HADRON-STRUCTURE DEPENDENT QED CORRECTIONS IN RARE EXCLUSIVE B DECAYS

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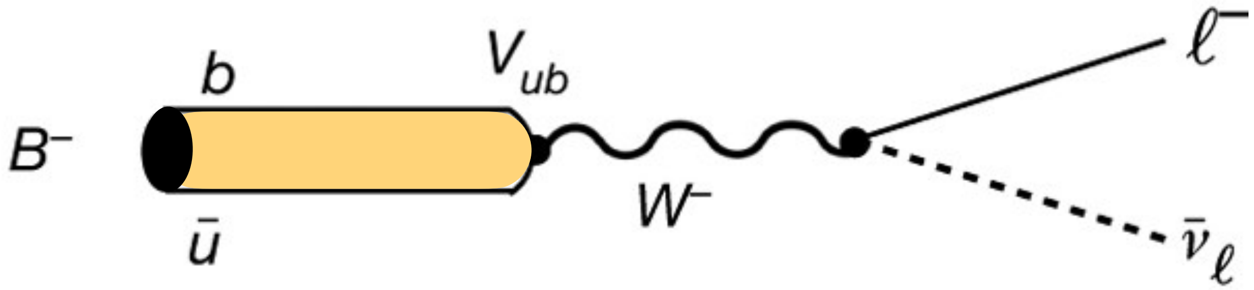
based on CC, M. König, MN: arXiv:2212.14430

MOTIVATION

Leptonic decays $B^- \to \ell^- \bar{\nu}_\ell$ are interesting for several reasons:

- **Determination of |V$_{ub}$|** largely unaffected by hadronic uncertainties



$$\Gamma \sim m_\ell^2 \, f_{B_u}^2 \, |V_{ub}|^2$$

B-meson decay constant

- Chiral suppression offers sensitive **probe of new scalar interactions**
- Comparing different lepton flavors yields **test of lepton universality**
  $\Rightarrow$ Belle II will measure $\ell = \mu, \tau$ channels with 5-7% uncertainty

[Belle II Physics Book]

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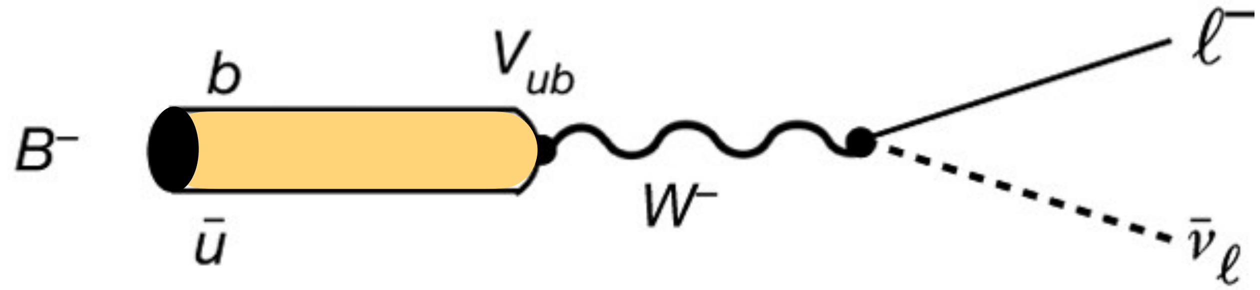

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B-meson decay constant

- QCD matrix element is known with <1% accuracy: [FNAL/MILC 2017]

$$\langle 0 | \, \bar{u} \gamma^\mu \gamma_5 b \, | B^-(p) \rangle = i f_{B_u} p^\mu \quad \text{with} \quad f_{B_u} = (189.4 \pm 1.4)\,\text{MeV}$$

- QED corrections can be of similar magnitude or even larger, due to presence of **large logarithms** $\alpha \ln(m_b/m_\ell)$ and $\alpha \ln(m_\ell/E_s)$

MOTIVATION

QED effects are well under control for $\mu > m_b$ and $\mu \ll \Lambda_{\rm QCD}$:

- Effective weak Hamiltonian contains all short-distance effects ($\mu > m_b$)

$$\mathcal{L}_{\rm eff} = -\frac{4G_F}{\sqrt{2}}\, K_{\rm EW}(\mu)\, V_{ub}\, (\bar{u}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu P_L \nu_\ell)$$

- Very soft photons see *B* meson as point-like particle

effective 4-fermion interaction
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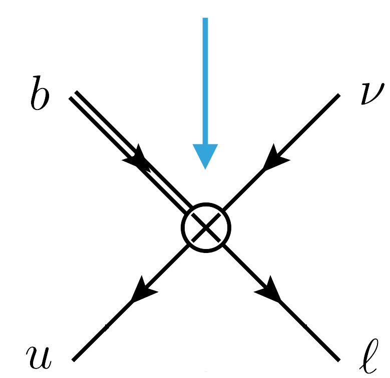

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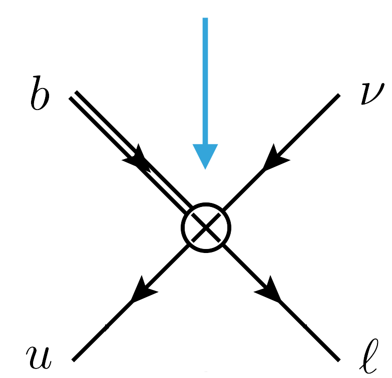

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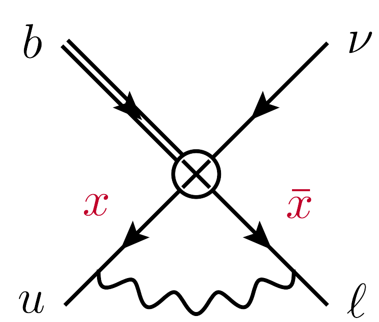
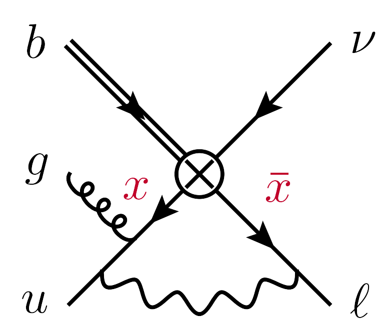
[CC, König, MN 2022]





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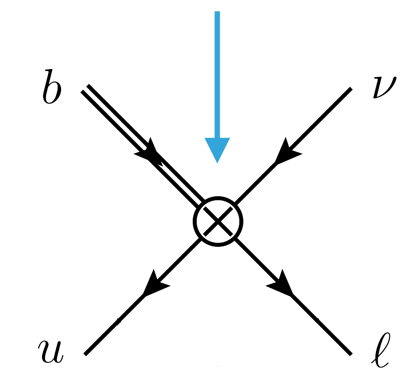

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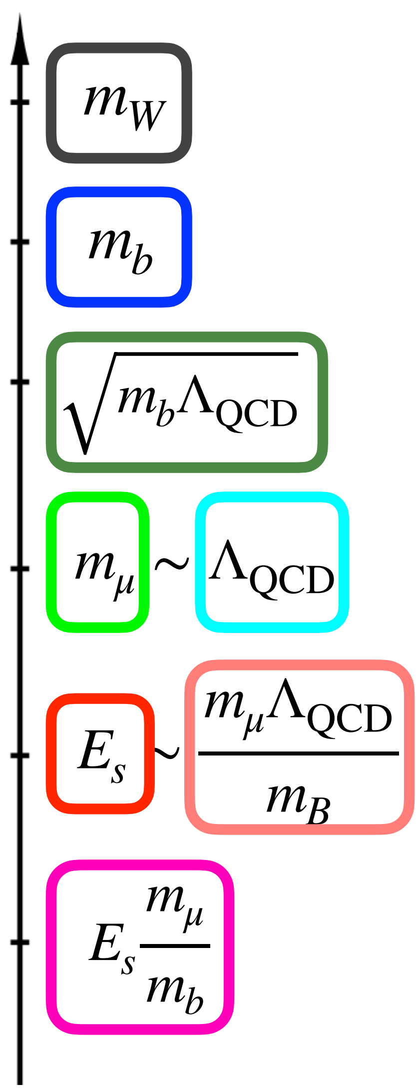
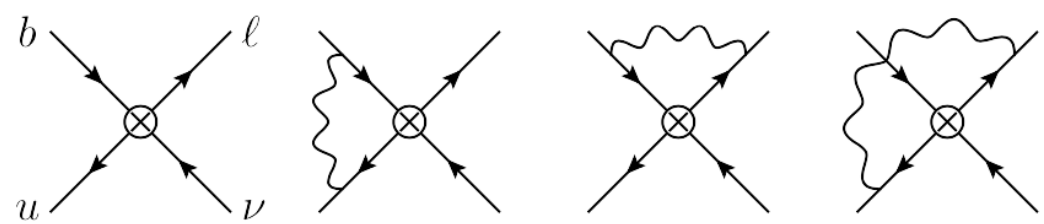
- Very soft photons see *B* meson as point-like particle
- Intermediate scale range $\Lambda_{\rm QCD} < \mu < m_b$ gives rise to more intricate effects, as virtual photons can resolve **inner structure of B meson**
  - $B_s \to \mu^+\mu^-$ [Beneke, Bobeth, Szafron 2017 & 2019]
  - $B \to K\pi, D\pi$ [Beneke, Böer, Toelstede, Vos 2020; Beneke, Böer, Finauri, Vos 2021]

RELEVANT SCALES

In the presence of QED effects, the decay $B^- \to \mu^- \bar{\nu}_\mu$ is sensitive to eight different energy scales:

