Proton radius “puzzle”

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Introduction

- The **proton** is a **fundamental building block** of visible matter (nucleons (p & n) → 99% of visible matter)
- Important to probe both **global properties** and **internal structure** of proton
- Example of **global property** is the **radius**
  - Use electromagnetic probe (photon) to determine ~ **spatial extent of the charge distribution of the proton** → charge radius
  - Can be measured using **elastic e-p scattering** or **hydrogen spectroscopy**

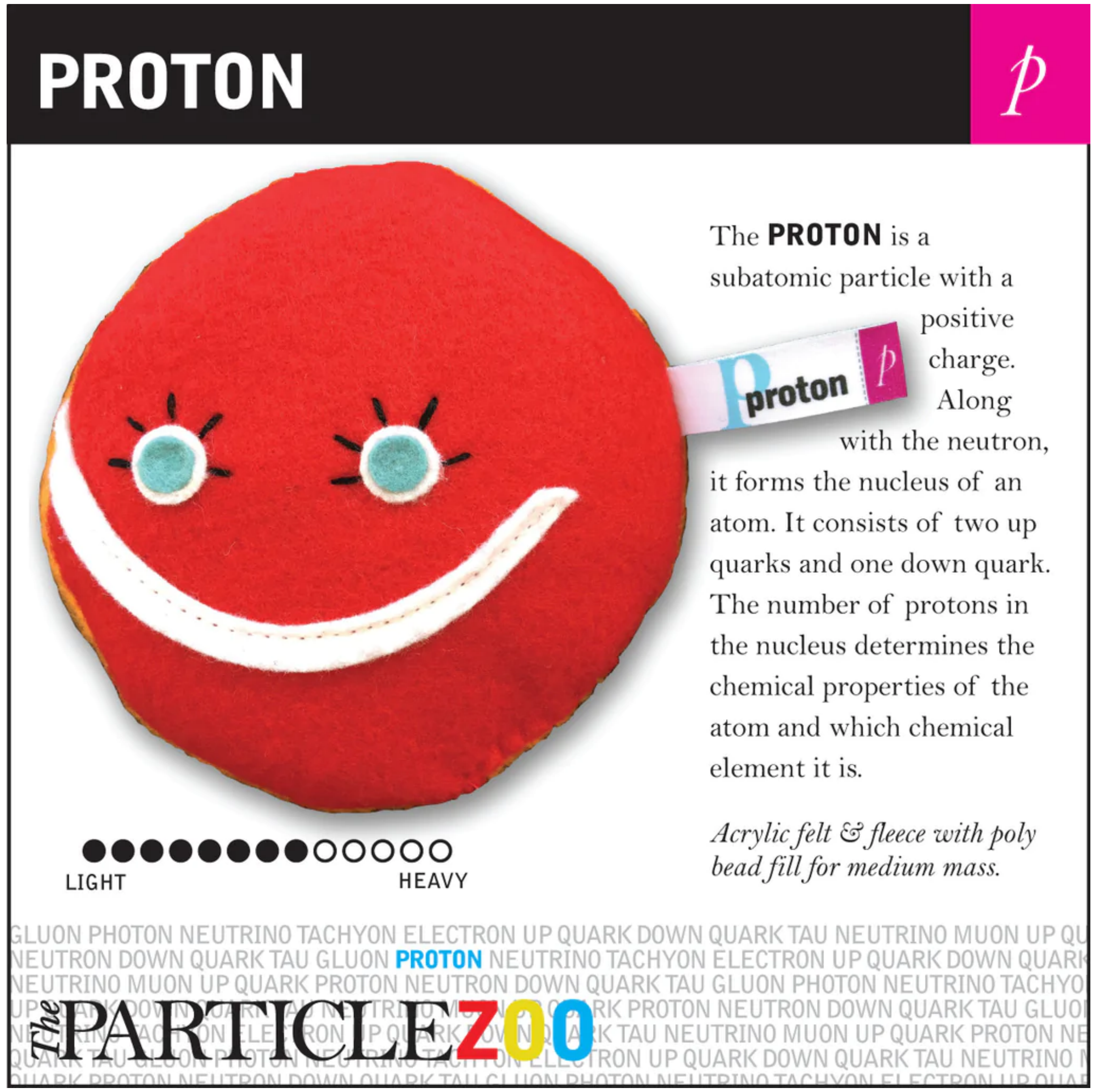


Photo credit: The Particle Zoo

Measuring the proton charge radius

From e-p scattering

- Elastic e-p scattering cross-section formula:

