Chiral Perturbation Theory: an EFT for QCD

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What is an Effective Field Theory?

- An Effective Field Theory (EFT) is a description of physics that contain the **correct degrees of freedom** for a chosen energy range

- EFTs can be used as alternative theories when the true underlying physics is excessively **complicated** or **inaccessible** by experiment

- Common uses of EFTs: Fermi interactions, Rayleigh scattering*

See A. Zee VIII.3

Key properties of EFTs

- To describe a system's physics usefully and accurately, EFT Lagrangians must contain:
  - Exactly all (and no more) of the **degrees of freedom** to be described
  - All of the terms allowed by the **symmetries** of the problem (∞ terms!)
  - Contain an **expansion parameter** that can be truncated when calculating observables

degree of freedom

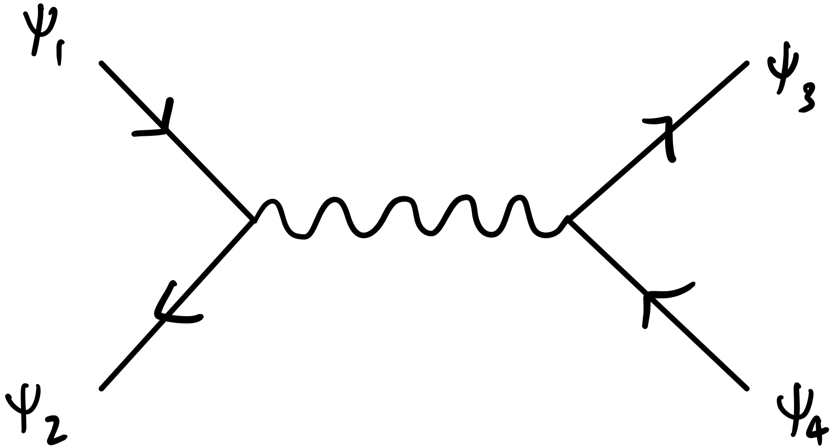
$$\mathcal{L}_{\text{EFT}} = \lambda^{(2)} M^2 \phi^2 + \lambda^{(4)} \phi^4 + \frac{\lambda^{(6)}}{\underbrace{M^2}_{= \#\left(\frac{E}{M}\right)^2}} \phi^6 + \dots$$

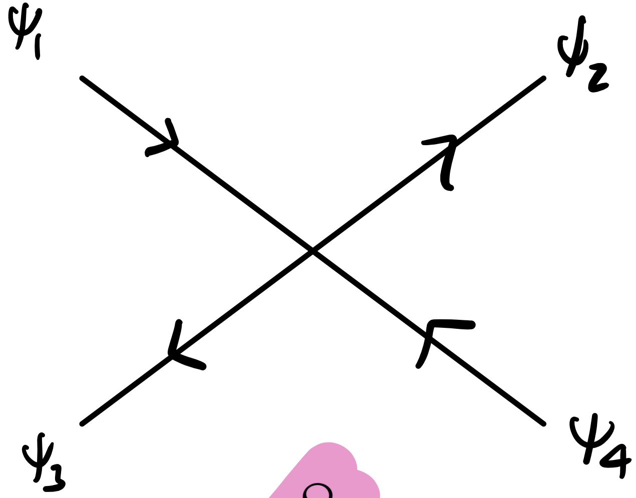
Symmetry: $\phi \to -\phi$

expansion parameter

The "classic" EFT: Fermi Theory

$$\mathcal{M} = -i\frac{e^2}{\sin^2\theta_w M_W^2} J_-^\mu J_{\mu+} + O(\frac{q^2}{M_W^2})$$





$$\mathcal{L}_F = \frac{8}{\sqrt{2}} G_F J_-^\mu J_{\mu+}$$

A toy EFT for proton decays*

- What is the simplest Lagrangian we could write down for proton decay?
  - Key properties: is Lorentz scalar, has mass dimension 4, obeys *SU(3)⊗SU(2)⊗U(1)*

necessary to be a Lorentz scalar

$$\mathcal{L} \supset \frac{qqql}{M^2}$$

necessary to be dimension 4

([fermion] = $\frac{3}{2}$)

