi}{2},-\frac{\pi}{4})}\right) - P\left(0_{(\frac{\pi}{2},\frac{\pi}{4})}\right), \\
S_{TT}^{xy} &= P\left(0_{(\frac{\pi}{2},\frac{\pi}{2})}\right) - P\left(0_{(\frac{\pi}{2},0)}\right).
\end{aligned}$$



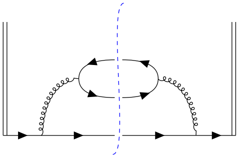
from: Bacchetta & Mulders, PRD 62(2000) 114004

Quark fragmentation

$$\langle 0| W_{\bar{n}} \psi_i^{\alpha}(b) |J/\psi(P), X\rangle \langle J/\psi(P), X| \bar{\psi}_i^{\alpha'}(b) W_{\bar{n}}^{\dagger} |0\rangle$$



Example diagram



- perform contractions between Wilson line, quark field, final state

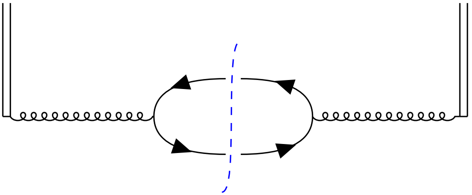
$$
=\frac{g^4}{4zM^4N_c}\int\frac{d^Dk}{(2\pi)^D}\int db^- e^{ib^-P^+/z}e^{-ib(k+P)}
\times \mathrm{Tr}\left[\not{k}\gamma^\mu\frac{\not{k}+\not{P}}{(k+P)^2+i\epsilon}\left(\Gamma\chi_{\mu\nu}\right)\frac{\not{k}+\not{P}}{(k+P)^2+i\epsilon}\gamma^\nu\right]\delta(k^2)
$$

where

$$
\chi_{\mu\nu}=\overline{u}(p')\gamma_\mu T^a v(p')\overline{v}(p')\gamma_\nu T^b u(p')
$$

- b^- integral yields delta function, which combined with $\delta(k^2)$ means all the integration is trivial

Gluon fragmentation



$$\langle 0| W_{\bar{n}} G^{+\alpha} |J/\psi(P), X\rangle \langle J/\psi(P), X| G^{+\alpha'} W_{\bar{n}}^{\dagger} |0\rangle$$
$$= \frac{-g^4}{2M^4 P_-}\delta(1-z)(P_- g^{\alpha\mu} - P^{\alpha}\bar{n}^{\mu})(P^- g^{\beta\nu} - P^{\beta}\bar{n}^{\nu}) I_0 \varepsilon^*_{\nu}\varepsilon_{\mu}$$

Defined $I_0 \equiv \frac{M^2}{(N_c^2-1)(d-1)}\xi^\dagger\sigma^i T^a \eta\eta^\dagger\sigma^i T^a\xi$ ... will eventually match onto LDME

Decomposing the FFs

$$\Phi_U(x,\boldsymbol{p}_T) = \frac{1}{4}\left\{ f_1(x,p_T^2)\, \not{n}_+ + \left(h_1^\perp(x,p_T^2)\,\sigma_{\mu\nu}\frac{p_T^\mu}{M}n_+^\nu \right)\right\},$$

$$\Phi_L(x,\boldsymbol{p}_T) = \frac{1}{4}\left\{ g_{1L}(x,p_T^2)\, S_L\,\gamma_5\,\not{n}_+ + h_{1L}^\perp(x,p_T^2)\, S_L\, \mathrm{i}\sigma_{\mu\nu}\gamma_5 n_+^\mu\frac{p_T^\nu}{M}\right\},$$

$$\begin{aligned}\Phi_T(x,\boldsymbol{p}_T) = \frac{1}{4}\Bigg\{ & g_{1T}(x,p_T^2)\,\frac{\boldsymbol{S}_T\cdot\boldsymbol{p}_T}{M}\,\gamma_5\,\not{n}_+ + h_{1T}(x,p_T^2)\,\mathrm{i}\sigma_{\mu\nu}\gamma_5 n_+^\mu S_T^\nu \\ & + h_{1T}^\perp(x,p_T^2)\,\frac{\boldsymbol{S}_T\cdot\boldsymbol{p}_T}{M}\,\mathrm{i}\sigma_{\mu\nu}\gamma_5 n_+^\mu\frac{p_T^\nu}{M} \\ & + \left(f_{1T}^\perp(x,p_T^2)\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu\frac{p_T^\rho}{M}S_T^\sigma\right)\Bigg\},\end{aligned}$$