→ Detailed derivation in Thomson chapter 7

Electric form factor

$$\frac{d\sigma}{d\Omega} = \left(\frac{G_E^2 + \tau G_M^2}{(1+\tau)} + 2\tau G_M^2 \tan^2 \frac{\theta}{2} \right) \cdot \frac{\alpha^2}{4E_1^2 \sin^4(\theta/2)} \left(\frac{E_3}{E_1} \right) \cos^2 \frac{\theta}{2},$$

Magnetic form factor

- $\tau = Q^2/(4m_{\mathrm{p}}^2)$
- $Q^2 \equiv -q^2 = 4E_1E_3 \sin^2(\theta/2)$
- $E_1(E_3)$ = in(out)going electron energy
- θ = scattering angle

→ **Form factors** account for **extended** distributions of **charge/magnetic moment**

[Approximation holds in the low-Q^2 limit ($Q^2 \ll 4m_{\mathrm{p}}^2$)]

$$G_E(Q^2) \approx G_E(\mathbf{q}^2) = \int e^{i\mathbf{q}\cdot\mathbf{r}} \rho(\mathbf{r}) \mathrm{d}^3\mathbf{r}$$

$$G_M(Q^2) \approx G_M(\mathbf{q}^2) = \int e^{i\mathbf{q}\cdot\mathbf{r}} \mu(\mathbf{r}) \mathrm{d}^3\mathbf{r}$$

Measuring the proton charge radius

From e-p scattering

- In low-Q^2 limit we have $G_E(Q^2) \approx G_E(\mathbf{q}^2) = \int e^{i\mathbf{q}\cdot\mathbf{r}}\rho(\mathbf{r})\mathrm{d}^3\mathbf{r}$
- Assuming a spherically symmetric charge distribution, a Taylor series expansion of the Fourier integral leads to
- $G_E(Q^2) \approx 1 - \frac{1}{6}Q^2\langle r^2\rangle + \frac{1}{120}Q^4\langle r^4\rangle + \ldots$
- **Define the charge radius** to be $r_E^2 = -6\frac{\mathrm{d}G_E(Q^2)}{\mathrm{d}Q^2}\Big|_{Q^2=0}$

Involves extrapolation!