*See A. Zee VIII.3

A toy EFT for proton decays*

- What is the simplest Lagrangian we could write down for proton decay?
    - Key properties: is Lorentz scalar, has mass dimension 4, obeys *SU(3)⊗SU(2)⊗U(1)*

$$\mathcal{L} \approx \frac{1}{M^2}[(\bar{l}_L C q_L)(u_R C d_R) + (e_R C u_R)(\bar{q}_L C q_L) + (\bar{l}_L C q_L)(\bar{q}_L C q_L) + (e_R C u_R)(u_R C d_R)]$$

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$$

- Conclusions: B and L violated, B-L conserved, lower bound on *M* set by the measured *p* lifetime

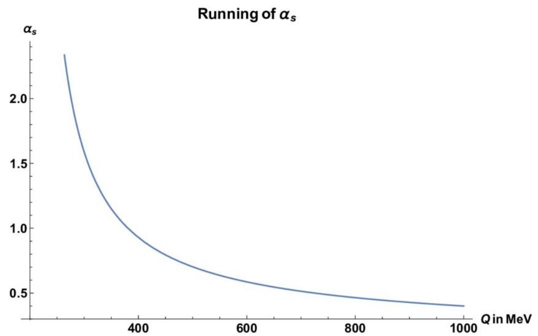
**See A. Zee VIII.3*

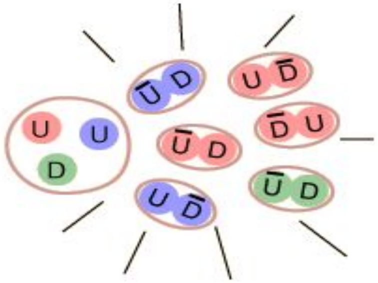
Why use EFTs for QCD?

- EFTs are often used to **describe low-energy physics** when some **high-energy element is inaccessible** (i.e. a particle)

- We would like a description of QCD at low-energies -- but in this realm, QCD is **non-perturbative**

- QCD is built around quarks and gluons, but due to color confinement, **we only observe pions and baryons**





The “full” solution:
Chiral Perturbation Theory

A refresher on chirality

- Chirality corresponds to the handedness of a particle
  - In the massless limit, the chiral and helicity eigenstates are identical

Chirality operator

$$\Gamma_{L,R} = \frac{1}{2}(1 \pm \gamma_5)$$

$$\psi_R = \Gamma_R \psi \qquad \psi_L = \Gamma_L \psi$$

$$\psi = \psi_L + \psi_R$$

QCD has an approximate chiral symmetry

$$\mathcal{L}_{QCD} = \bar{q}(i\not{D} - m)q - \frac{1}{2}\mathrm{tr}[G_{\mu\nu}G^{\mu\nu}]$$

- In the limit of massless quarks, the Lagrangian becomes

$$\mathcal{L}_{QCD,q} \approx \bar{q}_L i\not{D} q_L + \bar{q}_R i\not{D} q_R - \bar{q}_L m q_R - \bar{q}_R m q_L$$

