Weak Annihilation in Non-leptonic B Decays

Philipp Böer

work in progress with Matthias Neubert and Michel Stillger

partly based on: PB ’18 [PhD thesis] (form factor for NR bound states)

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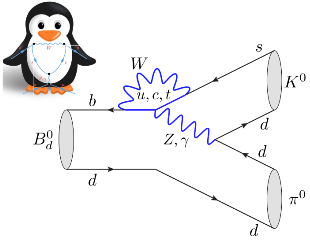
Non-Leptonic B Decays

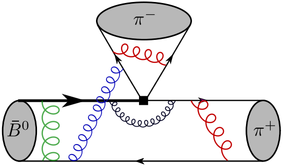


- provide insights into flavor-changing interactions
  - $\rightarrow$ CP-violation and extraction of CKM parameters
  - $\rightarrow$ sensitive to electroweak penguin contributions (probe of new physics)
  - $\rightarrow$ puzzles in $B \to D\pi$, DK and $B \to \pi\pi, \pi K$ decays   e.g. [Bordone et al. '20, Biswas et al. '23]

- challenging QCD dynamics due to purely hadronic final states
  - $\rightarrow$ exploit scale hierarchy $m_b \sim E_{\pi,K} \gg \Lambda_{\rm QCD}$
  - $\rightarrow$ factorization for $m_b \to \infty$ known since more than two decades

[BBNS '99, Bauer,Rothstein,Stewart '04]

QCD Factorization (charmless final states)



SCET$_\mathrm{I}$ operators:

$$\mathcal{O}^\mathrm{I} \sim [\bar{\chi}_C h_v][\bar{\chi}_{\bar{C}} \not{n} \gamma_5 \chi_{\bar{C}}]$$
$$\mathcal{O}^\mathrm{II} \sim [\bar{\chi}_C \not{\mathcal{A}}_{C,\perp} h_v][\bar{\chi}_{\bar{C}} \not{n} \gamma_5 \chi_{\bar{C}}]$$

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle \sim F^{B\to M_1}(q^2=0) \int_0^1 \mathrm{d}x\, \mathbf{T}_i^\mathrm{I}(x)\, \phi_{M_2}(x) + \int_0^\infty \frac{\mathrm{d}\omega}{\omega} \int_0^1 \mathrm{d}x\, \mathrm{d}y\; \mathbf{T}_i^\mathrm{II}(x,y,\omega)\, \phi_{M_1}(y)\, \phi_{M_2}(x)\, \phi_B^+(\omega)$$

- perturbative hard-scattering kernels $T_i^{\mathrm{I,II}}$ (hard/hard-collinear matching coefficients)
- convoluted with LCDAs for heavy and light mesons (soft/collinear functions in SCET$_\mathrm{II}$)
- form factor $F^{B\to M_1}$ absorbs all endpoint-divergent convolutions

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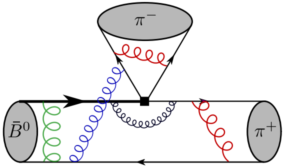

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State of the art:

- (N)NLO scattering kernels e.g. [Bell, Beneke, Huber, Li '07-20]
- 3-loop (2-loop) anomalous dimension for ϕ_M (ϕ_B^+) [Braun et al. '17+19]
- QEDF [Beneke, PB, Finauri, Toelstede, Vos '20-22]

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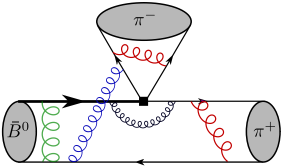

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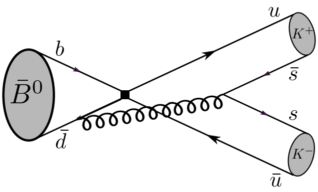
factorization of power-corrections $\sim \Lambda_\mathrm{had}/m_b \simeq 0.2$ unknown!