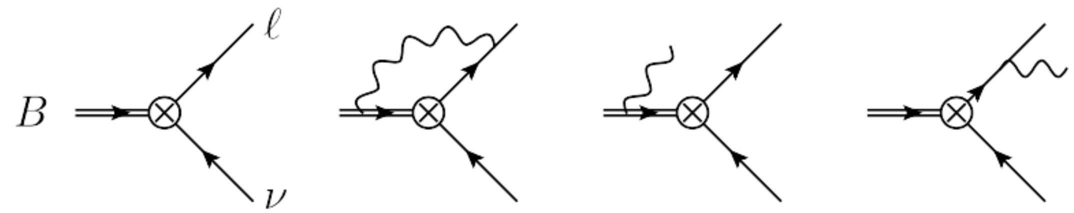




Fock-state description of B meson: $|\bar{u}b\rangle + |\bar{u}gb\rangle + \dots$

[see: Beneke, Bobeth, Szafron 2019]

description of B meson as a point-like pseudo-scalar boson



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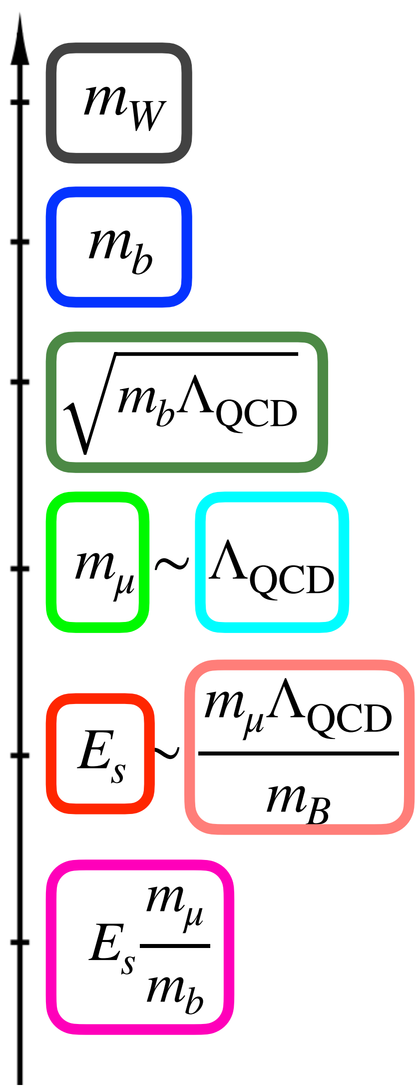

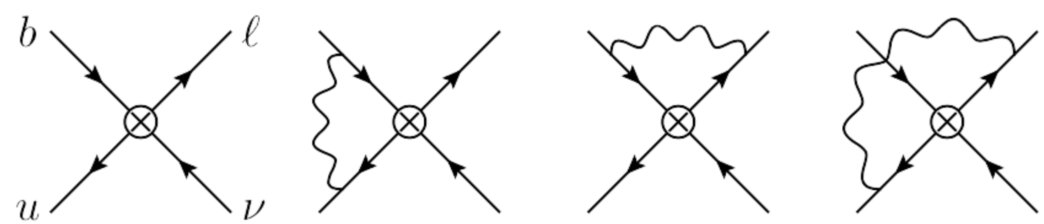

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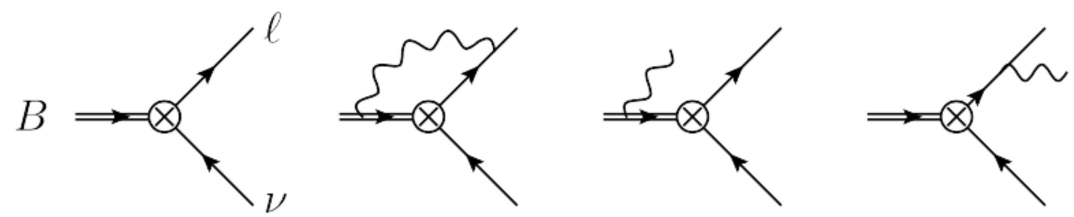

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QED CORRECTIONS IN LEPTONIC B DECAY

- Fact that external particles are charged under QED invalidates naive factorization, but scale separation can still be studied using SCET
- For power-suppressed processes such as $B^- \to \ell^- \bar{\nu}_\ell$, derivation of SCET factorization theorems is far more complicated than at leading power and has been understood only recently
  [Liu, MN 2019; Liu, Mecaj, MN, Wang 2020; Beneke *et al.* 2022; Liu, MN, Schnubel, Wang 2022]
- Our work is one of the first derivations of a **subleading-power factorization theorem** for a process involving **nonperturbative hadronic dynamics** [along with: Feldmann, Gubernari, Huber, Seitz 2022; see also: Hurth, Szafron 2023]

QED CORRECTIONS IN LEPTONIC B DECAY – CHALLENGES

- Quark current $\bar{u}\,\gamma^\mu P_L\,b$ is not gauge invariant under QED
  $\Rightarrow$ add a Wilson line to account for soft photon interactions with charged lepton
- One option: light-like Wilson line $\bar{u}\,\gamma^\mu P_L\,b\,S_n^{(\ell)\dagger}$ [Beneke, Bobeth, Szafron 2019]
  - anomalous dimension sensitive to IR regulators
  - matching onto point-like meson theory not fully understood

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- Our choice: **time-like Wilson line** $\bar{u}\,\gamma^\mu P_L\, b\, S_{v_\ell}^{(\ell)\dagger}$ [CC, König, MN 2022]
  - $S_{v_\ell}^{(\ell)} \triangleq S_n^{(\ell)} C_{v_\ell}^{(\ell)}$ arises since both soft and soft-collinear photons can resolve the structure of the B meson
  - soft-collinear photons interactions with charged lepton described in "boosted HLET" and decoupled via a field redefinition [Fleming, Hoang, Mantry, Stewart 2007]

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- Our choice: **time-like Wilson line** $\bar{u}\,\gamma^\mu P_L\,b\,S_{v_\ell}^{(\ell)\dagger}$ [CC, König, MN 2022]
  - well-defined anomalous dimension (after refactorization) 😌
  - consistent matching onto point-like meson theory 😌

QED CORRECTIONS IN LEPTONIC B DECAY – CHALLENGES

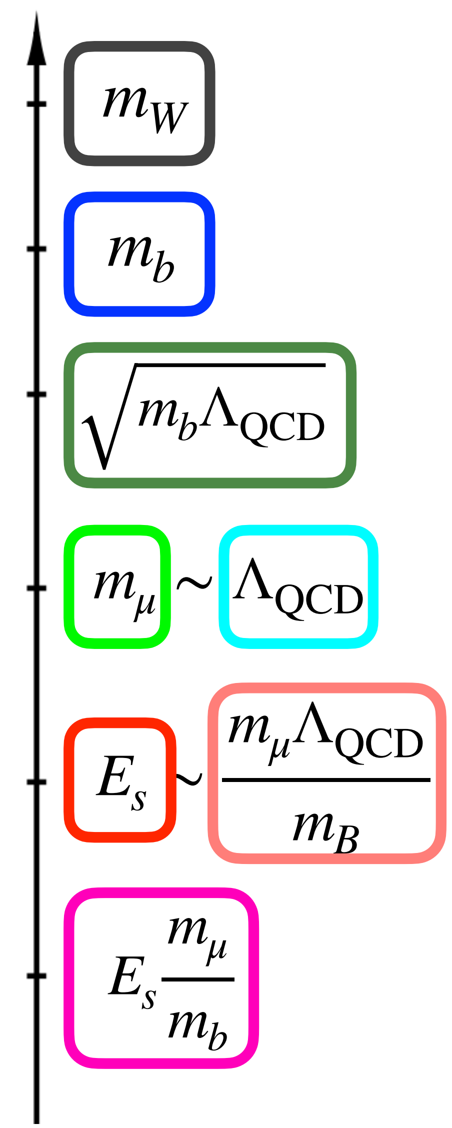
Have analyzed the factorization properties (scale separation) of the $B^- \to \mu^- \bar{\nu}_\mu$ amplitude including QED corrections in SCET

[CC, König, MN 2022]

- Relevant modes for **virtual QED corrections**:
  - hard
  - *hard-collinear*
  - soft
  - *collinear*
  - *soft-collinear*

resolve the light-cone structure of the *B* meson

- Relevant modes for **real QED corrections**:
  - ultra-soft
  - ultra-soft-collinear



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- Relevant modes for **virtual QED corrections**:
  - hard $(1, 1, 1)$
  - *hard-collinear* $(\lambda, 1, \lambda^{\frac{1}{2}})$
  - soft $(\lambda, \lambda, \lambda)$
  - *collinear* $(\lambda_\ell^2, 1, \lambda_\ell)$
  - *soft-collinear* $\lambda(\lambda_\ell^2, 1, \lambda_\ell)$
- Relevant modes for **real QED corrections**:
  - ultra-soft $(\lambda_E, \lambda_E, \lambda_E)$
  - ultra-soft-collinear $\lambda_E(\lambda_\ell^2, 1, \lambda_\ell)$

Expansion parameters:

$$\lambda = \frac{\Lambda_{\rm QCD}}{m_b}$$

$$\lambda_\ell = \frac{m_\mu}{m_b} \sim \lambda$$

$$\lambda_E = \frac{E_s}{m_b} \sim \lambda_\ell^2$$

STRATEGY

We want to disentangle all these scales by

- identifying the **appropriate EFT** description
- deriving a **factorization theorem** to describe the multi-scale problem in terms of convolutions of single-scale objects