Measuring the proton charge radius

From e-p scattering

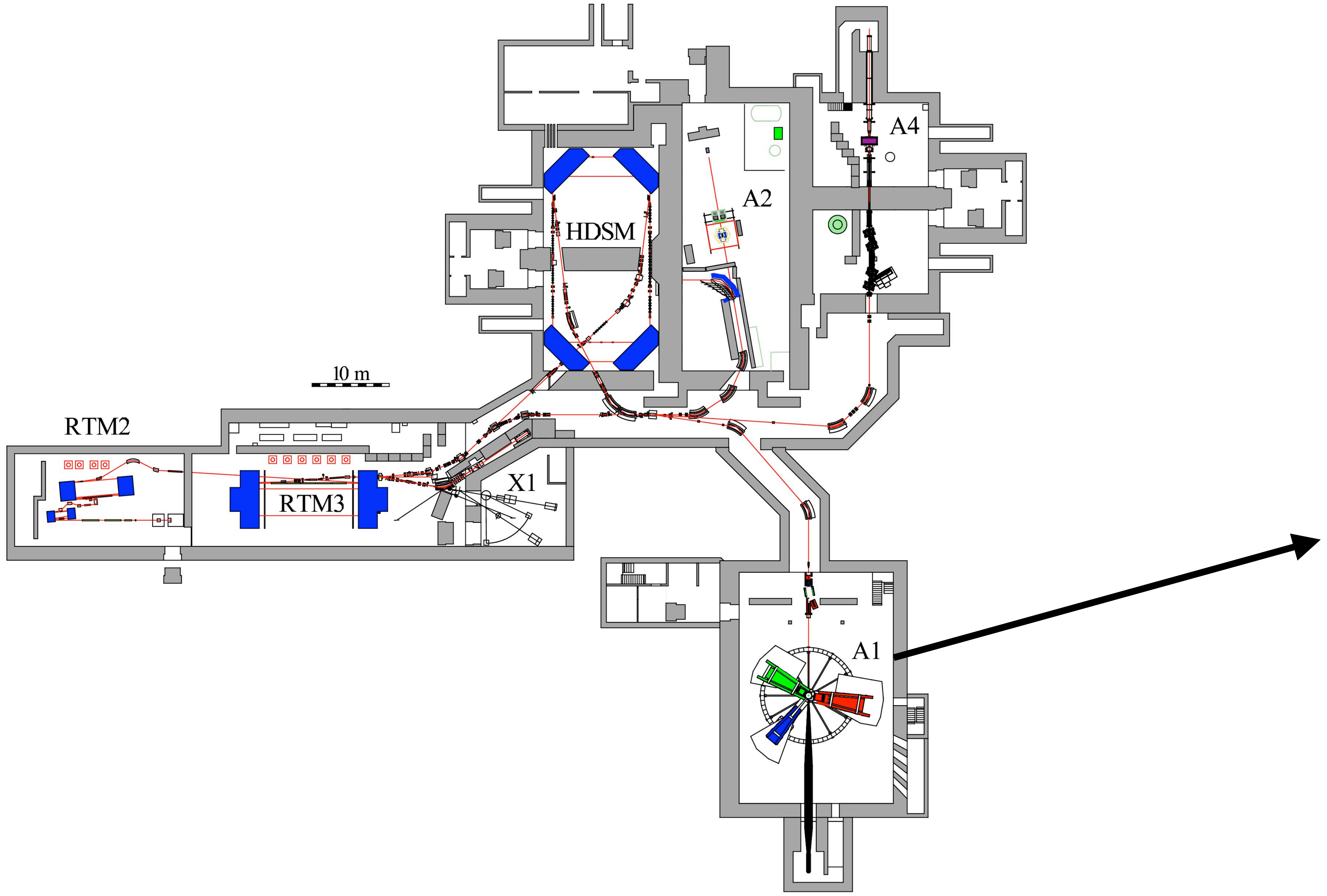
- Procedure to measure the proton charge radius:
  - **Measure e-p scattering cross-section** at (low) values of Q^2
  - Using e-p cross-section formula, **determine** $G_E(Q^2)$ at (low) values of Q^2
  - **Fit a functional form** to $G_E(Q^2)$ and **extrapolate** experimental data **to** $Q^2 = 0$
  - Determine the **charge radius** from the **slope of the tangent at** $Q^2 = 0$