Weak Annihilation

Topology where the constituents of the B are annihilated in weak vertex

$\rightarrow$ e.g. annihilation dominated decay $\bar{B}_d \rightarrow K^+ K^-$



- Why power-suppressed?
  - $\rightarrow$ purely **hard** interactions (no $1/\omega$ enhancement from hard-collinear spectator quark)
  - $\rightarrow$ B-meson described by decay constant f_B

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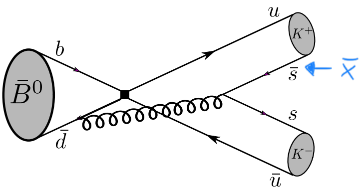

- Why power-suppressed?
  - $\rightarrow$ purely **hard** interactions (no $1/\omega$ enhancement from hard-collinear spectator quark)
  - $\rightarrow$ B-meson described by decay constant f_B
- standard hard-scattering approach gives endpoint-singularities: ($\phi(x) \simeq 6x\bar{x}$) [BBNS '01]

$$A \sim f_B f_K^2 \int_0^1 dx \int_0^1 dy \, \phi_{K^+}(x)\phi_{K^-}(y) \left[\frac{1}{y(1-x\bar{y})} + \frac{1}{\bar{x}^2 y} \right]$$

- consistent treatment would be a major step in controlling the underlying power-expansion

Interlude: non-relativistic bound states

In the limit $\boxed{m_b \gg m_q = m_c \gg \Lambda}$ the mesons are entirely dominated by 2-particle Fock states.

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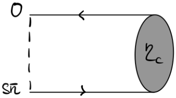
$\langle \eta_c | \, \bar{\chi}_c(0) \frac{\bar{\not{n}}}{2} \gamma_5 \chi_c(s\bar{n}) \, |0\rangle \quad =$



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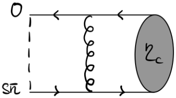

+ ...

$$\phi_{\eta_c}(x) = \delta(x - 1/2) \quad + \quad \frac{\alpha_s C_F}{4\pi} \left(\frac{\mu^2}{m_c^2} \right)^{\varepsilon} f(\varepsilon) \; (1 - x) \Big(1 + \mathcal{O}(\bar{x}) \Big) + \mathcal{O}(\alpha_s^2)$$

→ requires rapidity regulator α in endpoint-divergent inverse moments

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In [PB '18] I have shown that endpoint-divergences in α exponentiate

$$\int_0^1 \frac{dx}{\bar{x}^2} \phi_{\eta_c}(x) \sim \exp\left\{ \frac{1}{\alpha} \left(\frac{\nu}{2E}\right)^{\alpha} \times f(\mu/m_c) \right\}$$

"collinear anomaly" [Becher,Neubert '10]

- → they cancel in products with divergent moments of the B-LCDA in e.g. form factors
- → looks like standard $\mathrm{SCET_{II}}$ problem. . .

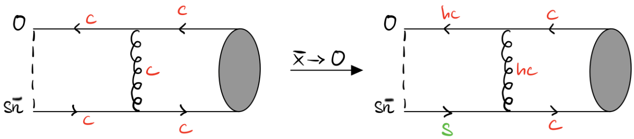
. . . it's **not**! It's a nested integral problem!

NR bound states - refactorization

What is the all-order $\bar{x} = 1 - x \to 0$ asymptotics of the bare charmonium LCDA?