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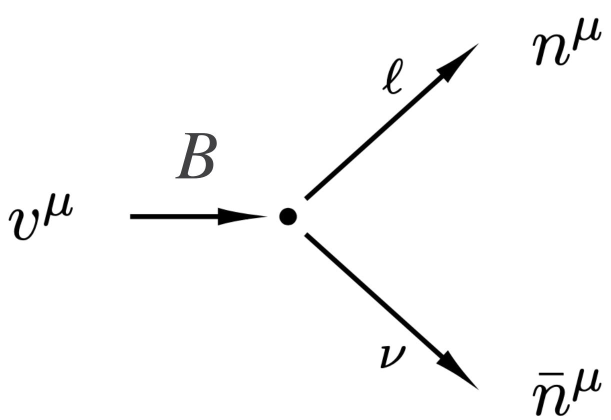
In the following, we will

- describe the **EFT construction** across all scales
- discuss the **factorization & refactorization** of the **"virtual" amplitude**
- sketch the **low-energy theory** describing real emissions

KINEMATICS

In the B-meson rest frame, the muon and the neutrino are back to back.
We identify the direction of the **muon** as the "**collinear**" one.



$$p_B^\mu = m_B v^\mu\,, \qquad v^\mu = (1,0,0,0)$$

$$p_\mu^\mu = \frac{m_B}{2}\left(1+\lambda_\mu^2, 0, 0, +1-\lambda_\mu^2\right) \approx \frac{m_B}{2}(1,0,0,\ +1) = \frac{m_B}{2} n^\mu$$

$$p_\nu^\mu = \frac{m_B}{2}\left(1-\lambda_\mu^2, 0, 0, -1+\lambda_\mu^2\right) \approx \frac{m_B}{2}(1,0,0,\ -1) = \frac{m_B}{2} \bar{n}^\mu$$

$$\uparrow$$

$$\lambda_\mu = \frac{m_\mu}{m_b} \ll 1$$

Initial state quark are bound in the meson, with residual momenta of $\mathcal{O}(\Lambda_{\mathrm{QCD}})$

$$p_b = m_b v + k_b\,, \quad p_q = k_q \qquad\qquad k_b, k_q \sim m_b(\lambda, \lambda, \lambda) \qquad\qquad \lambda = \frac{\Lambda_{\mathrm{QCD}}}{m_b}$$

$\mu \sim m_b$: FROM THE FERMI THEORY TO HQET $\times$ SCET-1

- Below m_b, radiation is too soft to affect the b-quark momentum
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- Remaining fields can have large momenta, but small invariant mass
  $\Rightarrow$ the right EFT is **SCET-1**
- In practice, the fields & power counting we need at this stage are:

$$\begin{aligned} \nu &\to \chi_{\overline{hc}}^{(\nu)} \\ \ell &\to \chi_{hc}^{(\ell)} \\ b &\to h_v \\ q &\to \chi_{hc}^{(q)},\ q_s \end{aligned}$$

(+ photons, gluons)

$$\chi_{hc}^{(\ell)}, \chi_{hc}^{(q)}, \chi_{\overline{hc}}^{(\nu)} \sim \lambda^{1/2}$$

$$h_v,\ q_s \sim \lambda^{3/2}$$

$$\mathcal{A}_{hc\perp}^{\mu},\ \mathcal{G}_{hc\perp}^{\mu} \sim \lambda^{1/2}$$

photons gluons

CONSTRUCTION OF SCET-1 BASIS

To keep in mind when building the basis:

- All operators stem from the Fermi Lagrangian
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  - for $\Gamma_\ell = 1$ the muon is right-handed $\Rightarrow$ the chirality flip gives a factor m_ℓ, included in the operator definition

CONSTRUCTION OF SCET-1 BASIS

Given this, we have **five classes of 4-fermion operators**:

A. Operators with soft spectator
B. Operators with hard-collinear spectator
C. Operators with soft spectator + hard-collinear photon
D. Operators with hard-collinear spectator + hard-collinear photon
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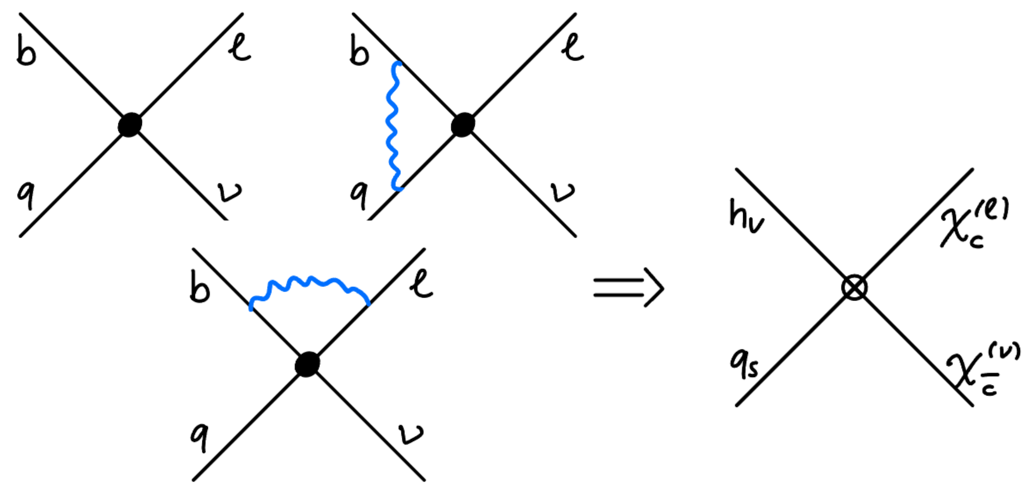
Only these are needed at $\mathcal{O}(\alpha)$.

CONSTRUCTION OF SCET-1 BASIS

A. Operators with soft spectator

- Obtained from hard matching:



power counting

$$\mathcal{O}_{A,1}^{(5)} = m_\ell \left(\bar{q}_s \frac{\not{\bar{n}}}{2} P_L h_v \right) \left(\bar{\chi}_{hc}^{(\ell)} \frac{1}{i\bar{n}\cdot\overleftarrow{\partial}} P_L \chi_{\overline{hc}}^{(\nu)} \right)$$

$$\mathcal{O}_{A,2}^{(5)} = m_\ell \left(\bar{q}_s \frac{\not{\bar{n}}}{2} P_L h_v \right) \left(\bar{\chi}_{hc}^{(\ell)} \frac{1}{in\cdot\overrightarrow{\partial}} P_L \chi_{\overline{hc}}^{(\nu)} \right)$$

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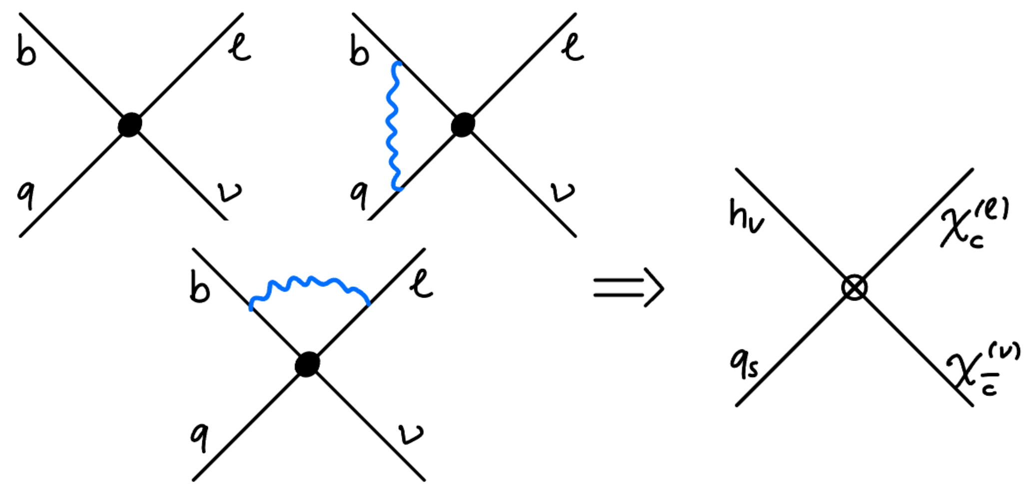

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- Only the first contributes at tree level:

no projection on *B* meson

$$i\mathcal{A}_{\text{tree}} = -\frac{4G_F}{\sqrt{2}} V_{ub} \left[\cancel{(\bar{v}_q \gamma_\perp^\mu P_L u_b)(\bar{u}_\ell \gamma_{\perp\mu} P_L v_\nu)} + \frac{2m_\ell}{m_B} \left(\bar{v}_q \frac{\not{\bar{n}}}{2} P_L u_b \right) (\bar{u}_\ell P_L v_\nu) \right]$$

lepton EOM: $$\bar{u}_\ell \frac{\not{n}}{2} = \frac{m_\ell}{m_B} \bar{u}_\ell - \frac{2m_\ell^2}{m_B^2} \bar{u}_\ell \frac{\not{\bar{n}}}{2} , \qquad \frac{\not{\bar{n}}}{2} v_\nu = 0$$

