Glauber Phases in Non-Global Observables

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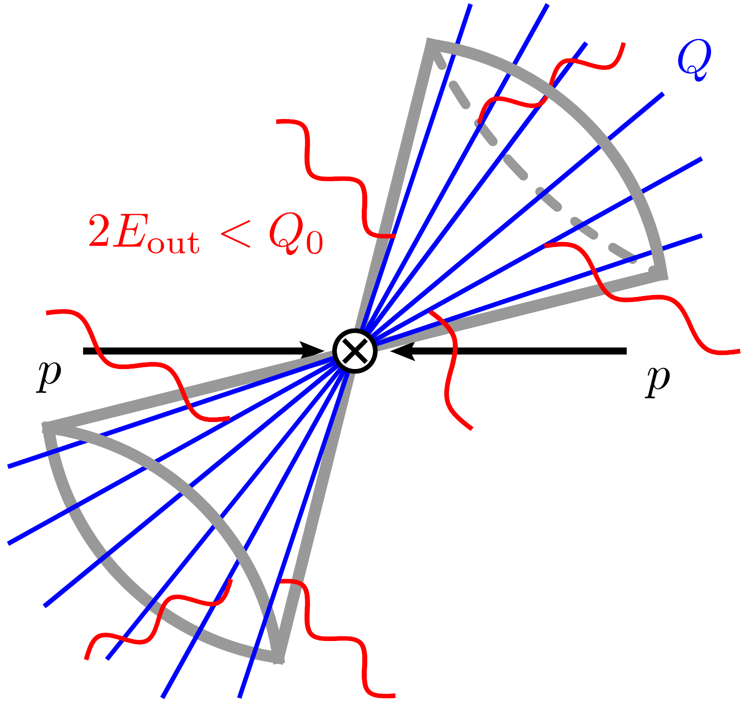
GAP-BETWEEN-JET CROSS SECTION



- veto radiation above a low scale Q_0 outside jets
- particles inside jet carry large energy $Q \sim \sqrt{\hat{s}}$
  - non-global logarithms $L = \ln(Q/Q_0)$

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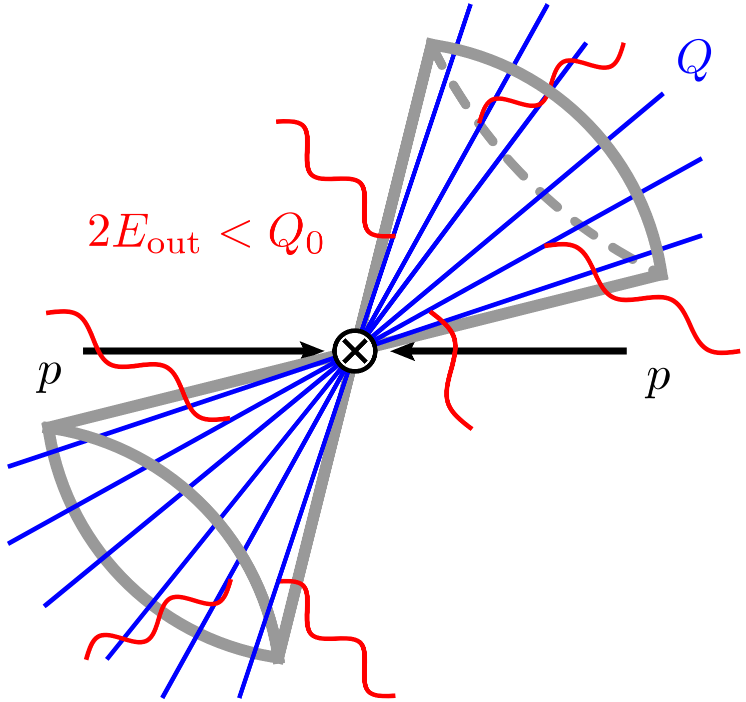

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- particles inside jet carry large energy $Q \sim \sqrt{\hat{s}}$
  - ➢ non-global logarithms $L = \ln(Q/Q_0)$
- Glauber gluon exchange between initial-state partons breaks color coherence
  - ➢ complicated structure at higher order
  - ➢ super-leading logarithms (SLLs)

$$\sigma \sim \sigma_{\text{Born}} \left\{1 + \alpha_s L + \alpha_s^2 L^2 + (\alpha_s \pi^2)\left(\alpha_s^2 L^3 + \alpha_s^3 L^5 + \alpha_s^4 L^7 + \dots\right)\right\}$$

[Forshaw, Kyrieleis, Seymour: JHEP **08** (2006) 059]
[Becher, Neubert, Shao: Phys. Rev. Lett. **127** (2021) 212002]

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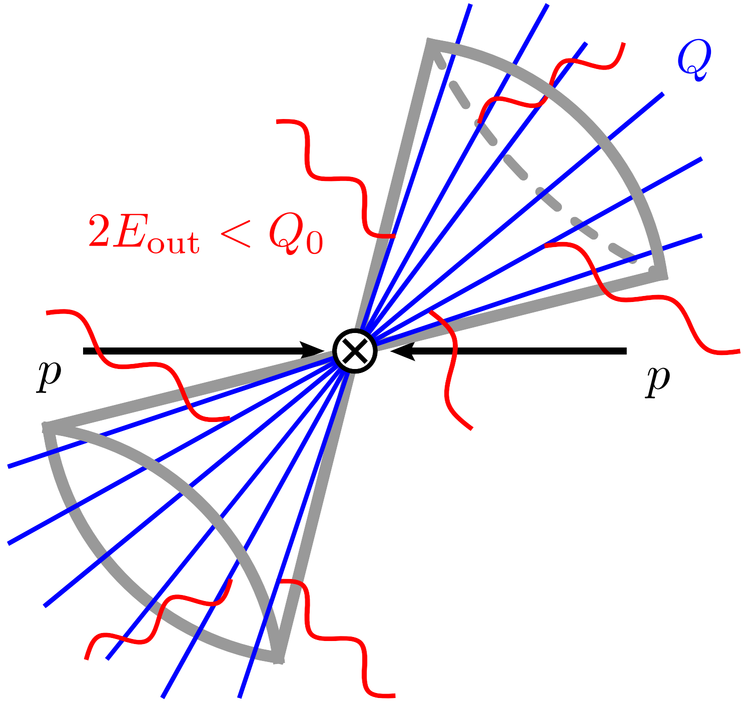