Measuring the proton charge radius

From e-p scattering

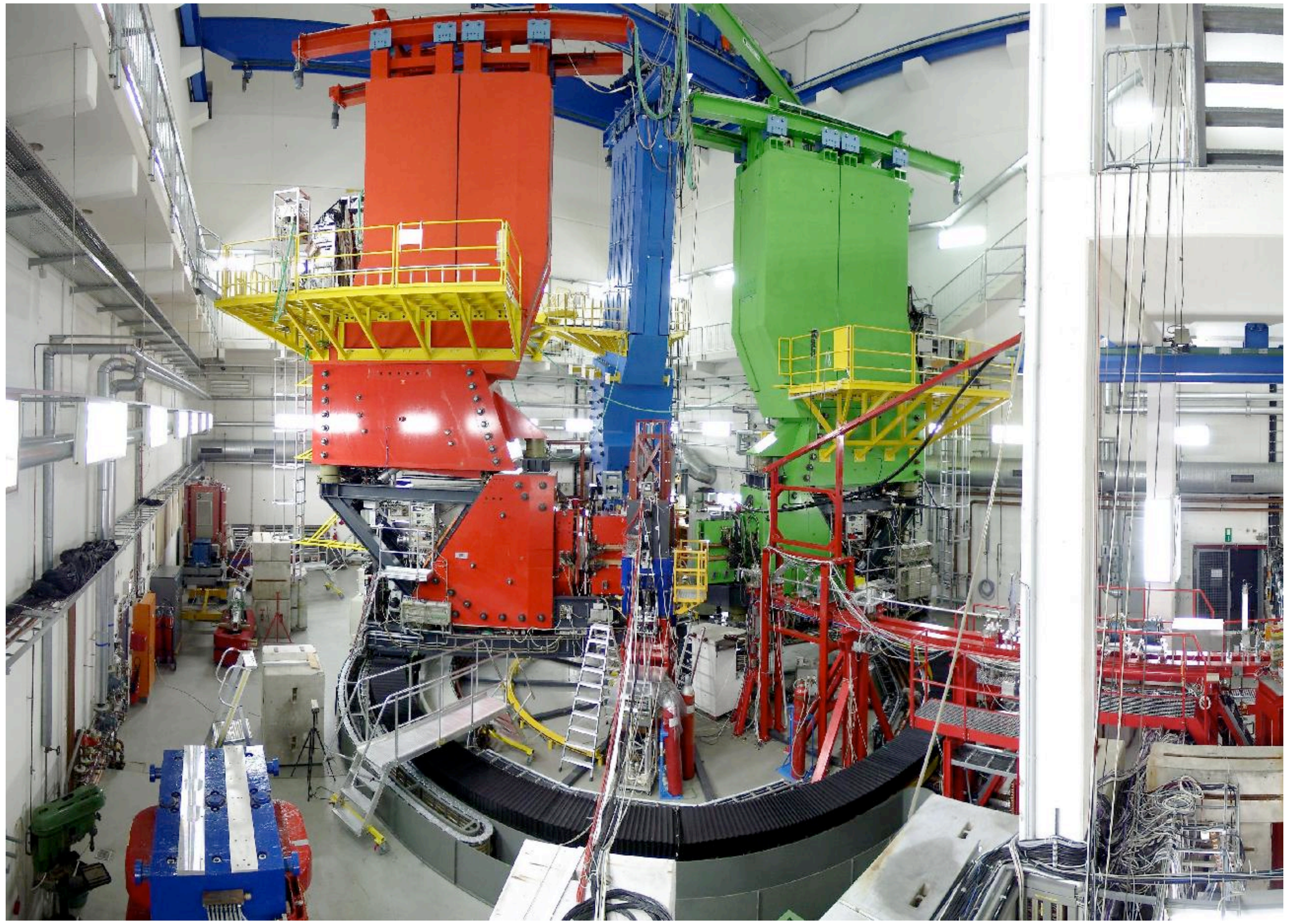
- Example: Mainz Microtron (MAMI), 2010

Electron beam with energies up to 1600 MeV



MAMI floorplan, image link

One spectrometer at fixed angle to measure the luminosity
Other two spectrometers moved as a function of scattering angle
Each spectrometer is 15 m / 200 tons