$\to$ determined by soft (or soft-collinear) limit of coll. field that carries momentum fraction $\bar{x}$:

$$\langle \eta_c(p) |\, \bar{\chi}_c(0) \frac{\bar{\not{n}}}{2} \gamma_5 \chi_c(s\bar{n})\, |0\rangle = -iEf_{\eta_c} \int_0^1 d\bar{x}\, e^{i\bar{x}s\bar{n}p}\, \phi_{\eta_c}(x)$$



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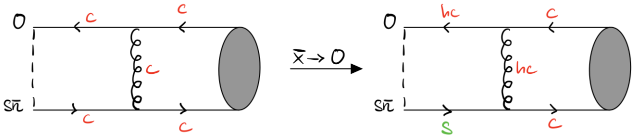
→ endpoint configuration $\sim [\bar{q}_s\psi_{hc}^{(5)}]$ involves a splitting of ψ_{hc} into a soft and two coll. quarks:

$$\phi(\bar{x}\to 0) \sim \int_0^1 d\hat{x}\,\phi(\hat{x})\int d\rho\, J(\hat{x},\rho)\, S(\rho,\bar{x})$$

[PB '18]

→ implicit ("iterative") integral equation with endpoint-divergent convolutions

cf. [Bell,PB,Feldmann '22]



Lessons from NR bound states

A non-additive cancellation of endpoint-singularities is a key characteristic of hadronic hard-exclusive reactions (process-independent), which is more complex than the recently understood cases.

An EFT formulation for such problems is unknown so far. Examples:

→ QED textbook process $e^- \mu^-$ backward scattering [Bell,PB,Feldmann '22]

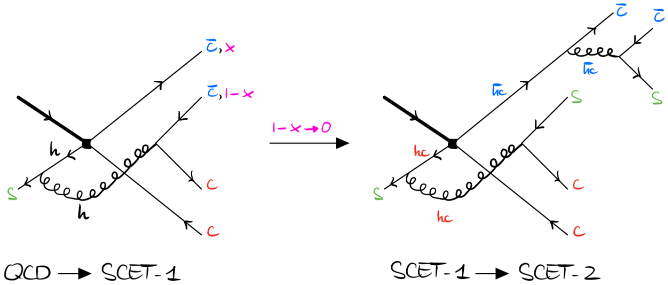
$$F \simeq \int_0^1 \frac{dx}{x} f_c(x) \int_0^1 \frac{dy}{y} f_{\bar{c}}(y)\, H(xy) \simeq \frac{I_1(2\sqrt{z})}{\sqrt{z}}\,, \qquad z = \frac{\alpha}{2\pi} \ln^2 \frac{m^2}{s}$$

→ Resummation of double-logarithms from energy-ordered ladder diagrams for the $B_c \to \eta_c$ soft-overlap form factor [Bell,PB,Feldmann,Horstmann, to appear]

Back to weak annihilation . . .

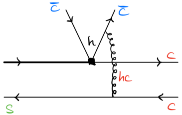
. . . need higher multiplicity states at the same order in λ:

(strict consequence from endpoint-divergence + power-counting)



LO Matching

BBNS @ l.p.: $$\left(\bar{\chi}_c \frac{\bar{\not{n}}}{\bar{n}\cdot\partial_c}\chi_c\right)\left(\bar{\chi}_{\bar{c}} \frac{\not{n}}{n\cdot\partial_{\bar{c}}}\chi_{\bar{c}}\right)\left(\bar{Q}_s \frac{\not{n}}{n\cdot\partial_s}H_v\right) \sim \lambda^{12}$$



(SCET$_{\rm II}$ counting: $\chi_c \sim \lambda^2$, $q_s \sim \lambda^3$, $A^{\perp}_{s,c} \sim \lambda^2$, and $\partial_s \sim \lambda^2$, with $\lambda^2 \sim \Lambda_{\rm QCD}$)

Goal: construct complete basis of boost and gauge-inv. operators $\sim \lambda^{14}$ in SCET$_{\rm II}$ with mass dim-6 that respect all quantum numbers in each sector.