- real and imaginary part of large logarithms

$$\ln\left(-Q^2/Q_0^2\right) = 2L - i\pi$$

- realistic values, e.g. $Q = 1$ TeV and $Q_0 = 40$ GeV

$$L \sim \pi$$

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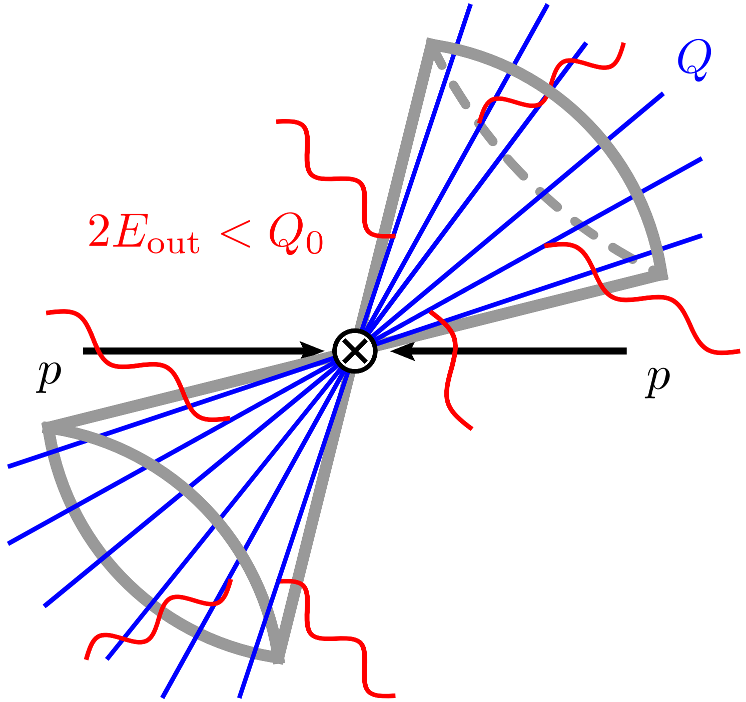

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- expansion in two $\mathcal{O}(1)$ parameters

$$\sigma \sim \sum_{\ell=1}^{\infty} \sum_{n=0}^{\infty} c_{\ell,n}\, w^{n+\ell}\, w_\pi^\ell \qquad w = \frac{N_c\,\alpha_s}{\pi} L^2 \qquad w_\pi = \frac{N_c\,\alpha_s}{\pi} \pi^2$$

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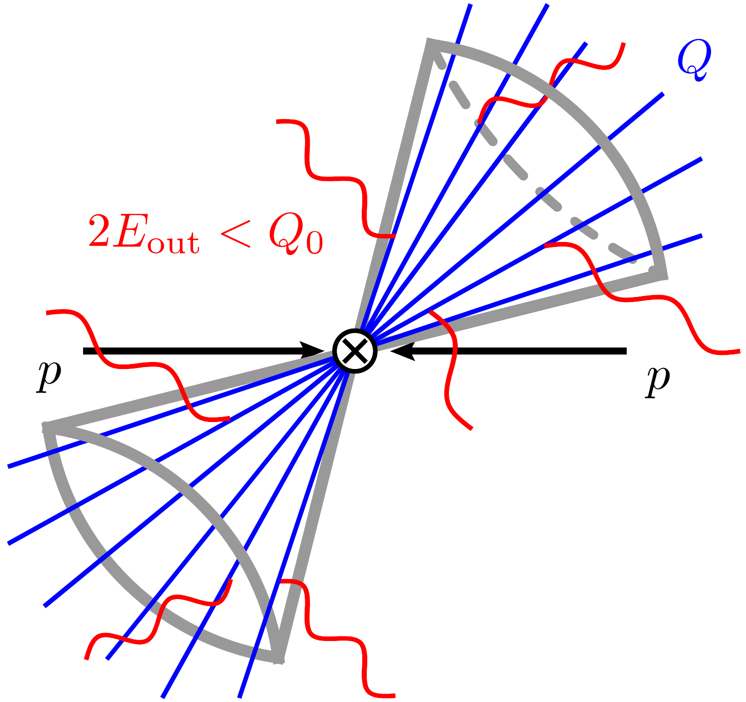

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- resummation of SLL ($\ell = 1$) done

[Becher, Neubert, Shao: Phys. Rev. Lett. **127** (2021) 212002]
[Becher, Neubert, Shao, MS: in preparation]

FACTORIZATION THEOREM

[Becher, Neubert, Rothen, Shao: Phys. Rev. Lett. **116** (2016) 192001]
[Becher, Neubert, Rothen, Shao: JHEP **11** (2016) 019]
[Becher, Neubert, Shao: Phys. Rev. Lett. **127** (2021) 212002]

- Factorization theorem for $pp \to M$ jets:

$$\sigma_{2\to M}(Q_0) = \int dx_1 \int dx_2 \sum_{m=M}^{\infty} \langle \boldsymbol{\mathcal{H}}_{2\to m}(\{\underline{n}\}, s, x_1, x_2, \mu) \otimes \boldsymbol{\mathcal{W}}_{2\to m}(\{\underline{n}\}, Q_0, x_1, x_2, \mu) \rangle$$

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- Hard function in color space

energy integrals including phase space constrains

$$\boldsymbol{\mathcal{H}}_{2\to m}(\{\underline{n}\}) = \int d\mathcal{E}_m \, |\mathcal{M}_{2\to m}(\{\underline{p}\})\rangle\langle\mathcal{M}_{2\to m}(\{\underline{p}\})|$$

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- Soft and collinear dynamics in lowest order at $\mu_s \sim Q_0$

$$\boldsymbol{\mathcal{W}}_{2\to m}(\mu_s) = f_{a_1}(x_1, \mu_s)\, f_{a_2}(x_2, \mu_s)\, \mathbf{1}$$

RENORMALIZATION GROUP EVOLUTION

- RG evolution from $\mu_h \sim Q$ down to $\mu_s \sim Q_0$

$$\boldsymbol{\mathcal{H}}(\mu_s) = \boldsymbol{\mathcal{H}}(\mu_h) + \int_{\mu_s}^{\mu_h} \frac{d\mu'}{\mu'} \boldsymbol{\mathcal{H}}(\mu_h)\, \boldsymbol{\Gamma}^H(\mu') + \int_{\mu_s}^{\mu_h} \frac{d\mu'}{\mu'} \int_{\mu'}^{\mu_h} \frac{d\mu''}{\mu''} \boldsymbol{\mathcal{H}}(\mu_h)\, \boldsymbol{\Gamma}^H(\mu'')\, \boldsymbol{\Gamma}^H(\mu') + \ldots$$