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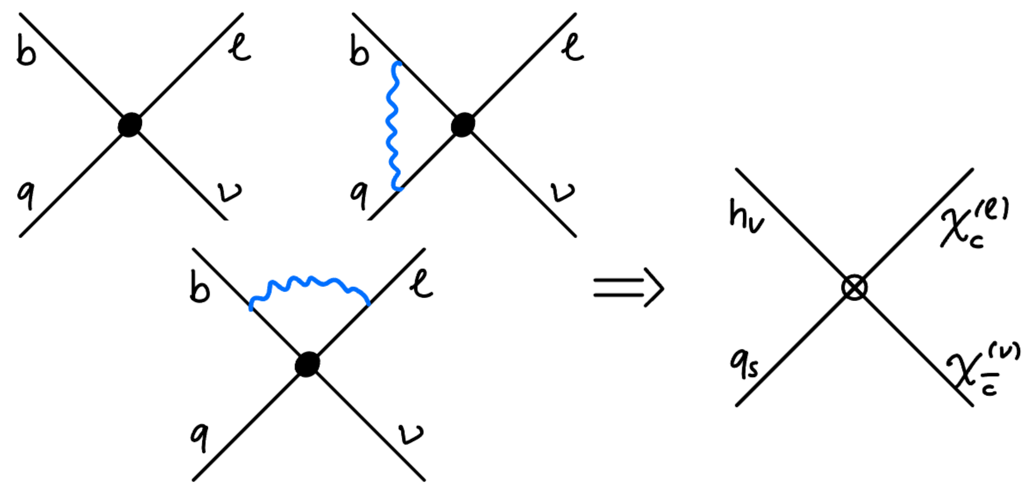

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$$\boxed{\mathcal{O}_{A,1}^{(5)} = m_\ell \left(\bar q_s \frac{\not n}{2} P_L h_v\right)\left(\bar\chi_{hc}^{(\ell)} \frac{1}{i\bar n\cdot\overleftarrow{\partial}} P_L \chi_{\overline{hc}}^{(\nu)}\right)}$$

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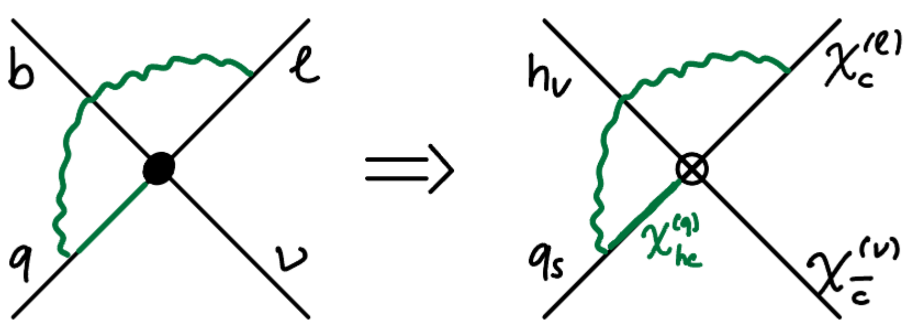
CONSTRUCTION OF SCET-1 BASIS

B. Operators with hard-collinear spectator



$$
\begin{aligned}
\mathcal{O}_{B,1}^{(7/2)} &= \left(\bar\chi_{hc}^{(q)} \frac{1}{i\bar n\cdot\overleftarrow{\partial}}\, i\overleftarrow{\not D}_{\perp s} \frac{\not{\bar n}}{2}\gamma_\perp^\mu P_L h_v\right)\left(\bar\chi_{hc}^{(\ell)}\gamma_{\perp\mu} P_L \chi_{\bar C}^{(\nu)}\right), \\
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\mathcal{O}_{B}^{(4)} &= m_\ell \left(\bar\chi_{hc}^{(q)} \frac{\not{\bar n}}{2} P_L h_v\right)\left(\bar\chi_{hc}^{(\ell)} \frac{1}{i\bar n\cdot\overleftarrow{\partial}} P_L \chi_{\overline{hc}}^{\nu}\right).
\end{aligned}
$$

- Their one-loop matrix elements reproduce the hard-collinear loops in the Fermi theory

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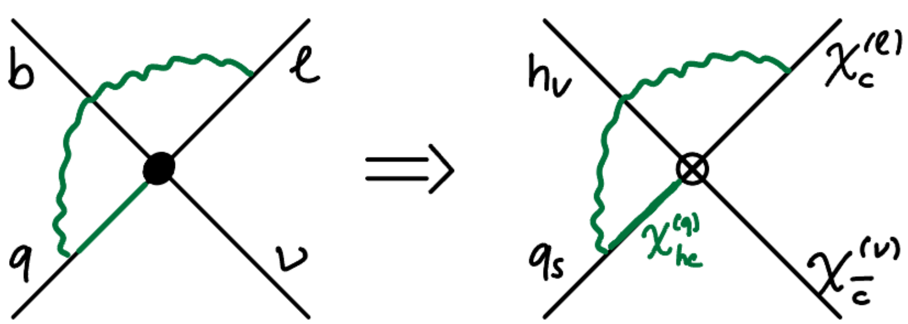
B. Operators with hard-collinear spectator



$$\mathcal{O}_{B,1}^{(7/2)} = \left(\bar{\chi}_{hc}^{(q)} \frac{1}{i\bar{n}\cdot\overleftarrow{\partial}} i\overleftarrow{\not{D}}_{\perp s} \frac{\not{\bar{n}}}{2} \gamma_\perp^\mu P_L h_v\right) \left(\bar{\chi}_{hc}^{(\ell)} \gamma_{\perp\mu} P_L \chi_{\bar{C}}^{(\nu)}\right),$$

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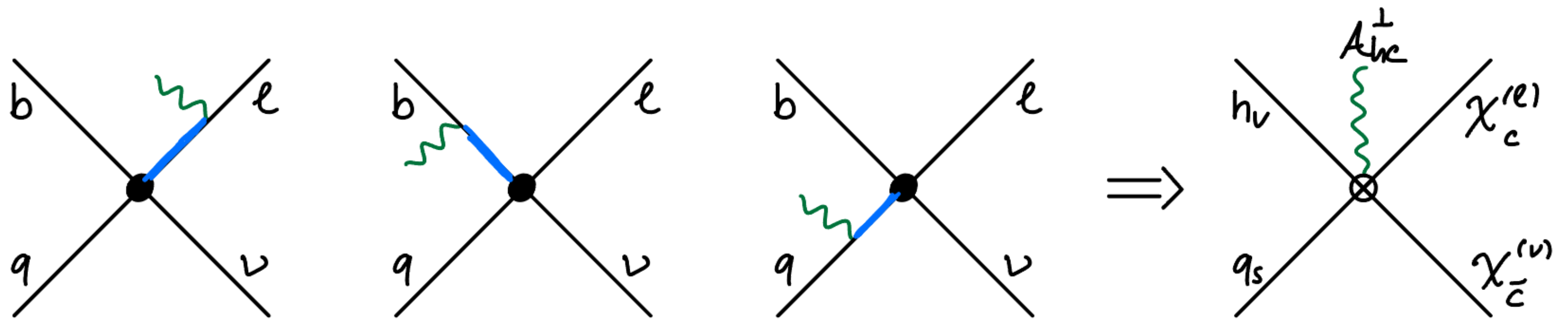
$$\mathcal{O}_{B}^{(4)} = m_\ell \left(\bar{\chi}_{hc}^{(q)} \frac{\not{\bar{n}}}{2} P_L h_v\right) \left(\bar{\chi}_{hc}^{(\ell)} \frac{1}{i\bar{n}\cdot\overleftarrow{\partial}} P_L \chi_{\overline{hc}}^{\nu}\right).$$

- Their one-loop matrix elements reproduce the hard-collinear loops in the Fermi theory
- B operators are power-enhanced with respect to A ones, but need one insertion of the (power-suppressed) soft-collinear interactions $\Rightarrow$ they contribute at the same order

CONSTRUCTION OF SCET-1 BASIS

C. Operators with with soft spectator + hard-collinear photon

- Arise from hard-collinear emission from muon, b or spectator quark



$$\mathcal{O}_{C,1}^{(5)} = \frac{1}{in\cdot\partial_{hc}}\left(\bar{q}_s\frac{\not{n}}{2}\gamma_\perp^\mu\mathcal{A}_{hc}^{(b)\perp}P_L h_v\right)\left(\bar{\chi}_{hc}^{(\ell)}\gamma_\mu^\perp P_L\chi_{\overline{hc}}^{(\nu)}\right)$$

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$$\mathcal{O}_{C,3}^{(5)} = \left(\bar{q}_s\mathcal{A}_{hc,\perp}^{(u)}\frac{1}{i\bar{n}\cdot\overleftarrow{\partial}_{hc}}\frac{\not{\bar{n}}}{2}\gamma_\perp^\mu P_L h_v\right)\left(\bar{\chi}_{hc}^{(\ell)}\gamma_\mu^\perp P_L\chi_{\overline{hc}}^{(\nu)}\right)$$