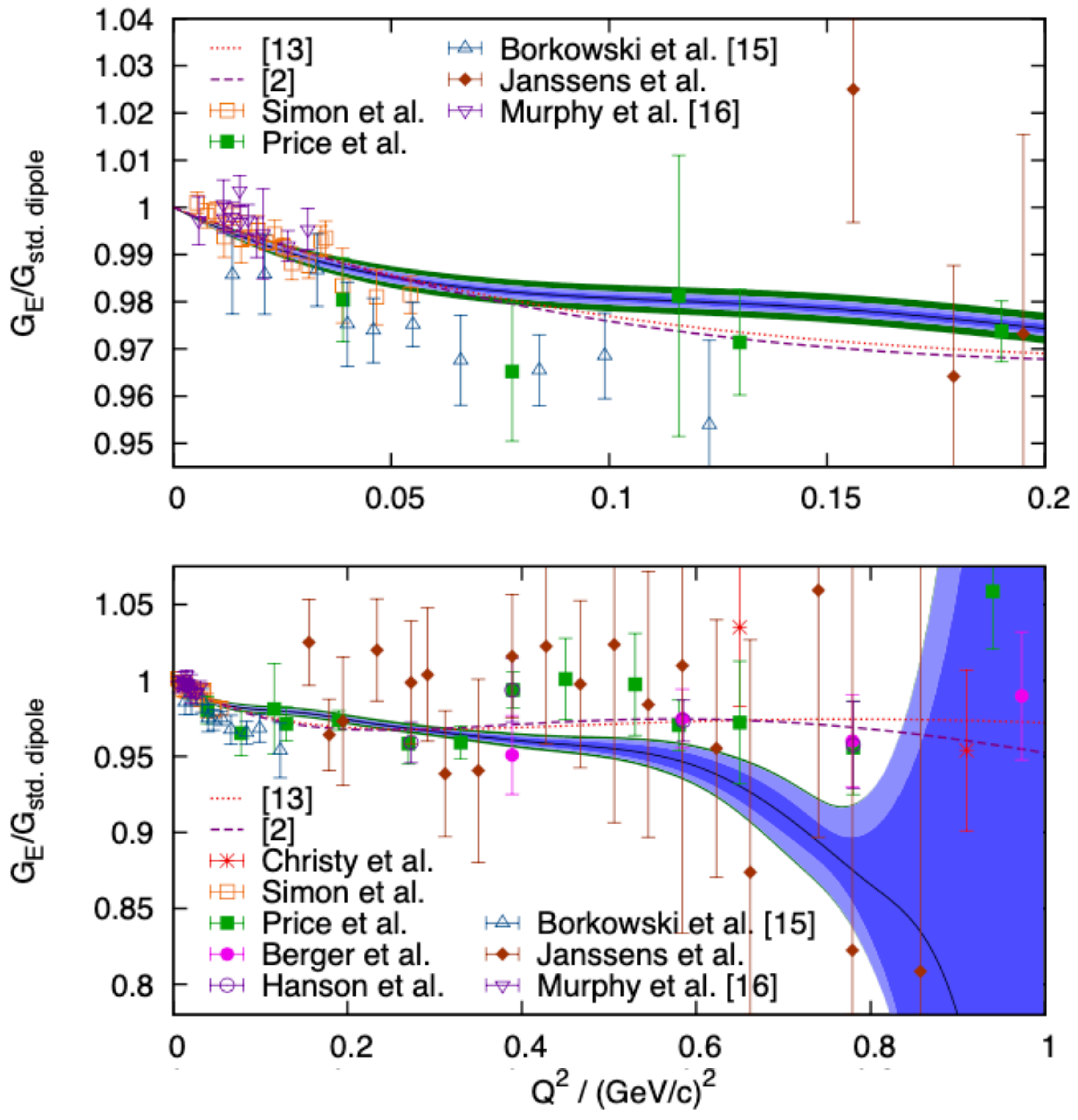


Spectrometer setup of A1 collaboration, image link

Measuring the proton charge radius

From e-p scattering

- Example: Mainz Microtron (MAMI), 2010
  - **1400** cross-section **measurements** with **statistical precision < 0.2%**
  - Q^2 coverage from **0.004 to 1 GeV**^{2}
  - Two fitting models, **spline** and **polynomial**
    - Slightly different results from the two models, so average together for final result
  - **Final result**: $r_E = 0.879(5)_{stat}(4)_{syst}(2)_{model}(4)_{group}$ fm
    - **Precision of ~1%**



Bernauer at al 2010

- black: best fit to data
- dark blue area: statistical 68% confidence band
- light blue area: experimental systematic error

Measuring the proton charge radius

From hydrogen spectroscopy

- **Energy levels** of hydrogen are given by

Rydberg constant — Corrections: relativistic, recoil, QED

$$E_{nlj} = hcR_\infty \left(-\frac{1}{n^2} + f_{nlj}\left(\alpha, \frac{m_e}{M}, \ldots\right) + \frac{C_{NS}}{n^3}\delta_{l0}\langle r_p^2 \rangle \right)$$

