NNLO jet and beam functions

Automation, distributions and jet clustering

Guido Bell, Kevin Brune, Goutam Das, Marcel Wald

Introduction

- Typical factorisation theorem in SCET

$$\mathrm{d}\sigma \simeq H\left(\mu_F\right) \cdot \prod_i B_i\left(\mu_F\right) \otimes \prod_k J_k\left(\mu_F\right) \otimes S\left(\mu_F\right)$$

- Resummation requires knowledge of anomalous dimensions and matching corrections

$$\underbrace{\Gamma_{\text{Cusp}}, \gamma_{\text{H}}, c_{\text{H}}}_{\text{Observable independent}} \qquad \underbrace{\gamma_{\text{B}}, \gamma_{\text{J}}, \gamma_{\text{S}}, c_{\text{B}}, c_{\text{J}}, c_{\text{S}}}_{\text{Observable dependent}}$$

- SoftSERVE: [Bell,Rahn,Talbert;19,20]
  - Automated framework to calculate NNLO soft functions
  - Applies to SCET-I and SCET-II observables
- We developed a similar framework for the beam and jet functions at NNLO
  - First application: p_T-veto for quark beam function [Bell,KB,Das,Wald;22]

Status-Previous SCET Workshop

Beam functions

- Framework
  - Focus on quark-to-quark matching kernel
  - SCET-I and SCET-II observables in Mellin-space
- Results
  - Beam Thrust
  - p_T-resummation
  - p_T-veto

Jet functions

- Framework
  - Complete framework for both quark and gluon jet functions
  - Only considered SCET-I observables
- Results
  - Thrust
  - Angularities
  - Transverse Thrust

Status-SCET 2023

Beam functions

- Framework
  - Focus on quark-to-quark matching kernel
  - SCET-I and SCET-II observables in Mellin-space
  - Complete framework for all beam function channels
  - SCET-I and SCET-II observables in x-space
- Results
  - Beam Thrust
  - p_T-veto

Jet functions

- Framework
  - Complete framework for both quark and gluon jet functions
  - Only considered SCET-I observables
  - Further generalisation of measurement function
  - Extended Framework to SCET-II observables
- Results
  - WTA-axis Broadening
  - WTA-axis p_T-resummation

Definitions

- Quark beam function

$$\frac{1}{2}\left[\frac{\not{n}}{2}\right]_{\beta\alpha} B_{q/h}(x,\tau,\mu) = \sum_X \delta\left((1-x)P_- - \sum_i k_i^-\right)$$
$$\langle h(P)|\,\bar{\chi}_\alpha\,|X\rangle\,\langle X|\,\chi_\beta\,|h(P)\rangle\mathcal{M}(\tau;\{k_i\})$$

- Quark jet function

$$\left[\frac{\not{n}}{2}\right]_{\beta\alpha} J_q(\tau,\mu) = \frac{1}{\pi}\sum_X (2\pi)^d \delta\left(Q - \sum_i k_i^-\right)\delta^{(d-2)}\left(\sum_i k_i^\perp\right)$$
$$\langle 0|\,\chi_\beta\,|X\rangle\,\langle X|\,\bar{\chi}_\alpha\,|0\rangle\mathcal{M}(\tau;\{k_i\})$$

- Similar definitions for the gluon initiated channels
- SCET-II observables require additional analytical regulator [Becher, Bell; 11]

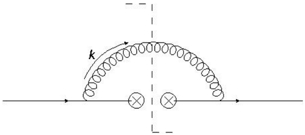
$$\prod_i \int \frac{\mathrm{d}^d k_i}{(2\pi)^d}\left(\frac{\nu}{k_i^- + k_i^+}\right)^\alpha \delta\left(k_i^2\right)\theta\left(k_i^{(0)}\right)$$