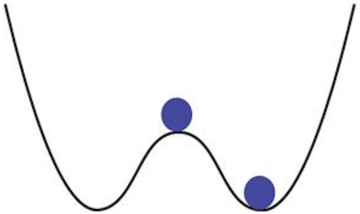
which obeys the symmetries

$$q_L \to e^{\sum_J \lambda_j \alpha_j} q_L$$

$$q_R \to e^{\sum_J \lambda_j \beta_j} q_R$$

Let's focus only on u, d, s quarks…

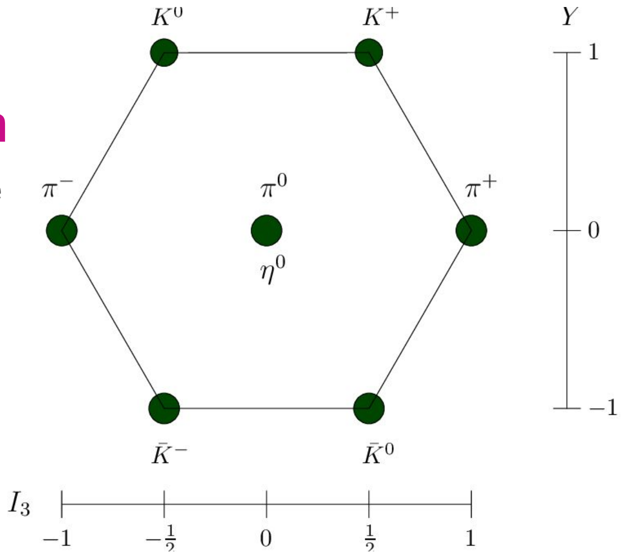
Motivating ChPT: the broken chiral symmetry



- The chiral symmetry of QCD is **spontaneously broken**
  - Light hadrons are not found in parity doublets of approximately equal masses
  - The spectrum obeys isospin *SU(2)* or flavor *SU(3)* representations, but not $SU(3)_L \otimes SU(3)_R$

- The chiral symmetry of QCD is **explicitly broken**
  - The light quarks have nonzero masses, so the Goldstone bosons do as well

- The result: 8 massive (but light) **Goldstone bosons *π, K, η***



Chiral Perturbation Theory (ChPT)

- ChPT uses the observed π DoF and expands about the ratio (m_π/Λ)

$$\mathcal{L}_\pi = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \dots$$

$$\Lambda = 4\pi F \approx 1\mathrm{GeV}$$

π decay constant

$$\mathcal{L}_2 = \frac{F^2}{4}\mathrm{tr}[D_\mu U (D^\mu U)^\dagger + \chi U^\dagger + U\chi^\dagger]$$

$$U = e^{i\phi/F}$$

$$\phi = \begin{pmatrix} \pi_0 & \sqrt{2}\pi_+ \\ \sqrt{2}\pi_- & -\pi^0 \end{pmatrix}$$

$$\chi = 2B\hat{m} = M^2$$

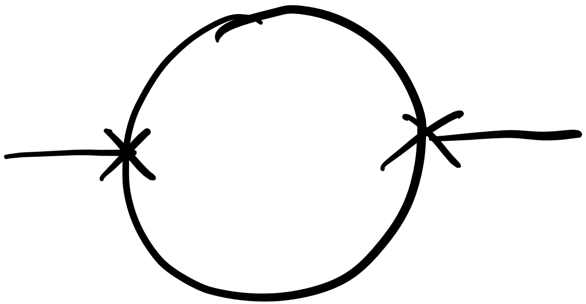
related to the quark condensate

$\hat{m} = m_d = m_u$

lowest order expansion of $(m_\pi)^2$

Chiral dimension and power counting

- In 4 dimensions, loop terms are suppressed



$k = 1$
$n = 4$
$N_L = 1$
$I_\pi = 2$
$N_{2k} = 2$

$\Rightarrow D = 4$

vertices from $\mathcal{L}_{2k}$

independent loop momenta

internal π lines

$$D = nN_L - 2I_\pi + \sum_k 2kN_{2k}^{\pi}$$

dimensions

Experimental tests of ChPT

- Common experimental routes to test ChPT for light mesons are numerous, and can involve measurements from:
  - $\pi\pi$ scattering
  - K decays involving π
  - π *polarizability*

- The main method is to calculate observables to some order in ChPT and compare with experimental results
  - ChPT predictions are sometimes compared with those of lattice QCD

**Disclaimer*: many of the summary papers I found were from pre-2010 and used very old data. Further, I am not an expert on the subject, so my choices of “good” summaries many not be in line with those of someone in the field.*

Experimental tests of ChPT: $\pi\pi$ scattering

- Summary paper: [hep-ph/0103088](hep-ph/0103088) (from 2001)