Bohr energy — Finite size of the proton

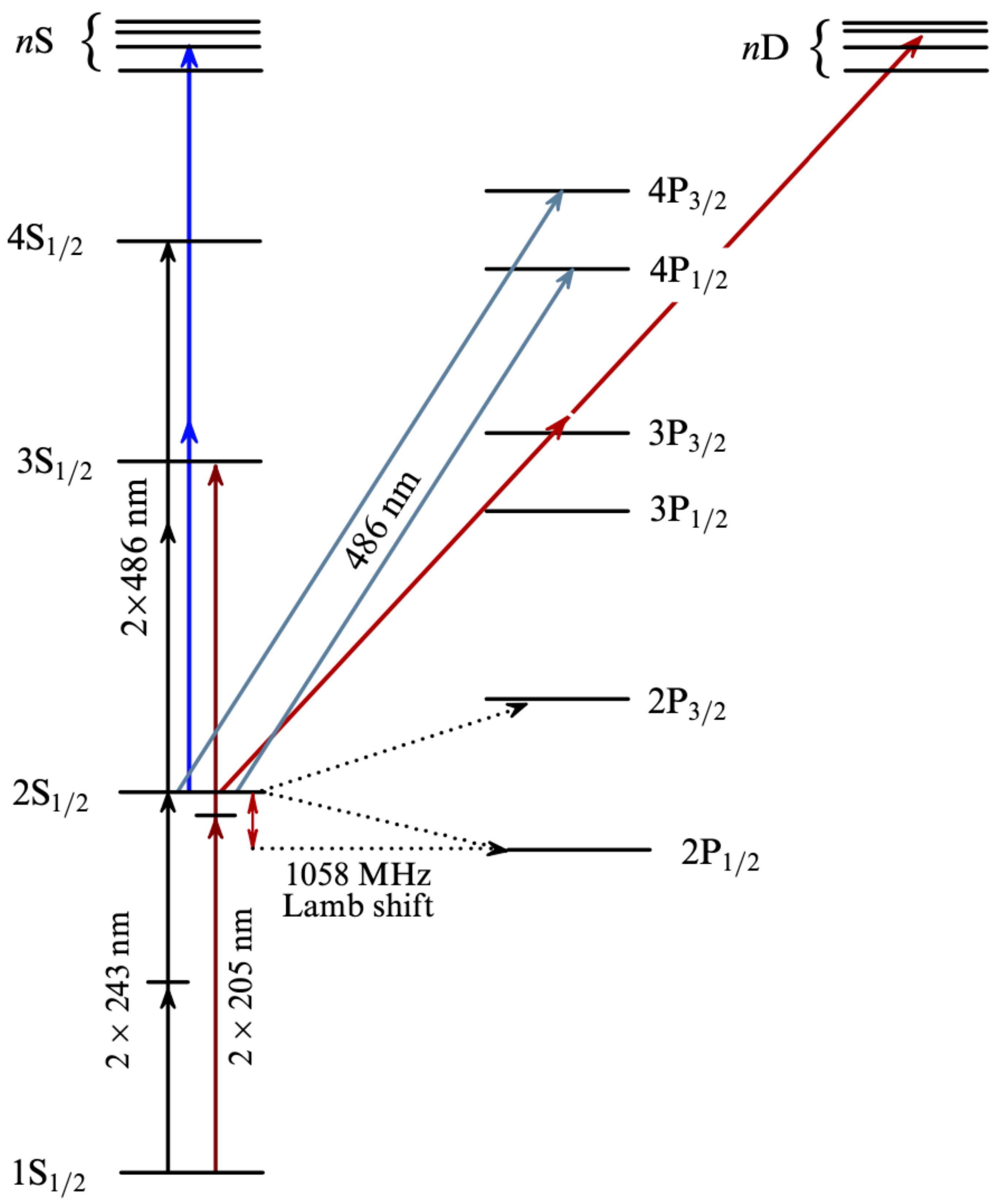
- $\langle r_p^2 \rangle$ in the last term is the **same as the charge radius** defined from the slope of the form factor $G_E(Q^2)$

- Can extract the charge radius from the **difference in energy levels**, where at least one of them is an S-state

  - Example: Lamb shift (energy difference between the $2S_{1/2}$ and $2P_{1/2}$ states)

nL_J

- n = principal quantum number
- L = orbital angular momentum
- J = total angular momentum