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- Anomalous dimension matrix $\boldsymbol{\Gamma}^H = \boldsymbol{\Gamma}^C + \boldsymbol{\Gamma}^S$ with

$$\boldsymbol{\Gamma}^S = \frac{\alpha_s}{4\pi} \begin{pmatrix} \boldsymbol{V}^S_{2\to M} & \boldsymbol{R}^S_{2\to M} & 0 & 0 & \dots \\ 0 & \boldsymbol{V}^S_{2\to M+1} & \boldsymbol{R}^S_{2\to M+1} & 0 & \dots \\ 0 & 0 & \boldsymbol{V}^S_{2\to M+2} & \boldsymbol{R}^S_{2\to M+2} & \dots \\ 0 & 0 & 0 & \boldsymbol{V}^S_{2\to M+3} & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} + \mathcal{O}(\alpha_s^2)$$

RENORMALIZATION GROUP EVOLUTION

- Different parts

$$\boldsymbol{V}^S_{2\to m} = \overline{\boldsymbol{V}}_{2\to m} + \boldsymbol{V}^G + \sum_{i=1,2} \boldsymbol{V}^c_i \ln\frac{\mu^2}{\mu_h^2}$$

$$\boldsymbol{R}^S_{2\to m} = \overline{\boldsymbol{R}}_{2\to m} + \sum_{i=1,2} \boldsymbol{R}^c_i \ln\frac{\mu^2}{\mu_h^2}$$

- Explicit expressions

$$\boldsymbol{V}^G = -8i\pi\left(\boldsymbol{T}_{1,L}\cdot\boldsymbol{T}_{2,L} - \boldsymbol{T}_{1,R}\cdot\boldsymbol{T}_{2,R}\right)$$

$$\boldsymbol{V}^c_i = 4C_i\,\mathbf{1}$$
$$\boldsymbol{R}^c_i = -4\,\boldsymbol{T}_{i,L}\circ\boldsymbol{T}_{i,R}\,\delta(n_k - n_i)$$

direction of emitted gluon

RENORMALIZATION GROUP EVOLUTION

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direction of emitted gluon

- Anomalous dimension matrix at 1-loop

$$\boldsymbol{\Gamma}^S = \frac{\alpha_s}{4\pi}\left(\overline{\boldsymbol{\Gamma}} + \boldsymbol{V}^G + \boldsymbol{\Gamma}^c\ln\frac{\mu^2}{\mu_h^2}\right) + \mathcal{O}(\alpha_s)$$

COLOR TRACES

- three important properties restrict possible combinations

$$\boldsymbol{\mathcal{H}}_{2\to m}\,\boldsymbol{\Gamma}^c\,\overline{\boldsymbol{\Gamma}} = \boldsymbol{\mathcal{H}}_{2\to m}\,\overline{\boldsymbol{\Gamma}}\,\boldsymbol{\Gamma}^c$$

$$\langle \boldsymbol{\mathcal{H}}_{2\to m}\,\boldsymbol{\Gamma}^c \otimes \mathbf{1}\rangle = 0$$

[Becher, Neubert, Shao: Phys. Rev. Lett. **127** (2021) 212002]

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- color trace including all double logarithms and Glauber phases

$$C^{\ell}_{\{\underline{r}\}} = \langle \boldsymbol{\mathcal{H}}_{2\to M}\,(\boldsymbol{\Gamma}^c)^{r_1}\,\boldsymbol{V}^G\,(\boldsymbol{\Gamma}^c)^{r_2}\,\boldsymbol{V}^G \ldots (\boldsymbol{\Gamma}^c)^{r_{2\ell-1}}\,\boldsymbol{V}^G\,(\boldsymbol{\Gamma}^c)^{r_{2\ell}}\,\boldsymbol{V}^G\,\overline{\boldsymbol{\Gamma}} \otimes \mathbf{1}\rangle$$

origin of π^2

- special case: $\ell = 1$ SLL

POSSIBLE COLOR STRUCTURES

- starting point for arbitrary representations

$$\langle \boldsymbol{\mathcal{H}}\, \boldsymbol{V}^G\, \overline{\boldsymbol{\Gamma}} \otimes \mathbf{1} \rangle = 64(i\pi) \sum_{j>2} J_j\, i f^{abc} \langle \boldsymbol{\mathcal{H}}\, \boldsymbol{T}_1^a\, \boldsymbol{T}_2^b\, \boldsymbol{T}_j^c \rangle$$

angular integral (anti-symmetric $1 \leftrightarrow 2$*)*

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angular integral (anti-symmetric $1 \leftrightarrow 2$)

- $\boldsymbol{V}^G$ and $\boldsymbol{\Gamma}^c$ are symmetric under $1 \leftrightarrow 2$ and act only on partons 1 and 2
  - ➢ two types of color structures

$$\boldsymbol{O}^{(j)} = (\boldsymbol{A}_1\, \boldsymbol{B}_2 - \boldsymbol{A}_2\, \boldsymbol{B}_1)\, \boldsymbol{T}_j \qquad \boldsymbol{S} = \boldsymbol{A}_1\, \boldsymbol{B}_2 + \boldsymbol{A}_2\, \boldsymbol{B}_1$$

come with angular integral J_{12} (symmetric $1 \leftrightarrow 2$)

POSSIBLE COLOR STRUCTURES - QUARKS

[Böer, Neubert, MS: arXiv:2304.xxxxx]

- (anti-)fundamental generators of $SU(N_c)$

$$\boldsymbol{T}_i^a \, \boldsymbol{T}_i^b = \frac{1}{2}\left([\boldsymbol{T}_i^a, \boldsymbol{T}_i^b] + \{\boldsymbol{T}_i^a, \boldsymbol{T}_i^b\} \right) = \frac{1}{2N_c}\delta^{ab}\,\mathbf{1}_i + \frac{1}{2}\left(i f^{abc} + \sigma_i\, d^{abc}\right)\boldsymbol{T}_i^c \qquad i = 1, 2$$

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- structures with $\boldsymbol{T}_j$ (anti-symmetric $1 \leftrightarrow 2$) and without $\boldsymbol{T}_j$ (symmetric $1 \leftrightarrow 2$)

$$\boldsymbol{O}_0^{(j)} = if^{abc}\,\boldsymbol{T}_1^a\,\boldsymbol{T}_2^b\,\boldsymbol{T}_j^c$$

← *only for odd number of* $\boldsymbol{V}^G$

$$\boldsymbol{O}_1^{(j)} = (\sigma_1 - \sigma_2)\, d^{abc}\,\boldsymbol{T}_1^a\,\boldsymbol{T}_2^b\,\boldsymbol{T}_j^c \qquad \boldsymbol{S}_1 = \boldsymbol{T}_1 \cdot \boldsymbol{T}_2$$

$$\boldsymbol{O}_2^{(j)} = (\boldsymbol{T}_1 - \boldsymbol{T}_2) \cdot \boldsymbol{T}_j \qquad \boldsymbol{S}_2 = \boldsymbol{1}$$

POSSIBLE COLOR STRUCTURES - GLUONS

- adjoint generators of $SU(N_c)$

$$\boldsymbol{F}_i^a\,\boldsymbol{F}_i^b = \frac{1}{2}\Big([\boldsymbol{F}_i^a,\boldsymbol{F}_i^b] + \{\boldsymbol{F}_i^a,\boldsymbol{F}_i^b\}\Big) = \frac{i}{2}\,f^{abc}\,\boldsymbol{F}_i^c + \frac{1}{2}\{\boldsymbol{F}_i^a,\boldsymbol{F}_i^b\} \qquad i = 1,2$$

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  - ➢ need different approach!