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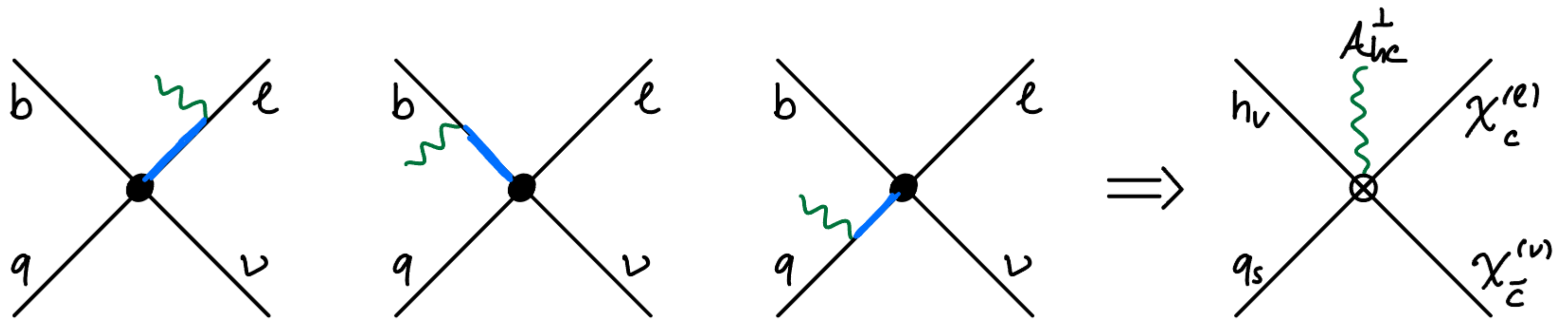

$$
\begin{aligned}
\mathcal{O}_{C,1}^{(5)} &= \frac{1}{in\cdot\partial_{hc}}\left(\bar{q}_s\frac{\not{n}}{2}\gamma_\perp^\mu\mathcal{A}_{hc}^{(b)\perp}P_L h_v\right)\left(\bar{\chi}_{hc}^{(\ell)}\gamma_\mu^\perp P_L\chi_{\overline{hc}}^{(\nu)}\right)\\
\mathcal{O}_{C,2}^{(5)} &= \frac{1}{i\bar{n}\cdot\partial_{hc}}\left(\bar{q}_s\frac{\not{\bar{n}}}{2}P_L h_v\right)\left(\bar{\chi}_{hc}^{(\ell)}\mathcal{A}_{hc,\perp}^{(\ell)}P_L\chi_{\overline{hc}}^{(\nu)}\right)\,,\\
\mathcal{O}_{C,3}^{(5)} &= \left(\bar{q}_s\mathcal{A}_{hc,\perp}^{(u)}\frac{1}{i\bar{n}\cdot\overleftarrow{\partial}_{hc}}\frac{\not{\bar{n}}}{2}\gamma_\perp^\mu P_L h_v\right)\left(\bar{\chi}_{hc}^{(\ell)}\gamma_\mu^\perp P_L\chi_{\overline{hc}}^{(\nu)}\right)
\end{aligned}
$$

- Moving to SCET-2, these reproduce the collinear loops of the Fermi theory

$\mu \sim \sqrt{m_b \Lambda}$: FROM SCET-1 TO SCET-2

- At $\mu \sim \sqrt{m_b \Lambda}$, integrate out hard-collinear modes and lower virtuality
  $\Rightarrow$ now collinear and soft modes live at the same scale: **SCET-2**

$$p_c \sim (1, \lambda^2, \lambda)\,, \qquad p_s \sim (\lambda, \lambda, \lambda)\,, \qquad p_c^2 \sim p_s^2 \sim \mathcal{O}\left(\lambda^2\right)$$

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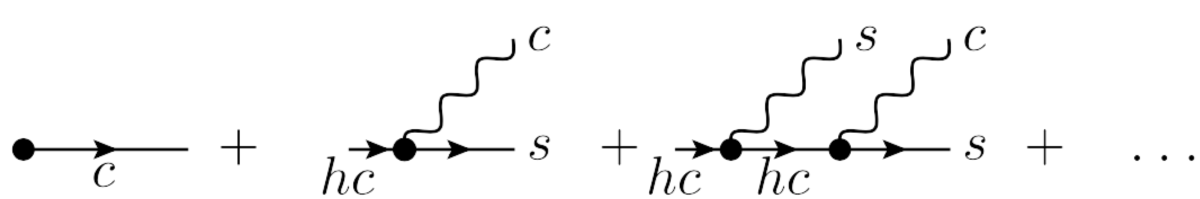
- When integrating out hard-collinear modes, intermediate propagators introduce **non-local operators**:

$$\psi_{hc} \quad \rightarrow \quad \psi_c \quad + \quad \psi_c \cdot \psi_s \quad + \quad \psi_c \cdot \psi_s^2 + \ldots$$



$$\frac{1}{n\cdot\partial} q_s\,, \quad \left(\frac{1}{n\cdot\partial}\mathcal{A}^\mu_{\perp s}\right)\left(\frac{1}{n\cdot\partial} q_s\right), \quad \ldots$$

$\Rightarrow$ now contain more fields, but are of the same order!

(NOT) OVERCOMING THE CHIRAL SUPPRESSION

- Inverse derivative operators can **probe the meson structure**, and **possibly overcome** the **chiral suppression**

$$\left\langle 0 \left| \left(\frac{1}{n \cdot \partial} \bar{q}_s \right) \ldots h_v \right| B \right\rangle \sim \frac{1}{\lambda_B} \sim \mathcal{O}\left(\Lambda_{\rm QCD}^{-1} \right)$$

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- Happens for $B_s \to \mu^+ \mu^-$, but not for $B \to \mu \bar{\nu}_\mu$ [Beneke, Bobeth, Szafron 2017, 2019]

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- Happens for $B_s \to \mu^+\mu^-$, but not for $B \to \mu\bar{\nu}_\mu$ [Beneke, Bobeth, Szafron 2017, 2019]
- For left-handed currents, these contributions come with **evanescent** Dirac structures:

$$\left(\bar{v}\frac{\not{n}}{2}\gamma_\perp^\mu\gamma_\perp^\nu P_L u\right)_h \left(\bar{u}\gamma_\mu^\perp\gamma_\nu^\perp\left[\frac{v-a\gamma_5}{2}\right]v\right)_\ell = 2(v-a)\left(\bar{v}\frac{\not{n}}{2}P_L u\right)_h \left(\bar{u}P_R v\right)_\ell + \mathcal{O}\left(\epsilon\right)$$

$\Rightarrow$ **structure-dependent contributions to $B \to \mu\bar{\nu}_\mu$ carry the same suppression as the tree level result!**

SCET-2 BASIS

$$\mathcal{Q}_{A,1} = \frac{m_\ell}{in\cdot\partial_c}\left(\bar{q}_s\frac{\not{\bar{n}}}{2}P_L h_v\right)\left(\bar{\chi}_c^{(\ell)}P_L\chi_{\bar{c}}^{(\nu)}\right)$$

$$\mathcal{Q}_{A,2} = \frac{m_\ell}{i\bar{n}\cdot\partial_c}\left(\bar{q}_s\frac{\not{n}}{2}P_L h_v\right)\left(\bar{\chi}_c^{(\ell)}P_L\chi_{\bar{c}}^{(\nu)}\right)$$

$$\mathcal{Q}_{B,1} = \left(\bar{q}_s\frac{1}{in\cdot\overleftarrow{\partial}_s}\mathcal{A}_c^{(u)\perp}\frac{i\overleftarrow{\not{D}}_{\perp,s}}{i\bar{n}\cdot\overleftarrow{\partial}_c}\frac{\not{n}\not{\bar{n}}}{4}\gamma_\perp^\mu P_L h_v\right)\left(\bar{\chi}_c^{(\ell)}\gamma_\mu^\perp P_L\chi_{\bar{c}}^{(\nu)}\right)$$

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$$\mathcal{Q}_{B,3} = \left(\bar{q}_s\frac{1}{in\cdot\overleftarrow{\partial}_s}\mathcal{A}_c^{(u)\perp}\frac{\not{n}\not{\bar{n}}}{4}P_L h_v\right)\left(\bar{\chi}_c^{(\ell)}\frac{m_\ell}{i\bar{n}\cdot\overleftarrow{\partial}_c}P_L\chi_{\bar{c}}^{(\nu)}\right)$$

$$\mathcal{Q}_{B,4} = \frac{1}{in\cdot\partial_c}\left(\bar{q}_s\frac{\not{n}}{2}\gamma_\perp^\mu\gamma_\perp^\nu P_L h_v\right)(\bar{\chi}_c^{(\ell)}\gamma_\mu^\perp\mathcal{A}_{c\nu}^{(b)\perp}P_L\chi_{\bar{c}}^{(\nu)})$$

$$\mathcal{Q}_{B,5} = \frac{1}{i\bar{n}\cdot\partial_c}\left(\bar{q}_s\frac{\not{\bar{n}}}{2}P_L h_v\right)\left(\bar{\chi}_c^{(\ell)}\mathcal{A}_c^{(\ell)\perp}P_L\chi_{\bar{c}}^{(\nu)}\right)$$

$$\mathcal{Q}_{B,6} = \left(\bar{q}_s\mathcal{A}_{c,\perp}^{(u)}\frac{1}{i\bar{n}\cdot\overleftarrow{\partial}_c}\frac{\not{\bar{n}}}{2}\gamma_\perp^\mu P_L h_v\right)\left(\bar{\chi}_c^{(\ell)}\gamma_\mu^\perp P_L\chi_{\bar{c}}^{(\nu)}\right)$$

SCET-2 BASIS

$$\boxed{\begin{aligned} \mathcal{Q}_{A,1} &= \frac{m_\ell}{i\bar{n}\cdot\partial_c}\left(\bar{q}_s\frac{\bar{\not{n}}}{2}P_L h_v\right)\left(\bar{\chi}_c^{(\ell)}P_L\chi_{\bar{c}}^{(\nu)}\right) \\ \mathcal{Q}_{A,2} &= \frac{m_\ell}{in\cdot\partial_c}\left(\bar{q}_s\frac{\not{n}}{2}P_L h_v\right)\left(\bar{\chi}_c^{(\ell)}P_L\chi_{\bar{c}}^{(\nu)}\right) \end{aligned}}$$

descend directly from A-type operators in SCET-1

$$\mathcal{Q}_{B,1} = \left(\bar{q}_s\frac{1}{in\cdot\overleftarrow{\partial}_s}\mathcal{A}_c^{(u)\perp}\frac{i\overleftarrow{\not{D}}_{\perp,s}}{i\bar{n}\cdot\overleftarrow{\partial}_c}\frac{\not{n}\bar{\not{n}}}{4}\gamma_\perp^\mu P_L h_v\right)\left(\bar{\chi}_c^{(\ell)}\gamma_\mu^\perp P_L\chi_{\bar{c}}^{(\nu)}\right)$$