Approach at NLO

Beam function

- Parametrisation:

$$k_- = (1-x)P_-,$$
$$|\vec{k}_\perp| = k_T,$$
$$\cos(\theta_k) = 1 - 2t_k$$



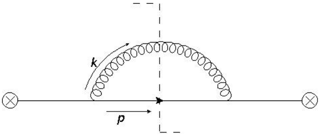
- Measurement function:

$$\mathcal{M}_1^B(\tau;k) = \exp\left[-\tau k_T \left(\frac{k_T}{(1-x)P_-}\right)^n f(t_k)\right]$$

Jet function

- Parametrisation:

$$k_- = zQ, \quad p_- = \bar{z}Q,$$
$$|\vec{k}_\perp| = |\vec{p}_\perp| = k_T,$$
$$\cos(\theta_k) = 1 - 2t_k$$



- Measurement function:

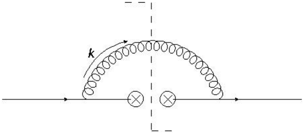
$$\mathcal{M}_1^J(\tau;k,p) = \exp\left[-\tau\,(k_T)^m \left(\frac{k_T}{Q}\right)^n (z\bar{z})^{-\frac{m+n}{1+n}n} f(z,t_k)\right]$$

Approach at NLO

Beam function

- Master formula:

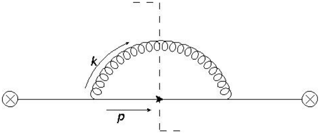
$$B^{(1)}_{i/j}(x,\tau) \simeq \frac{\Gamma\left(\frac{-2\epsilon}{1+n}\right)}{1+n}\,(1-x)^{-1-\frac{2n\epsilon}{1+n}-\alpha}\,W^{B}_{i/j}(x)$$
$$\times \int_0^1 \mathrm{d}t_k\,(4t_k\bar{t_k})^{-1/2-\epsilon} f(t_k)^{\frac{2\epsilon}{1+n}}$$



Jet function

- Master formula:

$$J^{(1)}_{i}(\tau) \simeq \frac{\Gamma\left(\frac{-2\epsilon}{m+n}\right)}{m+n}\int_0^1 \mathrm{d}z(z\bar{z})^{-1-\frac{2n\epsilon}{1+n}-\alpha}W^{J}_{i}(z)$$
$$\times \int_0^1 \mathrm{d}t_k\,(4t_k\bar{t_k})^{-1/2-\epsilon} f(z,t_k)^{\frac{2\epsilon}{m+n}}$$



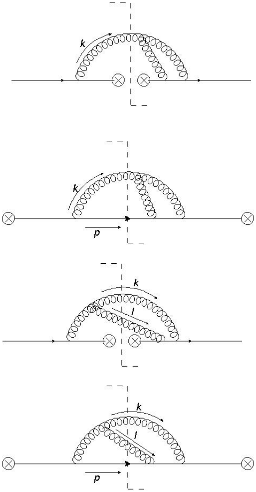
Approach at NNLO

- At NNLO we need to consider two different contributions
  1) Real-Virtual contribution
     - Matrix element: NLO collinear splitting functions
     - Same parametrisation as NLO
  2) Real-Real contribution
     - Matrix element: LO triple collinear splitting functions
     - Parametrisation:

$$q_T = \sqrt{(k_- + l_-)(k_+ + l_+)}, \quad b = \frac{k_T}{l_T}$$

$$a = \frac{k_- l_T}{k_T l_-}, \quad t_{kl} = \frac{1 - \cos\theta_{kl}}{2}$$

- Beam function: $x = \frac{k_- + l_-}{P_-}$
- Jet function: $z = \frac{k_- + l_-}{Q}$