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$$\boldsymbol{O}^{(j)} = ((\boldsymbol{A}_1)_{12}\,(\boldsymbol{B}_2)_{34} - (1 \leftrightarrow 2))\,\boldsymbol{T}_j^{5} \qquad\qquad \boldsymbol{S} = (\boldsymbol{A}_1)_{12}\,(\boldsymbol{B}_2)_{34} + (1 \leftrightarrow 2)$$

$$4! + \binom{5}{3}\,2! = 44 \qquad\qquad 3! + \frac{4!}{2^3} = 9$$

maximal number (without symmetry constrains)

POSSIBLE COLOR STRUCTURES - GLUONS

- adjoint matrices of $SU(N_c)$

$$(\boldsymbol{F}^a)_{bc} = -if^{abc} \qquad (\boldsymbol{\Delta}^{ab})_{cd} = \delta^{ac}\delta^{bd} + \delta^{bc}\delta^{ad}$$

$$(\boldsymbol{D}^a)_{bc} = d^{abc} \qquad (\boldsymbol{\nabla}^{ab})_{cd} = \delta^{ac}\delta^{bd} - \delta^{bc}\delta^{ad}$$

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- only need the following structures for one initial-state parton

$$\mathbf{1}\,, \quad \boldsymbol{F}^a\,, \quad \boldsymbol{D}^a\,, \quad \{\boldsymbol{F}^a, \boldsymbol{F}^b\}\,, \quad \{\boldsymbol{D}^a, \boldsymbol{D}^b\}\,, \quad \{\boldsymbol{F}^a, \boldsymbol{D}^b\}\,, \quad \boldsymbol{\nabla}^{ab}$$

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- example

$$\boldsymbol{O}^{(j)}_{5f,FF,\nabla} = if^{abc}\left(\{\boldsymbol{F}^a_1, \boldsymbol{F}^d_1\}\boldsymbol{\nabla}^{bd}_2 - \{\boldsymbol{F}^a_2, \boldsymbol{F}^d_2\}\boldsymbol{\nabla}^{bd}_1\right)\boldsymbol{T}^c_j$$

POSSIBLE COLOR STRUCTURES - GLUONS

- structures with $\boldsymbol{T}_j$ (anti-symmetric $1 \leftrightarrow 2$) and without $\boldsymbol{T}_j$ (symmetric $1 \leftrightarrow 2$)

$$\begin{aligned}
\boldsymbol{O}^{(j)}_{2,F} &= (\boldsymbol{F}_1 - \boldsymbol{F}_2)\cdot \boldsymbol{T}_j \\
\boldsymbol{O}^{(j)}_{2,D} &= (\boldsymbol{D}_1 - \boldsymbol{D}_2)\cdot \boldsymbol{T}_j \\
\boldsymbol{O}^{(j)}_{3f,F,F} &= if^{abc}\,\boldsymbol{F}^a_1\,\boldsymbol{F}^b_2\,\boldsymbol{T}^c_j \\
\boldsymbol{O}^{(j)}_{3f,D,D} &= if^{abc}\,\boldsymbol{D}^a_1\,\boldsymbol{D}^b_2\,\boldsymbol{T}^c_j \\
\boldsymbol{O}^{(j)}_{3f,F,D} &= if^{abc}\left(\boldsymbol{F}^a_1\boldsymbol{D}^b_2 - \boldsymbol{F}^a_2\boldsymbol{D}^b_1\right)\boldsymbol{T}^c_j \\
\boldsymbol{O}^{(j)}_{3d,F,D} &= d^{abc}\left(\boldsymbol{F}^a_1\boldsymbol{D}^b_2 - \boldsymbol{F}^a_2\boldsymbol{D}^b_1\right)\boldsymbol{T}^c_j
\end{aligned}$$

$$\#\boldsymbol{O}^{(j)} = 22$$

$$\begin{aligned}
\boldsymbol{O}^{(j)}_{4,F,\nabla} &= \left(\boldsymbol{F}^a_1\,\boldsymbol{\nabla}^{ab}_2 - \boldsymbol{F}^a_2\,\boldsymbol{\nabla}^{ab}_1\right)\boldsymbol{T}^b_j \\
\boldsymbol{O}^{(j)}_{4,F,FF} &= \left(\boldsymbol{F}^a_1\{\boldsymbol{F}^a_2,\boldsymbol{F}^b_2\} - \boldsymbol{F}^a_2\{\boldsymbol{F}^a_1,\boldsymbol{F}^b_1\}\right)\boldsymbol{T}^b_j \\
\boldsymbol{O}^{(j)}_{4,F,DD} &= \left(\boldsymbol{F}^a_1\{\boldsymbol{D}^a_2,\boldsymbol{D}^b_2\} - \boldsymbol{F}^a_2\{\boldsymbol{D}^a_1,\boldsymbol{D}^b_1\}\right)\boldsymbol{T}^b_j \\
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\boldsymbol{O}_{5f,\nabla,\nabla} &= if^{abc}\,\boldsymbol{\nabla}^{ad}_1\,\boldsymbol{\nabla}^{bd}_2\,\boldsymbol{T}^c_j \\
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\boldsymbol{O}_{5f,DD,DD} &= if^{abc}\{\boldsymbol{D}^a_1,\boldsymbol{D}^d_1\}\{\boldsymbol{D}^b_2,\boldsymbol{D}^d_2\}\,\boldsymbol{T}^c_j \\
&\ldots
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{S}_0 &= \mathbf{1} \\
\boldsymbol{S}_{2,F,F} &= \boldsymbol{F}_1\cdot\boldsymbol{F}_2 \\
\boldsymbol{S}_{2,D,D} &= \boldsymbol{D}_1\cdot\boldsymbol{D}_2 \\
\boldsymbol{S}_{2,F,D} &= \boldsymbol{F}_1\cdot\boldsymbol{D}_2 + \boldsymbol{F}_2\cdot\boldsymbol{D}_1 \\
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\end{aligned}$$