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$$\mathcal{Q}_{B,1} = \left(\bar{q}_s \frac{1}{i n\cdot\overleftarrow{\partial}_s}\mathcal{A}_c^{(u)\perp}\frac{i\overleftarrow{\not{D}}_{\perp,s}}{i\bar{n}\cdot\overleftarrow{\partial}_c}\frac{\not{n}\not{\bar{n}}}{4}\gamma_\perp^\mu P_L h_v\right)\left(\bar{\chi}_c^{(\ell)}\gamma_\mu^\perp P_L \chi_{\bar{c}}^{(\nu)}\right)$$

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descend directly from C-type operators in SCET-1

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descend directly from A-type operators in SCET-1

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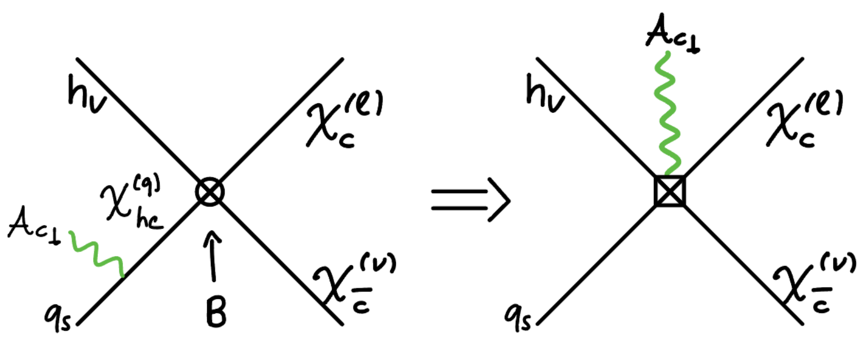
stem from matching B-type SCET-1 operators to SCET-2 at tree level



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descend directly from C-type operators in SCET-1

FACTORIZATION FORMULA (VIRTUAL CORRECTIONS)

convolution

$$\mathscr{A}^{\text{virtual}}_{B\to\ell\bar{\nu}} = \sum_j H_j\, S_j\, K_j + \sum_i H_i \otimes J_j \otimes S_i \otimes K_i\,,$$

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- hard function: matching corrections at $\mu \sim m_b$

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- soft (& soft-collinear) function: HQET B meson matrix elements

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convolution

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SCET-1 operators with soft spectator (A-type)

- hard function: matching corrections at $\mu \sim m_b$
- hard-collinear function: matching corrections at $\mu \sim (m_b \Lambda_{\text{QCD}})^{1/2}$
- collinear function: leptonic matrix elements, $\mu \sim m_\mu$
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FACTORIZATION FORMULA (VIRTUAL CORRECTIONS)

convolution

$$\mathscr{A}^{\text{virtual}}_{B\to\ell\bar{\nu}} = \sum_j \underbrace{H_j\ S_j\ K_j}_{\text{SCET-1 operators with soft spectator (A-type)}} + \sum_i H_i \underbrace{\otimes\ J_j \otimes S_i \otimes K_i}_{\text{SCET-1 operators with hc spectator (B-type)}},$$

- hard function: matching corrections at $\mu \sim m_b$
- hard-collinear function: matching corrections at $\mu \sim (m_b\Lambda_{\text{QCD}})^{1/2}$
- collinear function: leptonic matrix elements, $\mu \sim m_\mu$
- soft (& soft-collinear) function: HQET B meson matrix elements

FACTORIZATION FORMULA (VIRTUAL CORRECTIONS)

$$\mathcal{A}^{\text{virtual}}_{B\to\ell\bar{\nu}} = -\frac{4G_F}{\sqrt{2}} K_{\text{EW}}(\mu) V_{ub} \, \frac{m_\ell}{m_b} \, K_A(m_\ell) \, \bar{u}(p_\ell) P_L v(p_\nu) \left[H_A(m_b) S_A + \int d\omega \int_0^1 dx \, H_B(m_b, x) \, J_B(m_b\omega, x) S_B(\omega) \right]$$

$$\omega = n \cdot p_u$$

FACTORIZATION FORMULA (VIRTUAL CORRECTIONS)

$$\mathcal{A}_{B\to\ell\bar\nu}^{\text{virtual}} = -\frac{4G_F}{\sqrt{2}} K_{\text{EW}}(\mu) V_{ub}\,\frac{m_\ell}{m_b}\,K_A(m_\ell)\,\bar u(p_\ell) P_L v(p_\nu)$$
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$\omega = n\cdot p_u$

$$S_A = \langle O_A \rangle$$
$$O_A = \bar u_s \,\not{n}\, P_L h_v\, S_{v_\ell}^\dagger$$

$$S_B(\omega) = \langle O_B(\omega) \rangle$$
$$O_B(\omega) = \int \frac{dt}{2\pi} e^{i\omega t} \bar u_s(tn)[tn,0]\,\not{n}\, P_L h_v(0)\, S_{v_\ell}^\dagger(0)$$

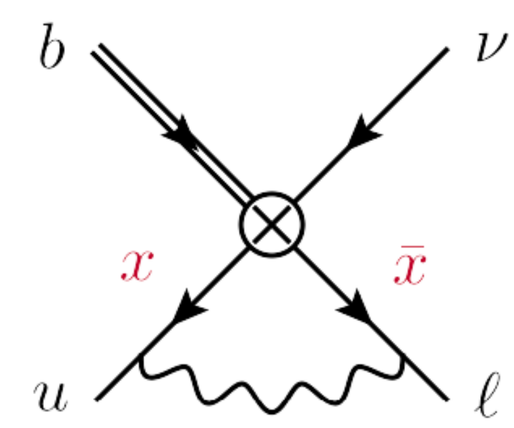
FACTORIZATION FORMULA (VIRTUAL CORRECTIONS)

$$\mathcal{A}^{\text{virtual}}_{B\to\ell\bar{\nu}} = -\frac{4G_F}{\sqrt{2}} K_{\text{EW}}(\mu) V_{ub} \frac{m_\ell}{m_b} K_A(m_\ell)\, \bar{u}(p_\ell) P_L v(p_\nu)$$
$$\left[H_A(m_b) S_A + \int d\omega \int_0^1 dx\, H_B(m_b, x)\, J_B(m_b\omega, x) S_B(\omega) \right]$$

$\omega = n \cdot p_u$

- Focus on second term:
  - **Hard and jet function share a variable** x = collinear momentum fraction carried by the spectator
  - They scale as $H_B \sim x^{-\epsilon}, J_B \sim x^{-1-\epsilon}$

    $\Rightarrow$ $H_B \otimes J_B$ **has an endpoint divergence in** $x = 0$**!**

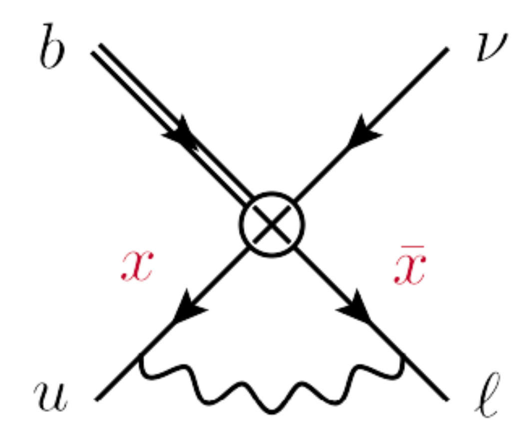


FACTORIZATION FORMULA (VIRTUAL CORRECTIONS)

$$\mathcal{A}^{\text{virtual}}_{B\to\ell\bar{\nu}} = -\frac{4G_F}{\sqrt{2}} K_{\text{EW}}(\mu) V_{ub} \frac{m_\ell}{m_b} K_A(m_\ell)\, \bar{u}(p_\ell) P_L v(p_\nu)$$
$$\left[H_A(m_b) S_A + \int d\omega \int_0^1 dx\, H_B(m_b, x)\, J_B(m_b\omega, x) S_B(\omega) \right] \qquad \omega = n\cdot p_u$$

- Focus on second term:
  - **Hard and jet function share a variable x = collinear momentum fraction carried by the spectator**
  - They scale as $H_B \sim x^{-\epsilon}, J_B \sim x^{-1-\epsilon}$

    $\Rightarrow$ $H_B \otimes J_B$ **has an endpoint divergence in** $x = 0$**!**



- This cannot be removed with standard RG techniques, but is systematically treatable with **refactorization-based subtraction (RBS) scheme**

[Liu, MN 2019; Liu, Mecaj, MN, Wang 2020; Beneke et al. 2022]

REFACTORIZATION

$$\mathcal{A}^{\text{virtual}}_{B\to\ell\bar{\nu}} = -\frac{4G_F}{\sqrt{2}} K_{\text{EW}}(\mu) V_{ub}\, \frac{m_\ell}{m_b}\, K_A(m_\ell)\, \bar{u}(p_\ell) P_L v(p_\nu)$$

$$\left[H_A(m_b) S_A + \int d\omega \boxed{\int_0^1 dx\, H_B(m_b, x)\, J_B(m_b\omega, x)} S_B(\omega) \right]$$

- Start from the second term

REFACTORIZATION

$$\mathcal{A}^{\text{virtual}}_{B\to\ell\bar\nu} = -\frac{4G_F}{\sqrt{2}} K_{\text{EW}}(\mu) V_{ub} \frac{m_\ell}{m_b} K_A(m_\ell)\, \bar u(p_\ell) P_L v(p_\nu)$$

$$\left[H_A(m_b) S_A + \int d\omega \boxed{\int_0^1 dx\, H_B(m_b, x)\, J_B(m_b\omega, x)} S_B(\omega) \right]$$

$$\boxed{\int_0^1 dx\, \left[H_B(m_b, x) J_B(m_b\omega, x) - \theta(\lambda - x)[[H_B(m_b, x)]][[J_B(m_b\omega, x)]] \right]}$$