$$\Phi_{LL}(x,\boldsymbol{p}_T) = \frac{1}{4}\left\{ f_{1LL}(x,p_T^2)\, S_{LL}\,\not{n}_+ + \left(h_{1LL}^\perp(x,p_T^2)\, S_{LL}\,\sigma_{\mu\nu}\frac{p_T^\mu}{M}n_+^\nu\right)\right\},$$

$$\begin{aligned}\Phi_{LT}(x,\boldsymbol{p}_T) = \frac{1}{4}\Bigg\{ & f_{1LT}(x,p_T^2)\,\frac{\boldsymbol{S}_{LT}\cdot\boldsymbol{p}_T}{M}\,\not{n}_+ + \left(g_{1LT}(x,p_T^2)\,\epsilon_T^{\mu\nu}S_{LT\,\mu}\frac{p_{T\,\nu}}{M}\,\gamma_5\,\not{n}_+\right) \\ & + \left(h'_{1LT}(x,p_T^2)\,\mathrm{i}\sigma_{\mu\nu}\gamma_5 n_+^\mu\epsilon_T^{\nu\rho}S_{LT\,\rho}\right) \\ & + \left(h_{1LT}^\perp(x,p_T^2)\,\frac{\boldsymbol{S}_{LT}\cdot\boldsymbol{p}_T}{M}\,\sigma_{\mu\nu}\frac{p_T^\mu}{M}n_+^\nu\right)\Bigg\},\end{aligned}$$

$$\begin{aligned}\Phi_{TT}(x,\boldsymbol{p}_T) = \frac{1}{4}\Bigg\{ & f_{1TT}(x,p_T^2)\,\frac{\boldsymbol{p}_T\cdot\boldsymbol{S}_{TT}\cdot\boldsymbol{p}_T}{M^2}\,\not{n}_+ \\ & - \left(g_{1TT}(x,p_T^2)\,\epsilon_T^{\mu\nu}S_{TT\,\nu\rho}\frac{p_T^\rho p_{T\,\mu}}{M^2}\,\gamma_5\,\not{n}_+\right) \\ & - \left(h'_{1TT}(x,p_T^2)\,\mathrm{i}\sigma_{\mu\nu}\gamma_5 n_+^\mu\epsilon_T^{\nu\rho}S_{TT\,\rho\sigma}\frac{p_T^\sigma}{M}\right) \\ & + \left(h_{1TT}^\perp(x,p_T^2)\,\frac{\boldsymbol{p}_T\cdot\boldsymbol{S}_{TT}\cdot\boldsymbol{p}_T}{M^2}\,\sigma_{\mu\nu}\frac{p_T^\mu}{M}n_+^\nu\right)\Bigg\}.\end{aligned}$$

- to go from PDFs to FFs: $\{f, g, h, x, p, n_+\} \to \{D, G, H, z, k, n_-\}$

from: Bacchetta & Mulders, PRD 62(2000) 114004

TMDFFs: table

		Parton polarization		
		Unpolarized	Longitudinal	Transverse
Hadron pol.	Unpolarized	D_1		$H_1^\perp$
	Longitudinal		G_{1L}	$H_{1L}^\perp$
	Transverse	$D_{1T}^\perp$	$G_{1T}^\perp$	H_1, $H_{1T}^\perp$
	LL	D_{1LL}		$H_{1LL}^\perp$
	LT	D_{1LT}	G_{1LT}	$H_{1LT}^\perp$, H'_{1LT}
	TT	D_{1TT}	G_{1TT}	$H_{1TT}^\perp$, H'_{1TT}

Results: quark TMDFFs

- all Hs, $D_{1T}^{\perp}$, $G_{1LT} = 0$
- $D_1 = \frac{g^4}{48\pi M^4 N_c z} \frac{\mathbf{k}_T^2 z^2 (z^2 - 2z + 2) + 2M^2 (z-1)^2}{[\mathbf{k}_T^2 z^2 - M^2 (z-1)]^2} I_0$ ← Echevarria et al. result
- $D_{1LL} = \frac{g^4}{48\pi M^4 N_c z} \frac{3M^2 (z-1)^2 - \mathbf{k}_T^2 z^2 (z^2 - 2z + 2)}{[\mathbf{k}_T^2 z^2 - M^2 (z-1)]^2} I_0$
- $D_{LT} = \frac{-g^4}{32\pi M^2 N_c} \frac{(z-2)(z-1)}{[\mathbf{k}_T^2 z^2 - M^2 (z-1)]^2} I_0$
- $D_{TT} = \frac{-g^4}{16\pi M^2 N_c} \frac{z(z-1)}{[\mathbf{k}_T^2 z^2 - M^2 (z-1)]^2} I_0$
- $G_{1L} = \frac{-g^4}{32\pi M^4 N_c} \frac{\mathbf{k}_T^2 z^2 (z-2)}{[\mathbf{k}_T^2 z^2 - M^2 (z-1)]^2} I_0$
- $G_{1T}^{\perp} = \frac{g^4}{16\pi M^2 N_c} \frac{z(z-1)}{[\mathbf{k}_T^2 z^2 - M^2 (z-1)]^2} I_0$
- G_{1TT}: too lengthy to display