$$\#\boldsymbol{S} = 7$$

PARTONIC CROSS SECTION

- reduced color trace

$n = \Sigma\, r_i$

$$C^{\ell}_{\{\underline{r}\}} = 64(i\pi)^{2\ell}(4N_c)^{n+2\ell}\left\{\sum_{j=3}^{2+M} J_j \sum_{i\in I^{(j)}} c_i^{(\ell|r_1\ldots r_{2\ell})}\langle \boldsymbol{\mathcal{H}}_{2\to M}\, \boldsymbol{O}_i^{(j)}\rangle + J_{12}\sum_{i\in I} d_i^{(\ell|r_1\ldots r_{2\ell})}\langle \boldsymbol{\mathcal{H}}_{2\to M}\, \boldsymbol{S}_i\rangle\right\}$$

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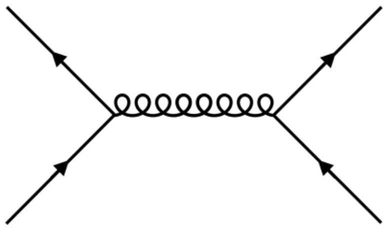
- combine color trace with nested scale integrals

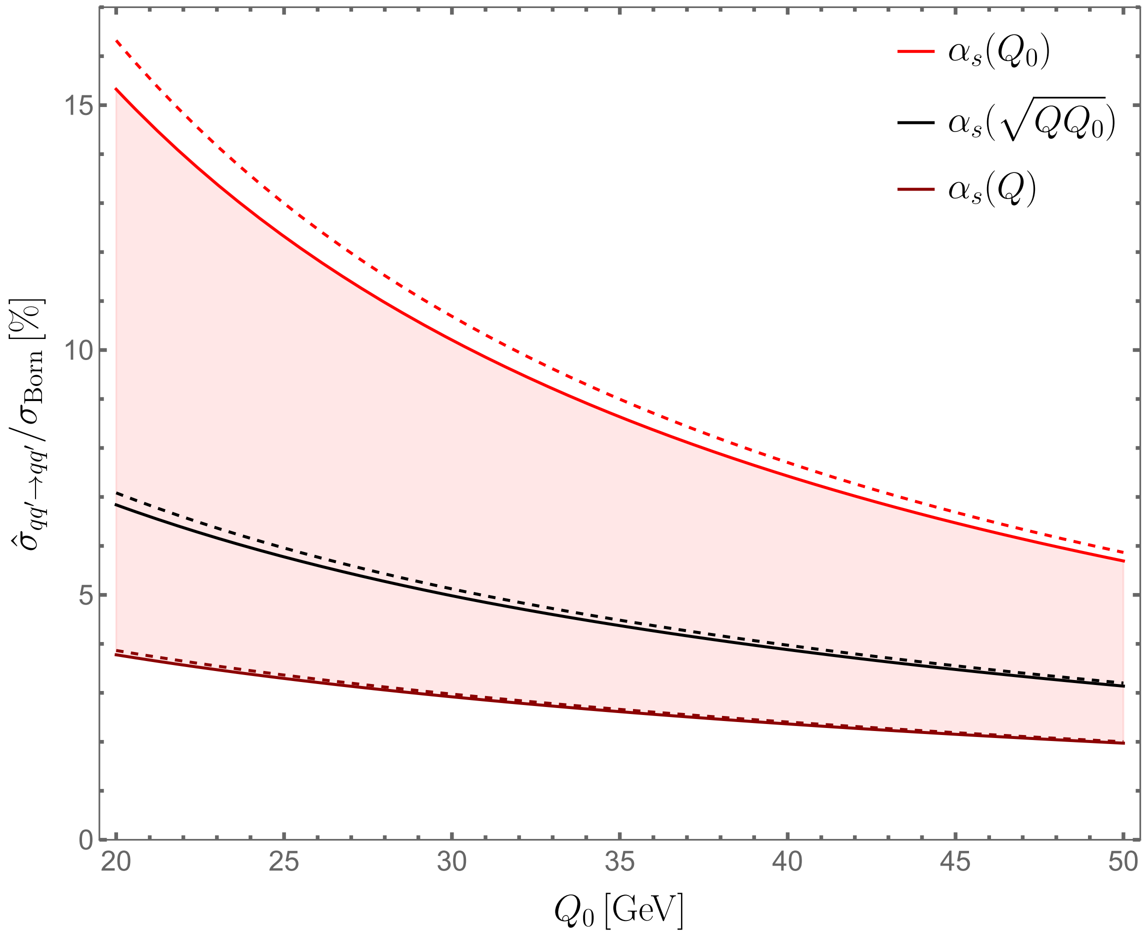
$(-2)!! \equiv (-1)!! \equiv 1$

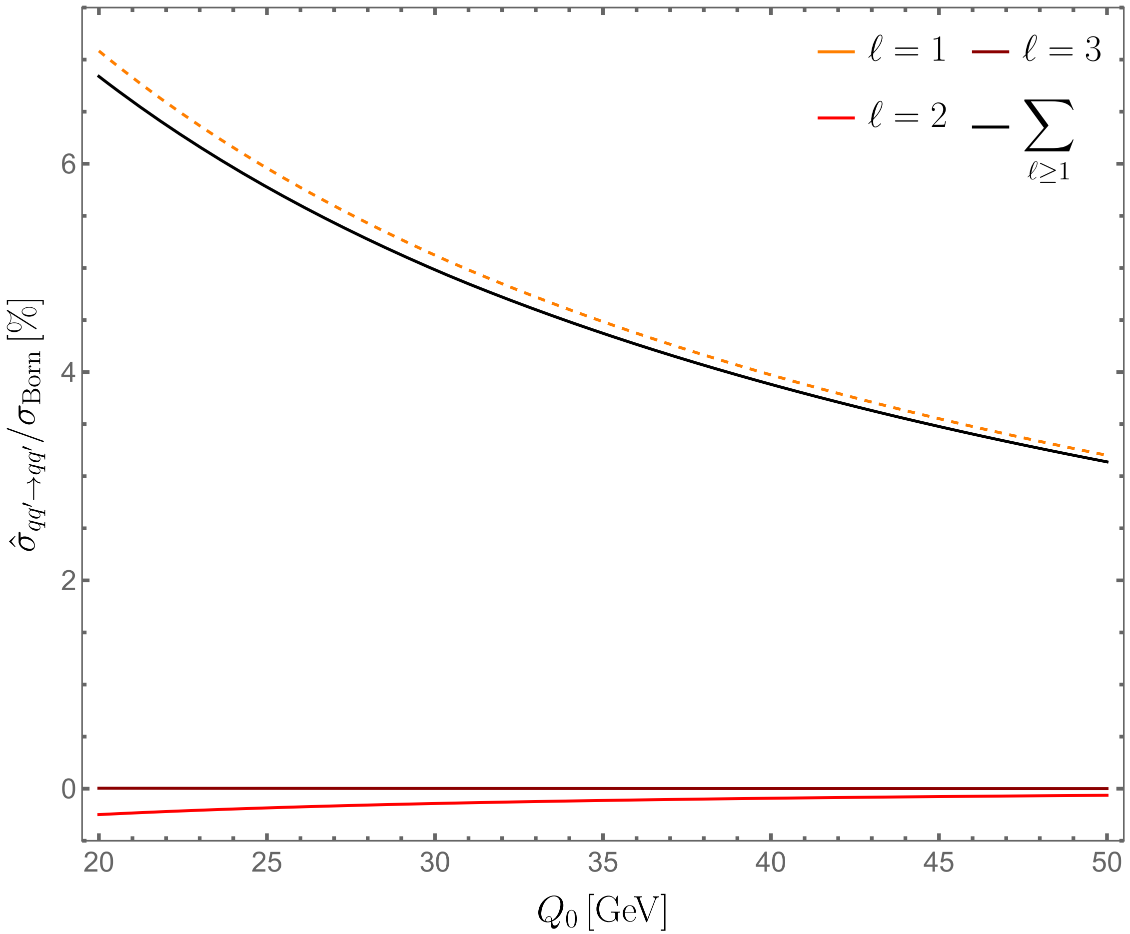
$$\hat{\sigma}^{\ell}_{\{\underline{r}\}} = \left(\frac{\alpha_s(\bar{\mu})}{4\pi}\right)^{2\ell+n+1} \frac{(-2)^n\, L^{2n+2\ell+1}}{(2n+2\ell)(2n+2\ell+1)} \prod_{k=1}^{2\ell} \frac{(2\sum_{i=1}^{k-1} r_i + k - 3)!!}{(2\sum_{i=1}^{k} r_i + k - 1)!!}\, C^{\ell}_{\{\underline{r}\}}$$

- ambiguity in the choice of the reference scale $\bar{\mu}$ can be avoided by using 1-loop running of α_s