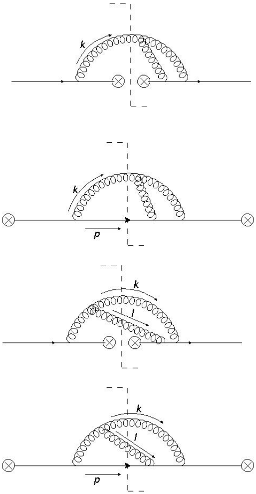


Approach at NNLO

- Measurement function at NNLO

$$\mathcal{M}_2^B(\tau;k,l) = \exp\left[-\tau q_T \left(\frac{q_T}{(1-x)P_-}\right)^n F(a,b,t_{kl},t_k,t_l)\right]$$

$$\mathcal{M}_2^J(\tau;k,l,p) = \exp\left[-\tau (q_T)^m \left(\frac{q_T}{(z\bar{z})^{\frac{m+n}{1+n}}\,Q}\right)^n F(z,a,b,t_{kl},t_k,t_l)\right]$$



- Many overlapping singularities remain
  - Sector decomposition
  - Selector functions
  - Non-linear transformations

→ All singularities factorised

Winner-take-all recombination scheme

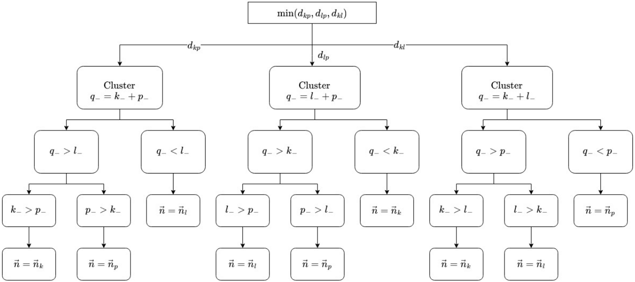
- Select the two closest particles and merge [Bertolini, Chan, Thaler; 13]

$$E_{(ij)} = E_i + E_j, \quad \vec{p}_{(ij)} = E_{(ij)} \left[\frac{\vec{p}_i}{|\vec{p}_i|} \theta(E_i - E_j) + \frac{\vec{p}_j}{|\vec{p}_j|} \theta(E_j - E_i) \right]$$

  → Massless pseudoparticle that points in the direction of the most energetic particle

  → This direction is the "winner-take-all" axis

- At leading power the soft function is the same as for the standard jet axis



SCET-II renormalisation for the jet function

- Additional scale $\nu \to$ Additional large logarithms
  - Resum them via collinear anomaly approach [Becher, Neubert; 13]

$$\left[J_i(\tau,\mu,\nu)\bar{J}_i(\tau,\mu,\nu)S(\tau,\mu,\nu)\right] \underset{\alpha=0}{\equiv} \left(Q^2\bar{\tau}^2\right)^{-F_i(\tau,\mu)} W_i(\tau,\mu)$$

- Anomaly exponent F_i and remainder function W_i fullfill RGEs

$$\frac{\mathrm{d}F_i(\tau,\mu)}{\mathrm{d}\ln\mu} = 2\Gamma_{\mathsf{Cusp}}$$

$$\frac{\mathrm{d}W_i(\tau,\mu)}{\mathrm{d}\ln\mu} = 2\left[\Gamma_{\mathsf{Cusp}}L - \gamma_H\right]W_i(\tau,\mu)$$

$\to$ Extraction of γ_H, anomaly coefficient d_i and c_i at NNLO

WTA-axis Broadening [Preliminary]

- Considered both quark- and gluon jet function
- Distance measure: k_T-algorithm
- Measurement: Scalar sum of the transverse momentum projected onto the WTA axis
- Anomaly exponent [Bell,Rahn,Talbert;19,20]

d_2	This work	SoftSERVE
$C_F T_F$	$-18.737(1)$	-18.737
C_F^2	$-0.116(230)$	0
$C_F C_A$	$15.950(154)$	$15.981(1)$

d_2	This work	SoftSERVE
$C_A T_F$	$-18.737(7)$	-18.737
C_A^2	$15.976(15)$	$15.981(2)$

- Remainder function

c_2	This work
$C_F T_F$	$62.958(3)$
C_F^2	$511.094(298)$
$C_F C_A$	$-240.436(274)$

c_2	This work
T_F^2	$17.435(1)$
$C_F T_F$	$-85.222(58)$
$C_A T_F$	$135.965(21)$
C_A^2	$293.955(71)$