Khabarova, Kolachevsky 2021

Measuring the proton charge radius

From hydrogen spectroscopy

- **CODATA** = Committee on Data for Science and Technology

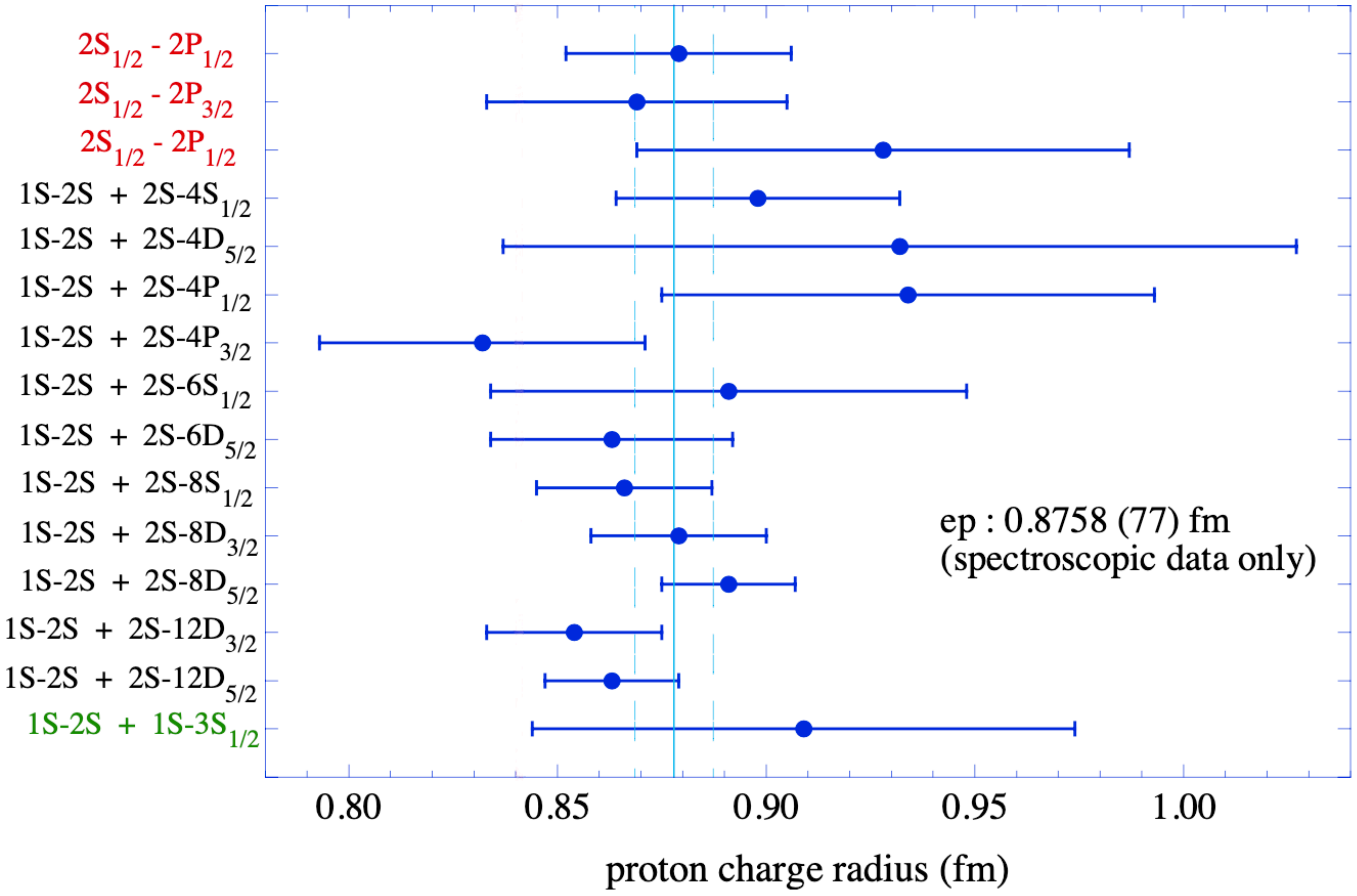
Summary

Every four years, the Committee on Data for Science and Technology (CODATA) issues recommended values of the fundamental physical constants. The values are determined by a least-squares adjustment, based on all the available theoretical and experimental information. The selection and assessment of data is done under the auspices of the CODATA Task Group on Fundamental Constants.

NIST website

- CODATA 2010 (spectroscopy only): $r_E = 0.8758(77)$ fm (sub-percent precision)
- CODATA 2010 + Mainz scattering result ($r_E = 0.879(8)$ fm): $r_E = 0.8775(51)$ fm

CODATA 2010



Adapted from Carlson 2015

Measuring the proton charge radius

From muonic hydrogen spectroscopy

- **Muonic hydrogen** = proton orbited by a muon
- The **finite size correction** term to the energy level has a m_r^3 **dependence**
  - Muon is **~200 times more massive** than electron
  - Proton radius effect is ~6.4×10^6 **larger** for muonic hydrogen that for ordinary hydrogen
- **Finite size correction** accounts for **~2% of muonic Lamb shift** compared to 0.01% of electronic Lamb shift