$0 < \lambda < 1$ $\qquad$ $[[f]] =$ singular part of f for $x \to 0$

- Start from the second term
- Remove the divergence from $H_B \otimes J_B$ with a plus subtraction

REFACTORIZATION

$$\mathcal{A}^{\text{virtual}}_{B\to\ell\bar\nu} = -\frac{4G_F}{\sqrt{2}} K_{\text{EW}}(\mu) V_{ub} \frac{m_\ell}{m_b} K_A(m_\ell)\, \bar u(p_\ell) P_L v(p_\nu)$$
$$\left[\boxed{H_A(m_b) S_A} + \int d\omega \boxed{\int_0^1 dx\, H_B(m_b,x)\, J_B(m_b\omega,x)} S_B(\omega)\right]$$

$$\Lambda = \lambda\, m_b \qquad \boxed{H_A(m_b) S_A^{(\Lambda)}}$$

$$\boxed{\int_0^1 dx\, \left[H_B(m_b,x) J_B(m_b\omega,x) - \theta(\lambda - x)[[H_B(m_b,x)]][[J_B(m_b\omega,x)]]\right]}$$

$0 < \lambda < 1$ $\qquad$ $[[f]]$ = singular part of f for $x \to 0$

- Start from the second term
- Remove the divergence from $H_B \otimes J_B$ with a plus subtraction
- Add the subtraction term back, combining it with the other term in the factorization formula

REFACTORIZATION

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$0 < \lambda < 1$ $\quad [[f]] =$ singular part of f for $x \to 0$

- The new soft function $S_A^{(\Lambda)}$ defines a renormalized decay "constant":

$$S_A^{(\Lambda)} = \langle 0 |\, O_A^{(\Lambda)} \,| B^-(v)\rangle = -\frac{i\sqrt{m_B}}{2} F(\mu,\Lambda,w)\, \langle 0 |\, S_{v_B}^{(B)} S_{v_\ell}^{(\ell)\dagger} \,| 0\rangle$$

$$w = v_B \cdot v_\ell \approx \frac{m_B}{2m_\ell}$$

$$O_A^{(\Lambda)} = \bar u_s \not{\bar n} P_L h_{v_B} S_{v_\ell}^{(\ell)\dagger}\left[1 + Q_\ell Q_u \frac{\alpha}{2\pi}\frac{e^{\epsilon\gamma_E}\Gamma(\epsilon)}{\epsilon(1-\epsilon)}\int d\omega\, \phi_-(\omega)\left(\frac{\mu^2}{\omega\Lambda}\right)^{\epsilon}\right]$$

VIRTUAL QED CORRECTIONS IN LEPTONIC B DECAY

Decay amplitude including virtual QED corrections at $\mathcal{O}(\alpha)$:

$$\mathcal{A}_{B\to\ell\bar{\nu}}^{\text{virtual}} = i\sqrt{2}G_F K_{\text{EW}}(\mu) V_{ub}\,\frac{m_\ell}{m_b}\sqrt{m_B}\,F(\mu,m_b,w)\,\bar{u}(p_\ell)P_L v(p_\nu)\Big[\mathcal{M}_{2p}(\mu)+\mathcal{M}_{3p}(\mu)\Big]$$

with:

$$\begin{aligned}
\mathcal{M}_{2p}(\mu) &= 1+\frac{C_F\alpha_s}{4\pi}\left[\frac{3}{2}\ln\frac{m_b^2}{\mu^2}-2\right] \\
&\quad+\frac{\alpha}{4\pi}\left\{Q_b^2\left[\frac{3}{2}\ln\frac{m_b^2}{\mu^2}-2\right]-Q_\ell Q_b\left[\frac{1}{2}\ln^2\frac{m_b^2}{\mu^2}+2\ln\frac{m_b^2}{\mu^2}-3\ln\frac{m_\ell^2}{\mu^2}+1+\frac{5\pi^2}{12}\right]\right. \\
&\quad\left.+2Q_\ell Q_u\int_0^\infty d\omega\,\phi_-(\omega)\ln\frac{m_b\omega}{\mu^2}+Q_\ell^2\left[\frac{1}{\epsilon_{\text{IR}}}\left(\ln\frac{m_B^2}{m_\ell^2}-2\right)+\frac{1}{2}\ln^2\frac{m_\ell^2}{\mu^2}-\frac{1}{2}\ln\frac{m_\ell^2}{\mu^2}+2+\frac{5\pi^2}{12}\right]\right\}
\end{aligned}$$

$$\mathcal{M}_{3p}(\mu) = \frac{\alpha}{\pi}Q_\ell Q_u\int_0^\infty d\omega\int_0^\infty d\omega_g\,\phi_{3g}(\omega,\omega_g)\left[\frac{1}{\omega_g}\ln\left(1+\frac{\omega_g}{\omega}\right)-\frac{1}{\omega+\omega_g}\right]$$

[CC, König, MN 2022]

$\Rightarrow$ significant hadronic uncertainties in $\mathcal{O}(\alpha)$ terms!

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Decay amplitude including virtual QED corrections at $\mathcal{O}(\alpha)$:

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with:

IR divergence cancels against real soft photon emission

$$\mathcal{M}_{2p}(\mu) = 1 + \frac{C_F\alpha_s}{4\pi}\left[\frac{3}{2}\ln\frac{m_b^2}{\mu^2} - 2\right]$$
$$+ \frac{\alpha}{4\pi}\Bigg\{Q_b^2\left[\frac{3}{2}\ln\frac{m_b^2}{\mu^2} - 2\right] - Q_\ell Q_b\left[\frac{1}{2}\ln^2\frac{m_b^2}{\mu^2} + 2\ln\frac{m_b^2}{\mu^2} - 3\ln\frac{m_\ell^2}{\mu^2} + 1 + \frac{5\pi^2}{12}\right]$$
$$+2Q_\ell Q_u\int_0^\infty d\omega\, \phi_-(\omega)\ln\frac{m_b\omega}{\mu^2} + Q_\ell^2\left[\frac{1}{\epsilon_{\text{IR}}}\left(\ln\frac{m_B^2}{m_\ell^2} - 2\right) + \frac{1}{2}\ln^2\frac{m_\ell^2}{\mu^2} - \frac{1}{2}\ln\frac{m_\ell^2}{\mu^2} + 2 + \frac{5\pi^2}{12}\right]\Bigg\}$$

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with:

large double logarithms

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$\mu < \Lambda \sim m_\mu$: UNDERSTANDING THE LOW-ENERGY THEORY

- Below $\mu \sim \Lambda_{\rm QCD}$ quarks hadronize: move to effective description with a **Yukawa theory**, with the meson treated as a **heavy scalar**:

$$\mathcal{L}_{\rm y} = y\, e^{-im_B(v\cdot x)} \varphi_B \left(\bar{\chi}_c^{(\ell)} P_L \chi_{\bar{c}}^{(\nu)} \right) + \text{h.c.}$$

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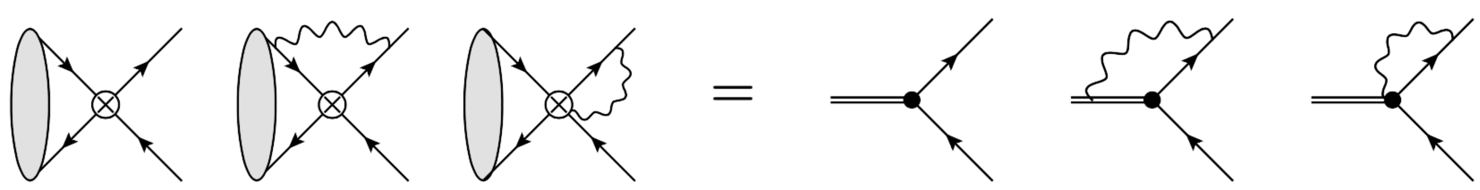
- Yukawa coupling is fixed by matching hadronic matrix elements between this and the previous description:

$$\langle \ell\, \nu \,|\, \mathcal{L}_{\rm SCET\,II \otimes HQET} \,|\, B \rangle \quad = \quad \langle \ell\, \nu \,|\, \mathcal{L}_{\rm SCET\,II \otimes HSET} \,|\, B \rangle$$



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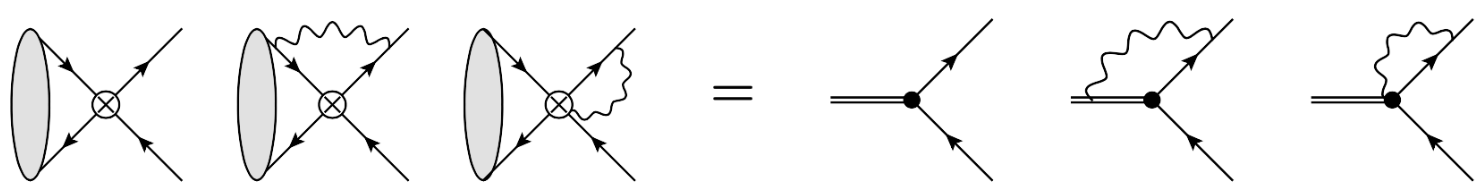

- Since $\Lambda_{\rm QCD} \sim m_\mu$, we integrate out the **muon** in the same step and describe it as a **boosted heavy lepton** field: $\ell(x) = e^{-i m_\ell v_\ell \cdot x} \chi_{v_\ell}(x)$