WTA-axis p_T-resummation [Preliminary]

- Considered only quark jet function
- Distance measure: k_T-algorithm, anti-k_T-algorithm and Cambridge-Aachen-algorithm
- Measurement: Vector sum of the transverse momentum projected onto the WTA axis
- Anomaly exponent agrees with literature [Bell,Rahn,Talbert;19,20]
- Remainder function comparison with `Event2` GSWZ:[Gutierrez-Reyes et al.;20]

c_2	This work	GSWZ
$C_F T_F$	$12.72(5)$	$-13.0(3)$
C_F^2	$-13.84(37)$	$12.2(11)$
$C_F C_A$	$14.40(80)$	$-9.3(2)$
k_T-algorithm		

c_2	This work	GSWZ
$C_F T_F$	$12.27(31)$	$-12.5(3)$
C_F^2	$-25.90(59)$	$25.3(6)$
$C_F C_A$	$3.72(131)$	$-6.3(2)$
anti-k_T-algorithm		

c_2	This work	GSWZ
$C_F T_F$	$12.29(31)$	$-12.5(3)$
C_F^2	$-25.76(60)$	$24.5(6)$
$C_F C_A$	$7.41(132)$	$-6.7(2)$
Cambridge-Aachen-algorithm		

Beam function approaches

1) Our previous approach
   - Resolve all distributions in Mellin space

$$\hat{B}_{i/j}(N,\tau) = \int_0^1 \mathrm{d}x\, x^{N-1} B_{i/j}(x,\tau)$$

   - Perform all integrals numerically → Similar to the jet function

2) Our new approach
   - Direct calculation in x-space
   - Expand in terms of distributions

$$(1-x)^{-1-2\alpha} f(x) = \frac{1}{-2\alpha} f(x)\delta(1-x) + \left[\frac{1}{1-x}\right]_+ f(x) + \dots$$
$$= \frac{1}{-2\alpha} f(1)\delta(1-x) + \left[\frac{1}{1-x}\right]_+ f(1) + \frac{f(x)-f(1)}{1-x} + \dots$$

SCET-II renormalisation for the beam function

- Matching on parton distribution functions

$$B_{i/h}(x,\tau,\mu) = \sum_k \int_x^1 \frac{\mathrm{d}z}{z} I_{i\leftarrow k}(x/z,\tau,\mu) f_{k/h}(z,\mu)$$

- Renormalisation follows as in the jet function case

$$\left[I_{i\leftarrow k}(x_1,\tau,\mu,\nu) I_{j\leftarrow l}(x_2,\tau,\mu,\nu) S_{ij}(\tau,\mu,\nu)\right] \underset{\alpha=0}{\equiv} \left(q^2\bar{\tau}^2\right)^{-F_{ij}(\tau,\mu)} I_{i\leftarrow k}(x_1,\tau,\mu) I_{j\leftarrow l}(x_2,\tau,\mu)$$

- Slightly different RGE for the matching kernel $I_{i\leftarrow k}(x,\tau,\mu)$

$$\frac{\mathrm{d}I_{i\leftarrow k}(x,\tau,\mu)}{\mathrm{d}\ln\mu} = 2\left[\Gamma_{\mathsf{Cusp}} L - \gamma_H\right] I_{i\leftarrow k}(x,\tau,\mu) - 2\sum_j I_{i\leftarrow j}(x,\tau,\mu) \otimes P_{j\leftarrow k}(x,\mu)$$

$\rightarrow$ Extraction of γ_H, anomaly coefficient d_i and $I_{i\leftarrow k}(x)$ at NNLO

Beam Thrust [Preliminary]