Abstract

We demonstrate that, together with the available experimental information, chiral symmetry determines the low energy behaviour of the $\pi\pi$ scattering amplitude to within very small uncertainties. In particular, the threshold parameters of the S–, P–, D– and F–waves are predicted, as well as the mass and width of the ρ and of the broad bump in the S–wave. The implications for the coupling constants that occur in the effective Lagrangian beyond leading order and also show up in other processes, are discussed. Also, we analyze the dependence of various observables on the mass of the two lightest quarks in some detail, in view of the extrapolations required to reach the small physical masses on the lattice. The analysis relies on the standard hypothesis, according to which the quark condensate is the leading order parameter of the spontaneously broken symmetry. Our results provide the basis for an experimental test of this hypothesis, in particular in the framework of the ongoing DIRAC experiment: The prediction for the lifetime of the ground state of a $\pi^+\pi^-$ atom reads $\tau = (2.90 \pm 0.10)\, 10^{-15}$ sec.

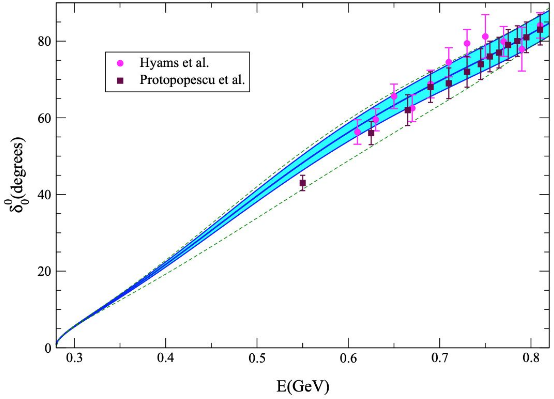


Figure 7: $I = 0$ S–wave phase shift. The full line results with the central values of the scattering lengths and of the experimental input used in the Roy equations. The shaded region corresponds to the uncertainties of the result. The dotted lines indicate the boundaries of the region allowed if the constraints imposed by chiral symmetry are ignored [6]. The data points are from refs. [49] and [50].

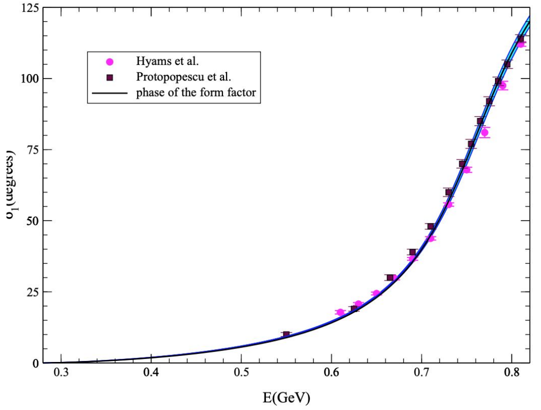


Figure 8: P–wave phase shift. The phase of the pion form factor is also shown, but it can barely be distinguished from the central result of our analysis. The data points are from refs. [49] and [50].

?? (B. Hyams et al, 1973)

SLAC, 1973

Experimental tests of ChPT: *K* decays

- Summary paper: from HEP-MAD (from 2013)

Abstract

Rare kaon decays allow to perform tests of the Chiral Perturbation Theory (ChPT) predictions and to measure combinations of the ChPT fundamental parameters. The rare $K^\pm \to \pi^\pm\gamma\gamma$ decay has the property that its amplitude vanishes at the lowest order in the ChPT. At the leading order, precise predictions for the decay rate and the di-photon invariant mass distribution are made in terms of a single coupling constant, $\hat{c}$. We describe the study of the $K^\pm \to \pi^\pm\gamma\gamma$ decay: we present the preliminary results of the fit to the $K^\pm \to \pi^\pm\gamma\gamma$ di-photon mass spectrum on data collected by the NA48/2 and NA62-R_K experiments at CERN and the final NA48/2 model-independent measurement of the branching ratio. We finally report the observation of the $K^\pm \to \pi^\pm\pi^0 e^+e^-$ decay on the full 2003 NA48/2 data sample.