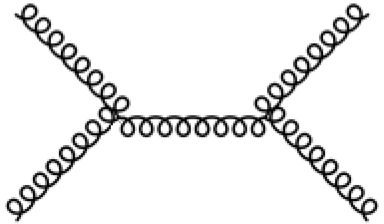
NUMERICAL ESTIMATES: $qq' \to qq'$

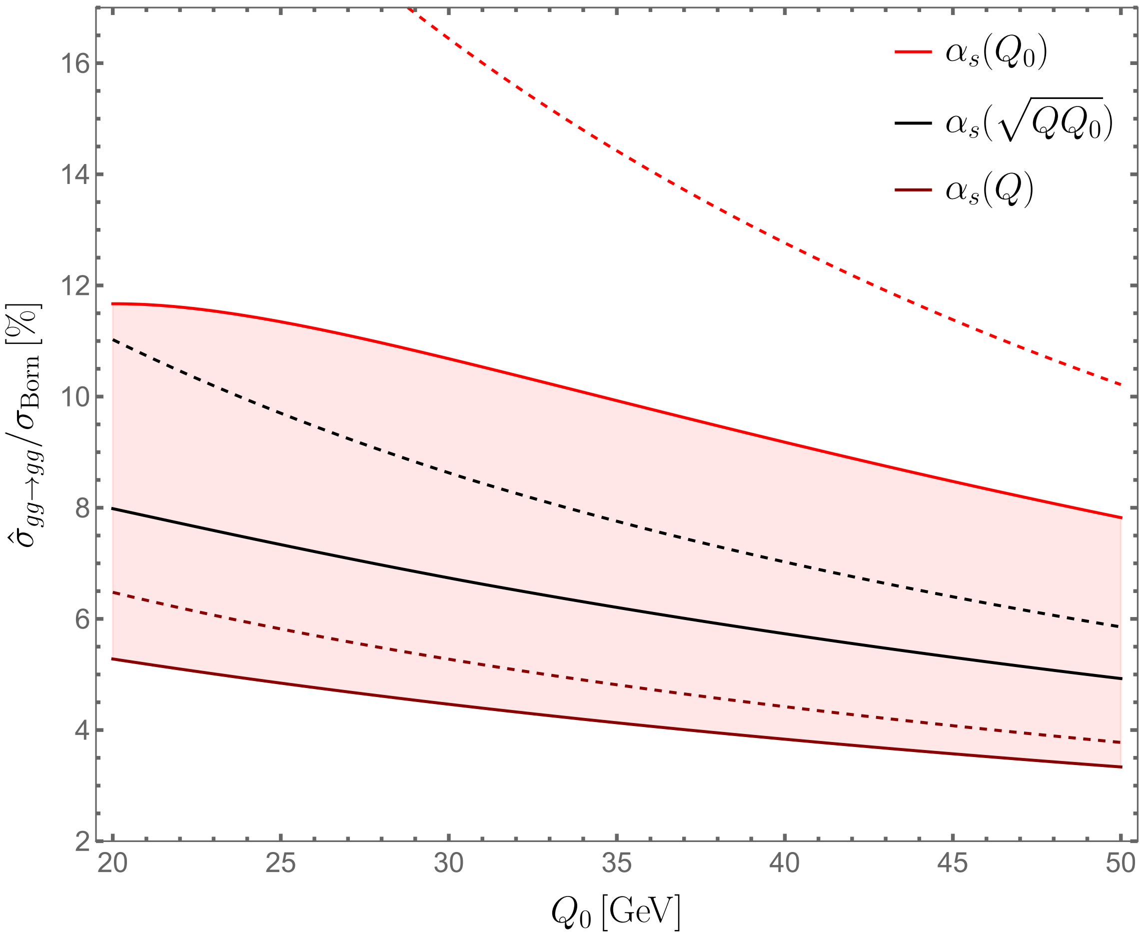


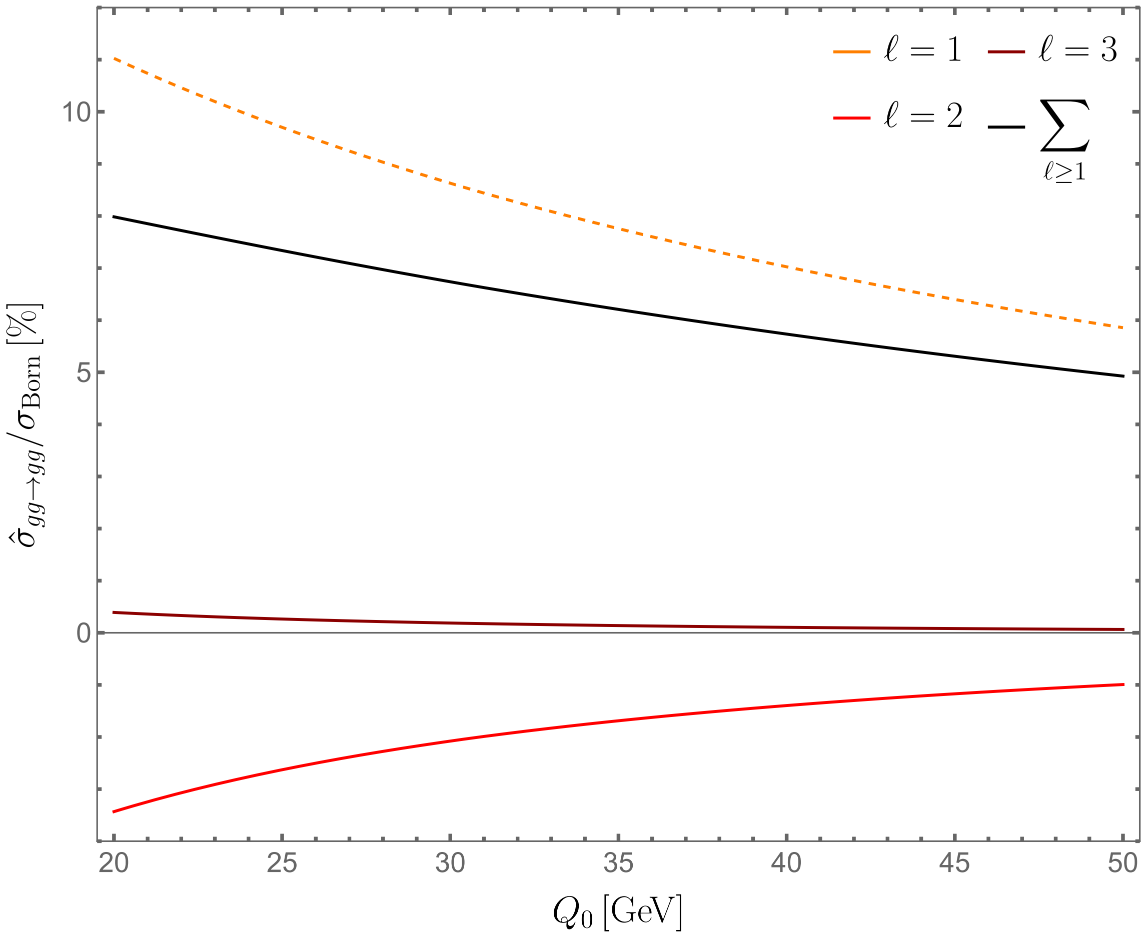




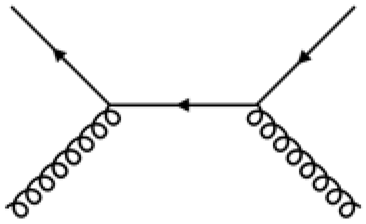
NUMERICAL ESTIMATES: $gg \to gg$

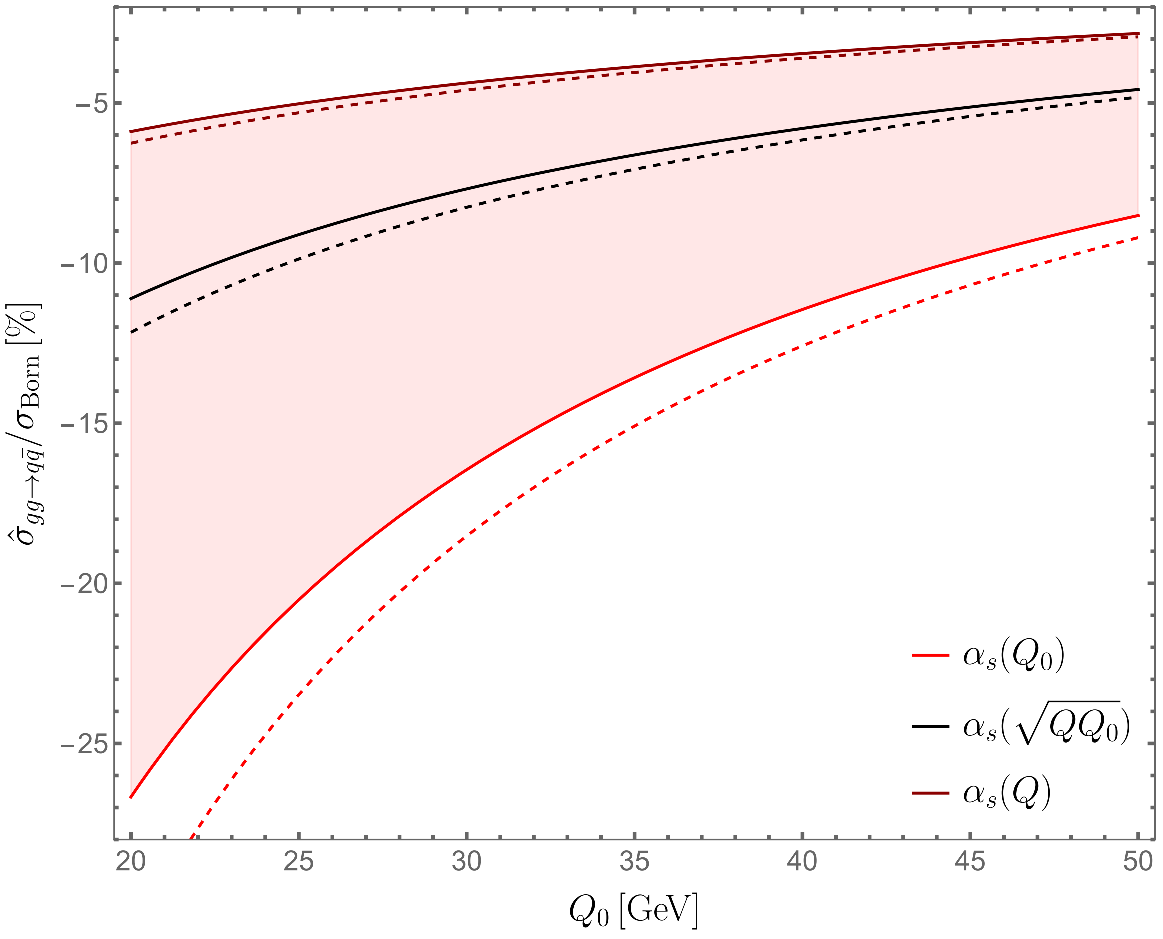


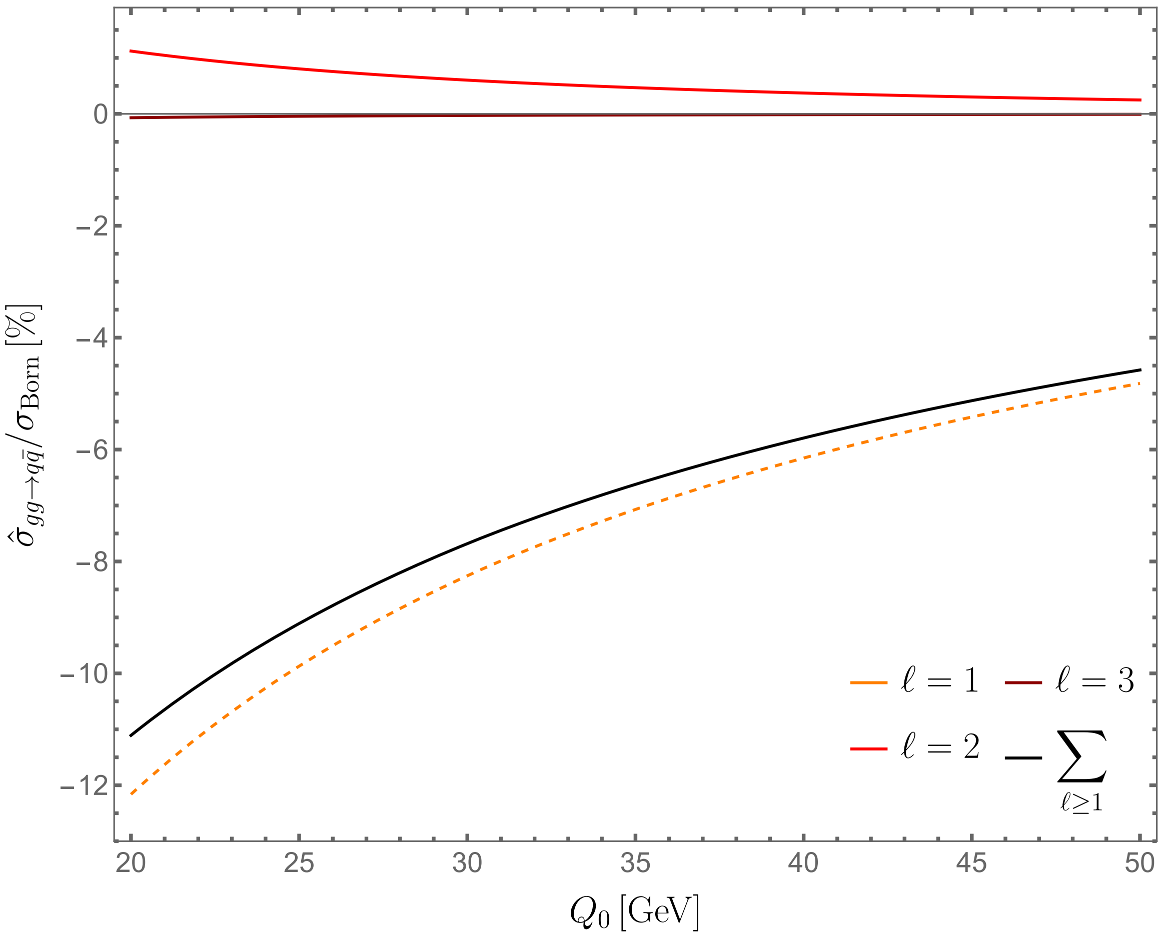




NUMERICAL ESTIMATES: $gg \to q\bar{q}$







CONCLUSION

- resummation of Glauber phases possible for quark and gluon induced processes

- contributions for quark induced processes very small
  - fortunately, enough to take SLL into account

- contributions for gluon induced processes can be sizeable