- Measurement: $\omega_2(k,l) = k_+ + l_+$
- Matching kernel can be written as

$$I_{i\leftarrow k}(x) = a_{-1}^{ik}\delta(1-x) + a_0^{ik}\left[\frac{1}{1-x}\right]_+ + a_1^{ik}\left[\frac{\log(1-x)}{1-x}\right]_+ + a_2^{ik}\left[\frac{\log^2(1-x)}{1-x}\right]_+ + a_3^{ik}\left[\frac{\log^3(1-x)}{1-x}\right]_+ + I_{i\leftarrow k}^{\text{Grid}}(x)$$

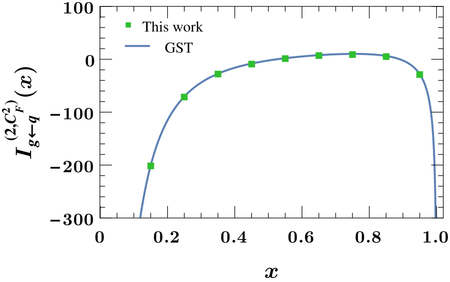
- Focus on the gluon-to-gluon and gluon-to-quark channels

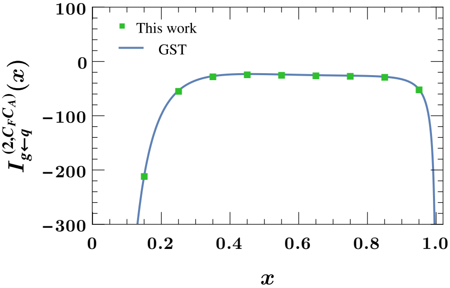
$C_A T_F n_F$	This work	GST
a_{-1}^{gg}	$-1.238(5)$	-1.238
a_0^{gg}	$3.910(3)$	3.910
a_1^{gg}	$-8.889(1)$	-8.889
a_2^{gg}	$2.667(1)$	2.667
a_3^{gg}	0	0

C_A^2	This work	GST
a_{-1}^{gg}	$-14.896(29)$	-14.897
a_0^{gg}	$35.027(17)$	35.027
a_1^{gg}	$-9.701(9)$	-9.701
a_2^{gg}	$-7.333(2)$	-7.333
a_3^{gg}	$8.000(1)$	8

GST: [Gaunt,Stahlhofen,Tackmann;14]

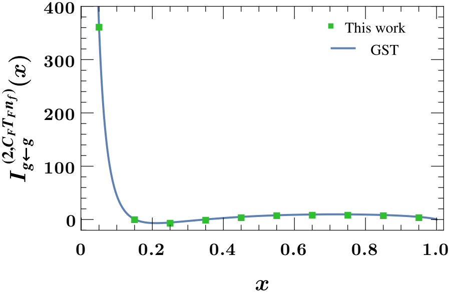
Beam Thrust-$I_{g\leftarrow q}^{\text{Grid}}(x)$ [Preliminary]

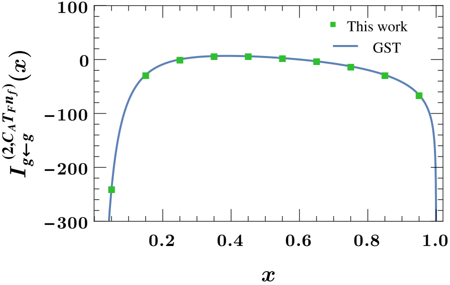


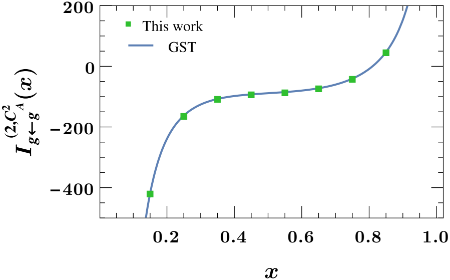


GST: [Gaunt,Stahlhofen,Tackmann;14]

Beam Thrust-$I^{\text{Grid}}_{g\leftarrow g}(x)$ [Preliminary]