NA48/2: CERN, 2003 - 2004

NA62: CERN, 2015 - present

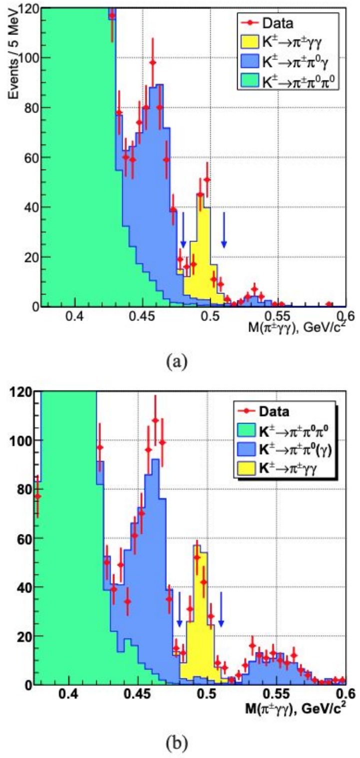
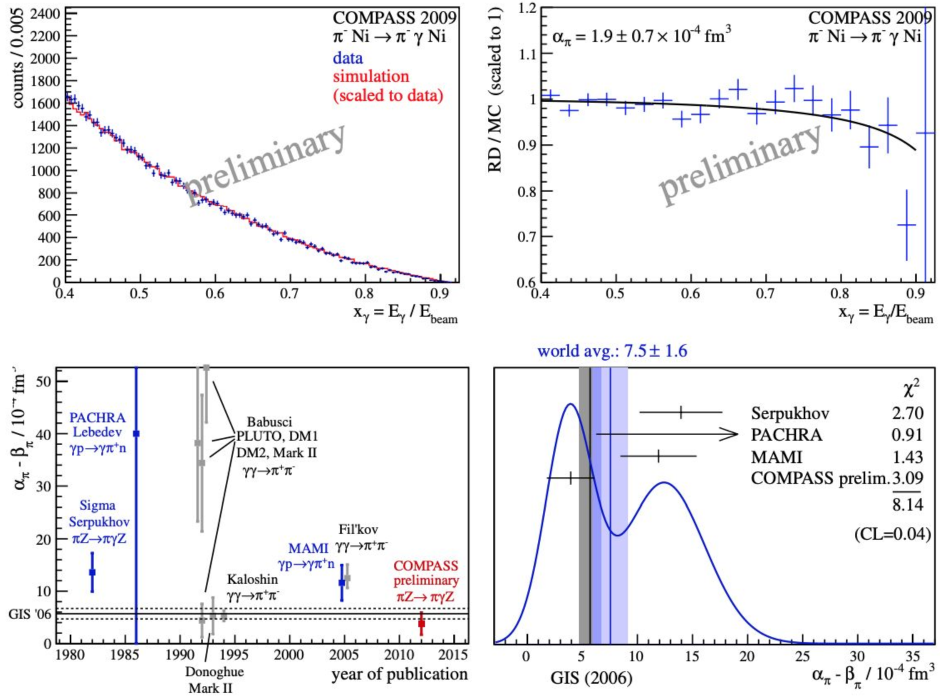


Figure 2: The reconstructed $\pi^\pm\gamma\gamma$ invariant mass compared with the sum of the estimated signal and background components for NA48/2 (a) and NA62 (b) data.

Experimental tests of ChPT: π polarizability

- Summary paper: from COMPASS (CERN, 2012-2021)

Abstract. The COMPASS experiment at CERN accesses pion-photon reactions via the Primakoff effect., where high-energetic pions react with the quasi-real photon field surrounding the target nuclei. When a single real photon is produced, pion Compton scattering is accessed and from the measured cross-section shape, the pion polarisability is determined. The COMPASS measurement is in contradiction to the earlier dedicated measurements, and rather in agreement with the theoretical expectation from ChPT. In the same experimental data taking, reactions with neutral and charged pions in the final state are measured and analyzed in the context of chiral perturbation theory.