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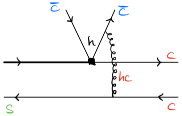

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Our starting point is the weak Hamiltonian with operators Q_{1-10} after Fierz: (also $u \to s$ but symmetric)

$$Q_1^{\pm,(u)} = [\bar{d}_i b_i]_{V-A}\,[\bar{u}_j u_j]_{V\pm A} \qquad \text{and} \qquad Q_2^{\pm,(u)} = [\bar{d}_i b_j]_{V-A}\,[\bar{u}_j u_i]_{V\pm A}$$

i) $m_b^2 \to \infty$: 3 classes of SCET$_{\text{I}}$ operators $(\bar{\psi}\psi)^2, (\bar{\psi}\psi)^2 A^{\mu}_{\perp}, (\bar{\psi}\psi)^3$

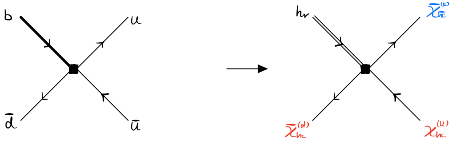
ii) $m_b\Lambda \to \infty$: need branchings of ψ_{hc} and $A^{\perp}_{hc}$ into soft and collinear partons up to $\mathcal{O}(\lambda^5)$

[Beneke,Feldmann '03]

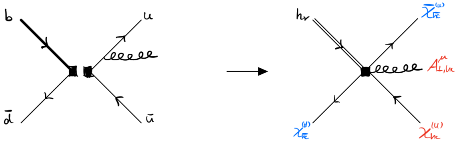
Matching onto SCET$_{\rm I}$

For $\bar{B}^0_d \to K^+K^-$ we need e.g. the matching relations (only some examples are shown!):

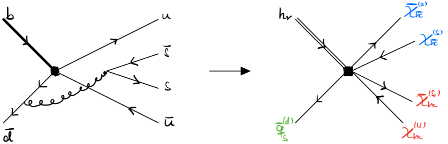
i) 4-quark operators:



ii) 4-quark+gluon operators:



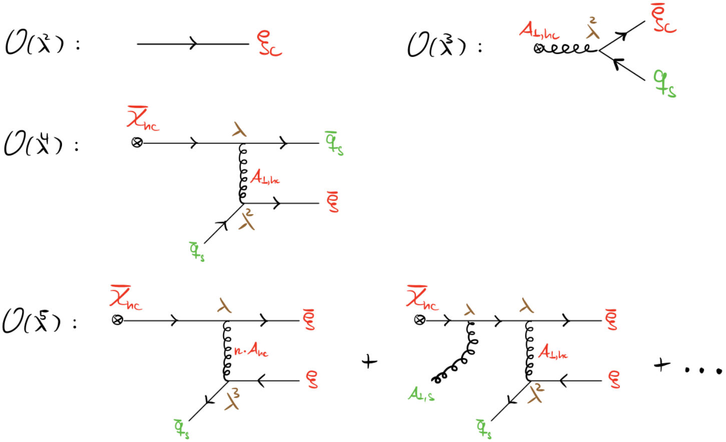
iii) 6-quark operators (BBNS):



$\to$ even more fields (or derivatives) in SCET$_{\rm I}$ give power-suppressed contribution in SCET$_{\rm II}$

Hard-collinear branchings up to $\mathcal{O}(\lambda^5)$

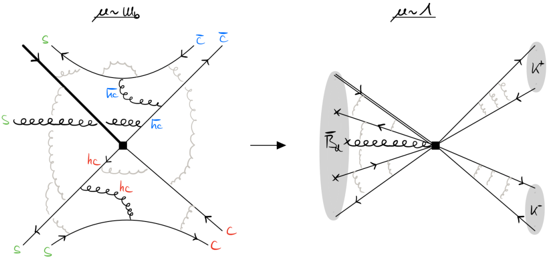
Splittings of SCET_I into SCET_II fields proceed via various sub-leading Lagrangian insertions. Relevant for our analysis are:



- hard modes integrated out $\rightarrow$ match hc and $\bar{hc}$ sector separately (use $\bar{n}\cdot A_{hc} = n\cdot A_{\bar{hc}} = 0$)
- Fierz relations $\rightarrow$ color-singlet/pseudo-scalar projections for SCET_II operators