Plotting a cross section

- example: unpolarized lepton beam/target, tensor polarized J/ψ

$$\frac{\mathrm{d}\sigma_{UU}(l + H \to l' + \vec{\vec{h}} + X)}{\mathrm{d}x_B\,\mathrm{d}z_h\,\mathrm{d}y\,\mathrm{d}^2\boldsymbol{P}_{h\perp}} =$$

$$\frac{4\pi\alpha^2 s}{Q^4}\left(1 - y - \frac{y^2}{2}\right) x_B \left\{ S_{h\,LL}\,\boldsymbol{I}\left[f_1\; D_{1LL}\right]\right.$$

$$+ |S_{h\,LT}|\,\cos(\phi^h_{h\,LT})\,\boldsymbol{I}\left[\frac{k^x}{M_h}\,f_1\;D_{1LT}\right]$$

$$\left. + |S_{h\,TT}|\,\cos(2\phi^h_{h\,TT})\,\boldsymbol{I}\left[\frac{(k^x)^2 - (k^y)^2}{M_h^2}\,f_1\;D_{1TT}\right]\right\},$$

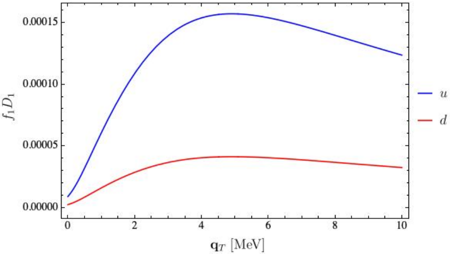
- numerically convolve the TMDFF with numerical PDFs from results of Bastami et al., JHEP 06 (2019) 007
- use Gaussian for transverse dependence of PDFs

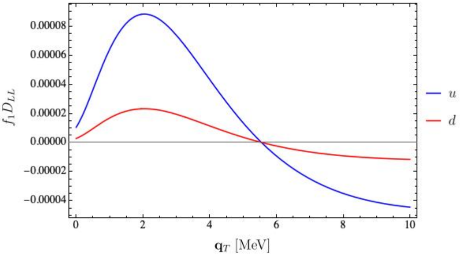
$$f_{\mathrm{TMD}} = \frac{1}{\pi\,\mathrm{av}}\exp(-\mathbf{k}_T^2/\mathrm{av}) f_{\mathrm{col}}$$

Bacchetta & Mulders, PRD 62(2000) 114004

Plots: quark

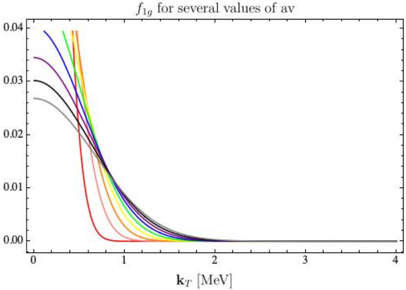
- left: unpolarized quark, unpolarized J/ψ
- right: unpolarized quark, LL polarized J/ψ

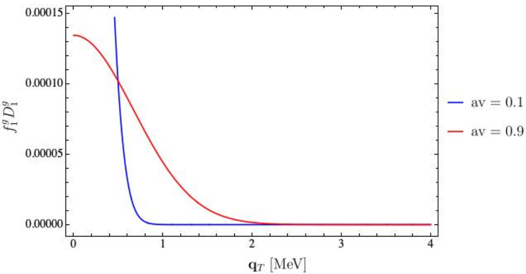




Plots: gluon

- left: varying av from 0.1 to 0.9
- right: unpolarized gluon, unpolarized J/ψ





Conclusions

- we can match TMDFFs onto NRQCD
  - only unpolarized have been calculated prior to now
  - we have calculated the leading-order 18 fragmentation functions for production of polarized J/ψ by polarized partons
- we are in the process of computing cross sections that can be used as a new set of observables to extract gluon TMDs & NRQCD LDMEs