But wait! Baryon masses aren't light…

- Naively, we could expand about the about the ratio $(m_p/\Lambda) \approx 1 \rightarrow$ very bad!
- A better treatment is Heavy Baryon Chiral Perturbation Theory (HBChPT)

$$\Psi(x) = e^{-imv\cdot x}(\mathcal{N}_v + \cancel{\mathcal{H}_v})$$

soft residual

large, non-shell

$$\mathcal{N}_v = e^{+imv\cdot x}\frac{1}{2}(1+\not{v})\Psi,$$

$$\mathcal{H}_v = e^{+imv\cdot x}\frac{1}{2}(1-\not{v})\Psi$$

$$\widehat{\mathcal{L}}^{(1)}_{\pi N} = \bar{\mathcal{N}}_v(iv\cdot D + g_A S_v \cdot u)\mathcal{N}_v + \mathcal{O}(1/m)$$

Experimental tests of HBChPT

- Summary paper: nucl-th/1212.3237 (from 2012)

MAMI, 2000s

A B S T R A C T

With the availability of the new neutral pion photoproduction from the proton data from the A2 and CB-TAPS Collaborations at Mainz it is mandatory to revisit Heavy Baryon Chiral Perturbation Theory (HBChPT) and address the extraction of the partial waves as well as other issues such as the value of the low-energy constants, the energy range where the calculation provides a good agreement with the data and the impact of unitarity. We find that, within the current experimental status, HBChPT with the fitted LECs gives a good agreement with the existing neutral pion photoproduction data up to ~170 MeV and that imposing unitarity does not improve this picture. Above this energy the data call for further improvement in the theory such as the explicit inclusion of the $\Delta(1232)$. We also find that data and multipoles can be well described up to ~185 MeV with Taylor expansions in the partial waves up to first order in pion energy.

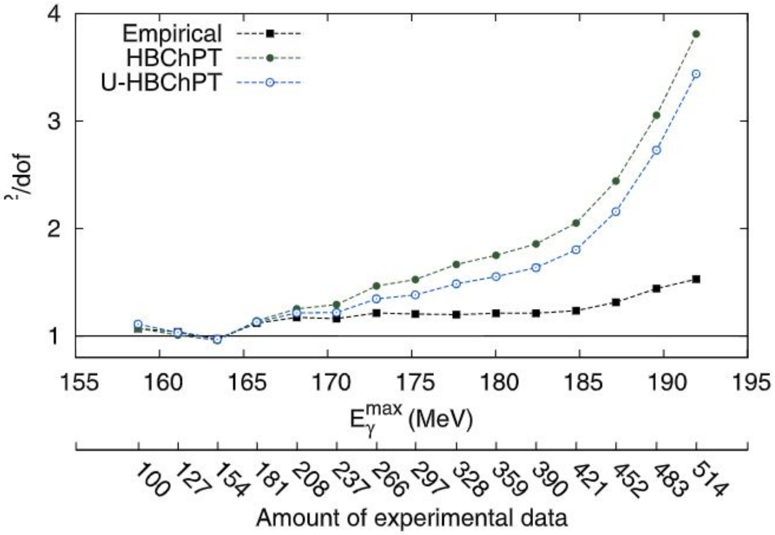


Fig. 1. (Color online.) χ^2/dof energy dependence for the empirical (full black squares), HBChPT (full green circles), and U-HBChPT (open blue circles) fits from a minimum photon energy of 151.68 MeV up to a variable maximum energy E_γ^{max}. Each point represents a separate fit and the connecting lines are drawn to guide the eye. The points are plotted at the central energy of each bin, although the calculations take the energy variation inside of each bin into account. The value $\chi^2/\text{dof} = 1$ is highlighted with a solid line.

References

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    - (specifically VIII.3)
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5. Sherer, Chiral perturbation theory: Success and challenge (2005)
6. Sherer, Introduction to Chiral Perturbation Theory (2002)
7. Guo, Introduction to Effective Field Theories for Hadron Physics (2016)