GST: [Gaunt,Stahlhofen,Tackmann;14]

p_T-veto

- Measurement: $\omega_2(k,l) = \theta(\Delta - R)\max(|\vec{k}^{\perp}|, |\vec{l}^{\perp}|) + \theta(R-\Delta)|\vec{k}^{\perp} + \vec{l}^{\perp}|$
  with $\Delta = \sqrt{\frac{1}{4}\ln^2\frac{k^- l^+}{k^+ l^-} + \theta_{kl}^2}$
- Matching kernel can be written as

$$I_{i\leftarrow k}(x) = a_{-1}^{ik}\delta(1-x) + a_0^{ik}\left[\frac{1}{1-x}\right]_+ + I_{i\leftarrow k}^{\mathsf{Grid}}(x)$$

- Independent calculation by Dingyu Shao
  - Based on computing the difference wrt to a known reference observables
  - Asymmetric phase-space regulator
    $\rightarrow$ Method is similar to the one from Abreu et al.[2207.07037]

p_T-veto [Preliminary]

- Matching kernel can be written as

$$I_{i\leftarrow k}(x) = a_{-1}^{ik}\delta(1-x) + a_0^{ik}\left[\frac{1}{1-x}\right]_+ + I_{i\leftarrow k}^{\text{Grid}}(x)$$

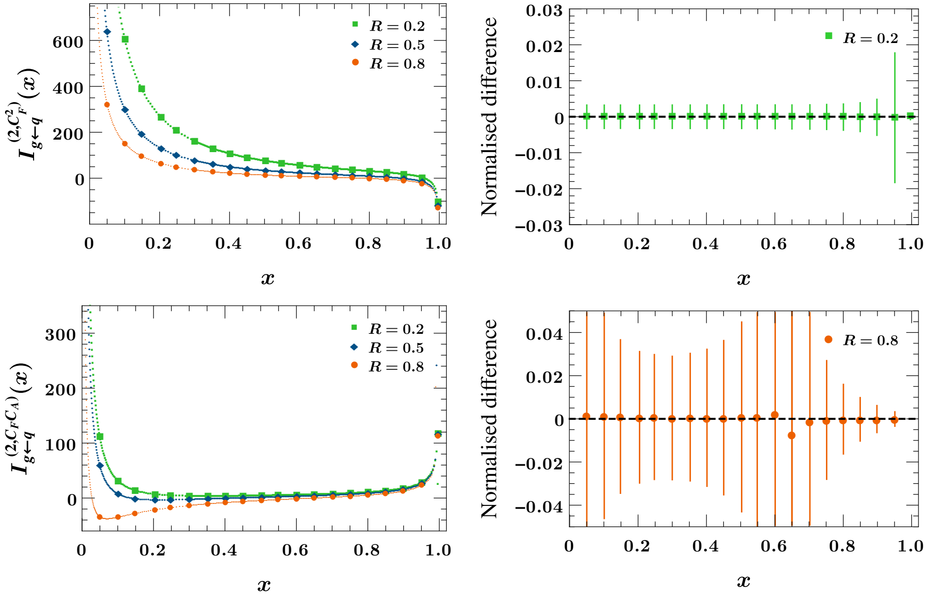
$R=0.2$	This work	AGMRS
a^{gg}_{-1,C_AT_F}	$0.31(1)$	0.32
a^{gg}_{-1,C_A^2}	$-23.32(9)$	-23.32
a^{gg}_{0,C_AT_F}	$16.44(1)$	16.44
a^{gg}_{0,C_A^2}	$79.02(6)$	79.02

$R=0.5$	This work	AGMRS
a^{gg}_{-1,C_AT_F}	$1.38(1)$	1.38
a^{gg}_{-1,C_A^2}	$-16.98(9)$	-16.98
a^{gg}_{0,C_AT_F}	$11.40(1)$	11.40
a^{gg}_{0,C_A^2}	$43.20(6)$	43.20