  - depending on partonic process, need to take higher Glauber phases into account

- outlook: quark-gluon induced processes, e.g. $gq \to gq$
  compare with results from exponentiating $\boldsymbol{V}^G$ in a parton shower [Nagy, Soper: 2019]

THANK YOU FOR YOUR ATTENTION!

BACKUP

MASTER FORMULA – QUARKS

$$\sigma^{\ell}_{\{\underline{r}\}} = \frac{\alpha_s L}{4\pi N_c^2} (-w)^{n+\ell} \, w_\pi^\ell \, \frac{2^{n+4}}{(2n+2\ell)(2n+2\ell+1)} \prod_{j=1}^{2\ell} \frac{\left(2\sum_{i=1}^{j-1} r_i + j - 3\right)!!}{\left(2\sum_{i=1}^{j} r_i + j - 1\right)!!} \prod_{i=2}^{\ell} \left[\frac{(\sigma_1 - \sigma_2)^2}{4} \left(1 - \frac{4}{N_c^2}\right) + \frac{2^{2-r_{2i-1}}}{N_c^2} \right]$$

$$\times \Bigg\{ \sum_{j=3}^{2+M} J_j \left(2^{-r_1} \langle \boldsymbol{\mathcal{H}}_{2\to M} (\boldsymbol{T}_1 - \boldsymbol{T}_2) \cdot \boldsymbol{T}_j \rangle - \frac{N_c}{2} (\sigma_1 - \sigma_2) d^{abc} \langle \boldsymbol{\mathcal{H}}_{2\to M} \, \boldsymbol{T}_1^a \, \boldsymbol{T}_2^b \, \boldsymbol{T}_j^c \rangle \right)$$

$$- 2(1 - \delta_{0 r_1}) J_{12} \left(2^{-r_1} C_F \langle \boldsymbol{\mathcal{H}}_{2\to M} \, \boldsymbol{1} \rangle + (1 - 2^{-r_1}) \langle \boldsymbol{\mathcal{H}}_{2\to M} \, \boldsymbol{T}_1 \cdot \boldsymbol{T}_2 \rangle \right) \Bigg\}$$

$$w = \frac{N_c \alpha_s}{\pi} L^2, \qquad w_\pi = \frac{N_c \alpha_s}{\pi} \pi^2$$