Matching onto $\mathrm{SCET_{II}}$

- We did the LO QCD $\to$ $\mathrm{SCET_I}$ $\to$ $\mathrm{SCET_{II}}$ matching for Q_{1-10} (construct physical basis ✓)
- Example: A leading contribution to the decay amplitude in the heavy-quark limit



$\to$ full-theory (QCD) analysis = horrible mess, but SCET gives strict rules

$\to$ involves new 4-quark(+g) soft functions with fields delocalized on two light cones n and $\bar{n}$

$\to$ . . . compared to $\sim f_B$ for the 6-quark operators

$\to$ configuration is of "leading-power" due to soft inverse derivatives

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- The $(\bar{\psi}\psi)^2 A^\perp$ operators ...
  - $\rightarrow$ seem to be free of endpoint divergences in $\mathrm{SCET_{II}}$ in soft* and coll sector
  - $\rightarrow$ they give rise to two (being agnostic to color) soft operators for $Q^{(u)}$:

$$\langle 0|\left[\bar{\mathcal{Q}}_n^{(d)}\frac{\not{n}\not{\bar{n}}}{4}\gamma_\perp^\mu(1-\gamma_5)\mathcal{H}_n^{(v)}\right]\left[\bar{\mathcal{Q}}_{\bar{n}}^{(s)}\frac{\not{\bar{n}}\not{n}}{4}\gamma_\perp^\mu(1-\gamma_5)(S_{\bar{n}}^\dagger S_n)\mathcal{Q}_n^{(s)}\right]|\bar{B}_d^0\rangle$$

  - $\rightarrow$ new and likely complex soft functions (no LCDAs!)
    (cf. "negative energies" in QED [Beneke,PB,Toelstede,Vos '22])
  - $\rightarrow$ soft Wilson lines connect all fields to hard vertex via finite segments
  - $\rightarrow$ non-vanishing contribution from Q_1, which was absent for $(\bar{\psi}\psi)^3$ operators (BBNS)

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- The $(\bar{\psi}\psi)^2$ operators ...
  - $\rightarrow$ give rise to six additional soft functions for $Q^{(u)}$ ($\sim \not{A}_s^\perp$ and different Dirac structure)
  - $\rightarrow$ they seem to appear in endpoint-divergent convolutions*
  - $\rightarrow$ they multiply endpoint-divergent integrals of tw-2 and tw-3 collinear LCDAs

…work in progress!

Conclusion

1.) Endpoint singularities are a generic problem of SCET@NLP, and a consistent treatment would be a major breakthrough in controlling the underlying power-expansion!

2.) In certain cases factorization can be cured via refactorization conditions [PB '18, Neubert/Liu '19] and additive rearrangements [Neubert/Liu '19], e.g. in

- → bottom induced $h \to \gamma\gamma(gg)$ decay [Neubert et al. '19-22]
- → off-diagonal gluon thrust [Beneke et al. '22]
- → QED corrections in leptonic B decays [Feldmann et al. '22, Cornella et al. '23]
- → power-corrections in inclusive $\bar{B} \to X_s\gamma$ [Hurth, Szafron '23]

3.) However, endpoint-singularities manifest in a non-additive way in power corrections to hadronic hard-exclusive processes (aka Brodsky-Lepage), e.g. in

- → heavy-to-light form factors [PB '18] + [Bell,PB,Feldmann,Horstmann, to appear]
- → μ-e backscattering [Bell,PB,Feldmann '22]

4.) Weak annihilation amplitudes belong to this class, and a proper SCET analysis reveals the complexity of the problem, which shares some similarities with the form factor but with more complicated soft physics. A better understanding of this problem remains key to further progress. Whether (a generalization of) the above-mentioned methods can be applied in this case remains to be investigated.

Thank you!

Backup-Slides

Modes in SCET_{II}