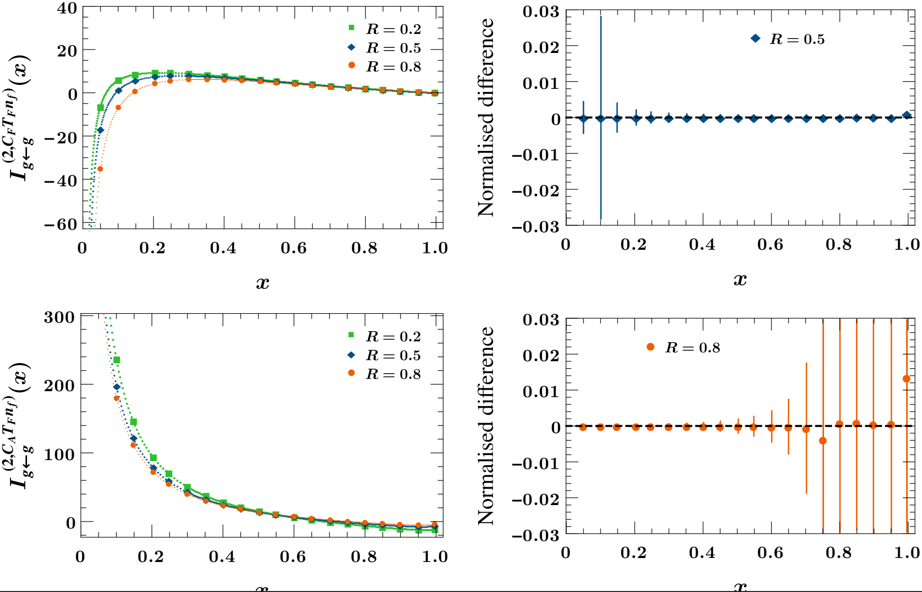
$R=0.8$	This work	AGMRS
a^{gg}_{-1,C_AT_F}	$1.94(1)$	1.94
a^{gg}_{-1,C_A^2}	$-12.29(9)$	-12.29
a^{gg}_{0,C_AT_F}	$9.01(1)$	9.01
a^{gg}_{0,C_A^2}	$20.19(6)$	20.19

AGMRS: [Abreu et al.;22]

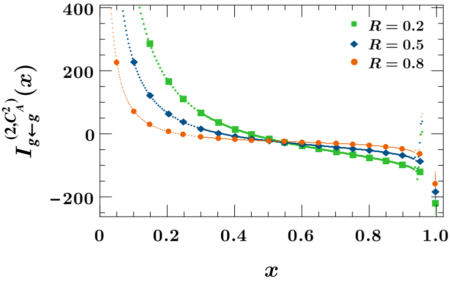
Comparison p_T-veto $I^{\text{Grid}}_{g\leftarrow q}(x)$ [Preliminary]

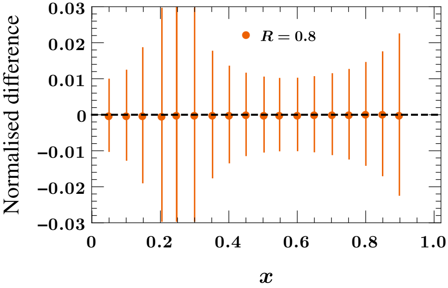


Comparison p_T-veto $I^{\text{Grid}}_{g\leftarrow g}(x)$ [Preliminary]



Comparison p_T-veto $I^{\text{Grid}}_{g\leftarrow g}(x)$ [Preliminary]





Conclusion

- We have reported on progress of our automated setup for the calculation of beam and jet functions at NNLO

Beam functions

- Setup extended to all matching kernels
- Direct calculation in x-space
- Beam thrust (SCET-I), p_T-veto (SCET-II)
- Comparison to Abreu et al.

Jet functions

- Setup extended to SCET-II observables
- Measurement functions based on jet algorithms
- WTA-axis Broadening, WTA-axis p_T-resummation

$\rightarrow$ We are ready to compute other jet and beam functions!