$\Rightarrow$ **low-E theory is a heavy scalar effective theory $\otimes$ bHLET**

$\mu < \Lambda \sim m_\mu$: UNDERSTANDING THE LOW-ENERGY THEORY

- It's a **theory of Wilson lines**: all interactions of the B and the muon with ultrasoft and ultrasoft-collinear photons can be moved into Wilson lines, and decoupled via field redefinitions:

$$Y_v^{(\mathrm{s})}(x) = \mathcal{P}\exp\left\{ie\int_{-\infty}^{0} ds\, v\cdot A_s(x+sv)\right\}$$

$$Y_v^{(\mathrm{sc})}(x) = \mathcal{P}\exp\left\{ie\int_{-\infty}^{0} ds\, v\cdot A_{\mathrm{sc}}(x+sv)\right\}$$

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- Real corrections are matrix elements of these Wilson lines:

$$W_s(\omega_s,\mu) = \left[\sum_{n_s=0}^{\infty}\prod_{i=1}^{n_s}\int d\Pi_i(q_i)\right]\left|\langle n_s\gamma_s(q_i)|\, Y_v^{(\mathrm{s})} Y_n^{(\mathrm{s})\dagger}\,|0\rangle\right|^2 \delta\left(\omega_s - q_0^{(\mathrm{s})}\right),$$

$$W_{sc}(\omega_{sc},\mu) = \left[\sum_{n_{sc}=0}^{\infty}\prod_{j=1}^{n_s}\int d\Pi_j(q_j)\right]\left|\langle n_{sc}\gamma_{sc}(q_j)|\, Y_{\bar{n}}^{(\mathrm{sc})\dagger} Y_{v_l}^{(\mathrm{sc})}\,|0\rangle\right|^2 \delta\left(\omega_{sc} - q_0^{(\mathrm{sc})}\right)$$

$\mu < \Lambda \sim m_\mu$: UNDERSTANDING THE LOW-ENERGY THEORY

- Convoluted with the **measurement function** involving the experimental cut, they yields the complete radiative function:

$$S(E_s,\mu) = \int_0^\infty d\omega_s \int_0^\infty d\omega_{sc} \boxed{\theta\left(\frac{E_s}{2} - \omega_s - \omega_{sc}\right)} W_s(\omega_s,\mu) W_{sc}(\omega_{sc},\mu)$$

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with:

$$\begin{aligned}\mathcal{M}_{2p}(\mu) &= 1+\frac{C_F\alpha_s}{4\pi}\left[\frac{3}{2}\ln\frac{m_b^2}{\mu^2}-2\right]\\ &\quad+\frac{\alpha}{4\pi}\left\{Q_b^2\left[\frac{3}{2}\ln\frac{m_b^2}{\mu^2}-2\right]-Q_\ell Q_b\left[\frac{1}{2}\ln^2\frac{m_b^2}{\mu^2}+2\ln\frac{m_b^2}{\mu^2}-3\ln\frac{m_\ell^2}{\mu^2}+1+\frac{5\pi^2}{12}\right]\right.\\ &\quad\left.+2Q_\ell Q_u\int_0^\infty d\omega\,\phi_-(\omega)\ln\frac{m_b\omega}{\mu^2}+Q_\ell^2\left[\frac{1}{\epsilon_{\text{IR}}}\left(\ln\frac{m_B^2}{m_\ell^2}-2\right)+\frac{1}{2}\ln^2\frac{m_\ell^2}{\mu^2}-\frac{1}{2}\ln\frac{m_\ell^2}{\mu^2}+2+\frac{5\pi^2}{12}\right]\right\}\\ \mathcal{M}_{3p}(\mu) &= \frac{\alpha}{\pi}Q_\ell Q_u\int_0^\infty d\omega\int_0^\infty d\omega_g\,\phi_{3g}(\omega,\omega_g)\left[\frac{1}{\omega_g}\ln\left(1+\frac{\omega_g}{\omega}\right)-\frac{1}{\omega+\omega_g}\right]\end{aligned}$$

⇒ significant hadronic uncertainties in $\mathcal{O}(\alpha)$ terms!

HADRONIC QUANTITIES

Generalization of the decay "constant" in presence of QED effects

- Matching relation (with X_γ an *n*-soft-photon state):

$$\langle X_\gamma | O_A^{(\Lambda)} | B^- \rangle = -\frac{i}{2}\sqrt{m_B}\, F(\mu, \Lambda, w)\, \langle X_\gamma | S_{v_B}^{(B)} S_{v_\ell}^{(\ell)\dagger} | 0 \rangle \quad \text{with} \quad w \equiv v_B \cdot v_\ell \approx \frac{m_B}{2m_\ell}$$

$\Rightarrow$ a **form factor** (like the Isgur-Wise function in $B \to D^{(*)}$ transitions)

[CC, König, MN 2022]

- Defining *F* as a Wilson coefficient implements the **nonperturbative matching of SCET onto the point-like meson effective theory** envisioned in [Beneke, Bobeth, Szafron 2019]

HADRONIC QUANTITIES

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$\Rightarrow$ a **form factor** (like the Isgur-Wise function in $B \to D^{(*)}$ transitions)

[CC, König, MN 2022]

- Evolution equations:

$$\frac{d\ln F}{d\ln\mu} = C_F\,\frac{3\alpha_s}{4\pi} - \frac{3\alpha}{4\pi}\left(Q_\ell^2 - Q_b^2 + \frac{2}{3}Q_\ell Q_u \ln\frac{\Lambda^2}{\mu^2}\right)$$

$$\frac{d\ln F}{d\ln\Lambda} = Q_\ell Q_u\,\frac{\alpha}{2\pi}\left[\int d\omega\,\phi_-(\omega)\ln\frac{\omega\Lambda}{\mu^2} - 1 + \dots\right]$$

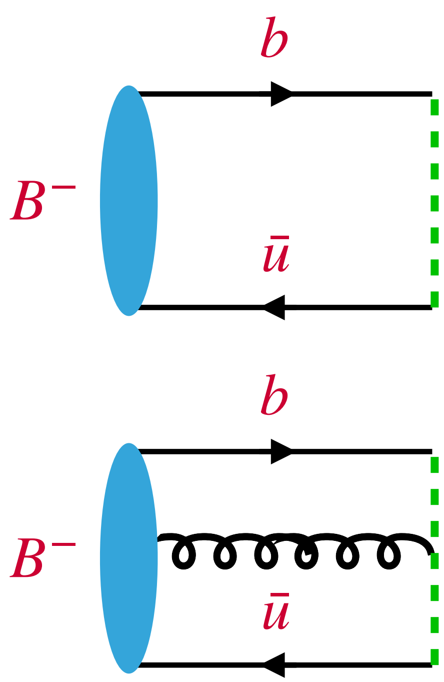
well-defined and insensitive to IR regulators

HADRONIC UNCERTAINTIES

Nonperturbative hadronic matrix elements:

$F(\mu, m_b, v_B \cdot v_\ell)$ a **form factor** rather than a decay "constant"

LCDAs



$\phi_\pm(\omega)$ [Grozin, MN 1996]

$\phi_{3g}(\omega, \omega_g)$ [Kawamura, Kodaira, Qiao, Tanaka 2001; Braun, Ji, Manashov 2017]

Several model LCDAs have been proposed, e.g.:

$$\phi_-(\omega) = \frac{1}{\omega_0}\, e^{-\omega/\omega_0}\,, \qquad \phi_{3g}(\omega, \omega_g) = \frac{\lambda_E^2 - \lambda_H^2}{3\omega_0^5}\, \omega\omega_g\, e^{-(\omega+\omega_g)/\omega_0}$$

HADRONIC UNCERTAINTIES

Model-independent results for the form factor $F(\mu, m_b, v_B \cdot v_\ell)$

- Relation to lattice QCD results for the *B*-meson decay constant:

$$\sqrt{m_B}\, f_B^{\mathrm{QCD}} = \left[1 - C_F\, \frac{\alpha_s(m_b)}{2\pi}\right] F(m_b, m_b, w)\Big|_{\alpha\to 0}$$

- For $w \gtrsim 1$, it would be possible to determine *F* using lattice QCD, in analogy with the Isgur-Wise function
- However, this seems illusive for $2w = m_B/m_\mu \approx 50$ (cf. the $B \to \pi$ form factor at $q^2 = 0$, corresponding to maximum recoil)
- However, unlike in QCD, it is sufficient to work to first order in α

HADRONIC UNCERTAINTIES

Model-independent results for the form factor $F(\mu, m_b, v_B \cdot v_\ell)$

- ***Preliminary*** finding:

$$F(\mu, \Lambda, w) \propto f_{B_u}^{\mathrm{QCD}} \left\{ 1 + \frac{\alpha}{4\pi} \left[c_1 \boxed{\ln(2w)} + c_0(\Lambda, \mu) \right] \right\} \qquad \ln \frac{m_B}{m_\ell}$$

with nonperturbative parameters $c_0(\Lambda, \mu)$ and c_1

- This may offer a path to a lattice determination of *F* by varying *w*

CONCLUSIONS

- Subleading-power factorization theorem with endpoint divergences subtracted in a nonperturbative context
- First consistent matching of SCET onto point-like meson theory
- Structure-dependent QED corrections a generic feature resulting from contributions of (hard-, soft-) collinear modes in SCET
  - important source of large logarithmic corrections
  - missed in previous treatments based on point-like meson model
- Results are relevant for consistent analyses of QED effects also in other rare *B* decays and allow for a precision determination of $|V_{ub}|$