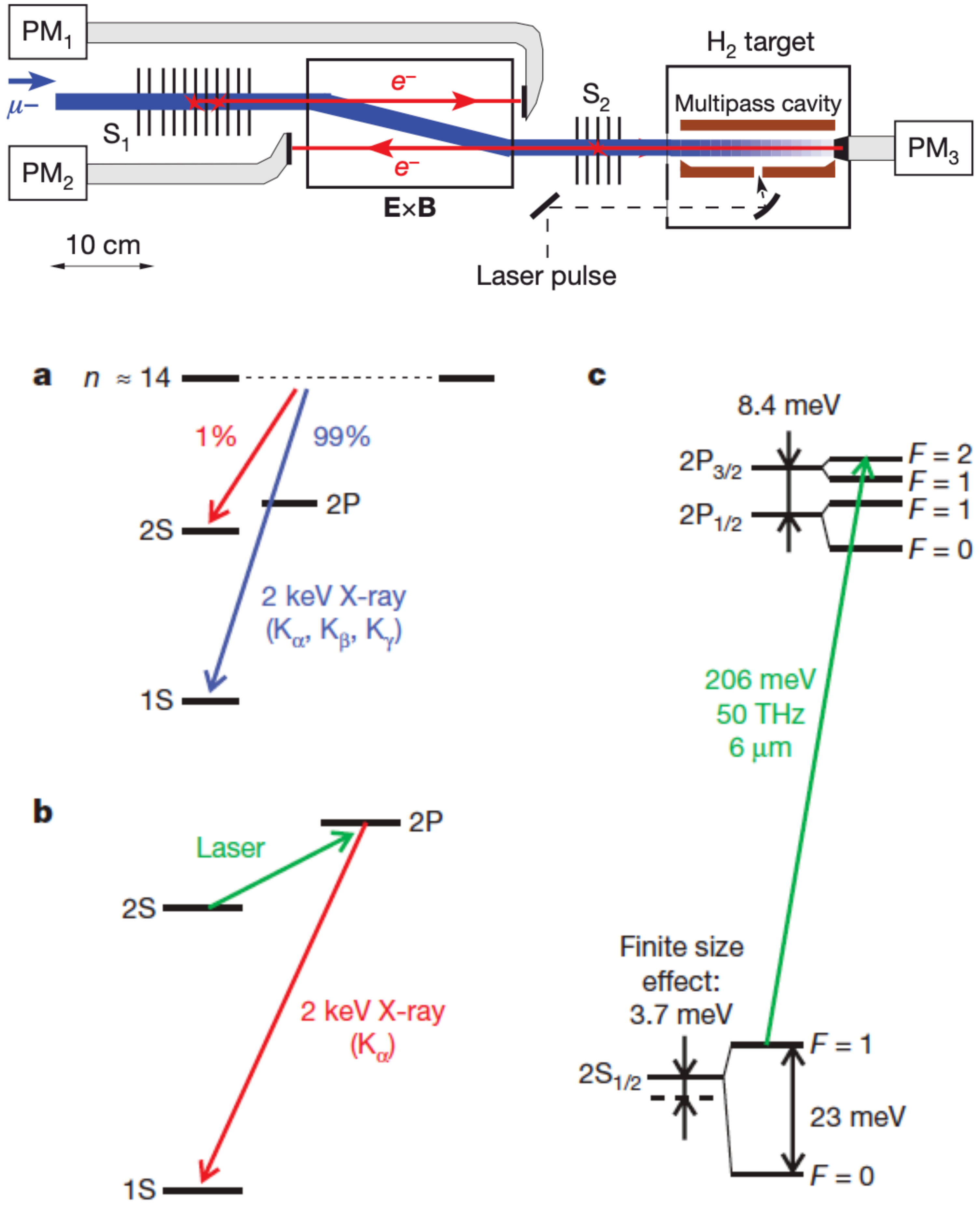
L_{2S}	Radiative correction	Vacuum polarization	Contribution of r_p	Total shift
ep	1085 MHz	−27 MHz	0.14 MHz	1057 MHz
μp	0.1 THz	−45 THz	0.93 THz	−49 THz

Khabarova, Kolachevsky 2021

Measuring the proton charge radius

From muonic hydrogen spectroscopy

- Pohl et al 2010 result — **first measurement muonic Lamb shift**, at Paul Scherrer Institute (PSI)
    - Muons stopped in H_2 gas → μp **atoms** with $n \approx 14$ (~1% in the 2S state)
    - Laser pulse induces **2S → 2P transitions**
    - **2P atoms go to ground state** by emitting 1.9keV X-rays
    - Generate a **resonance curve** by measuring the number of X-rays in coincidence with the laser pulse as a function of laser frequency/wavelength



Pohl et al 2010

Charge Radius Experiment with Muonic Atoms — CREMA collaboration

Measuring the proton charge radius

From muonic hydrogen spectroscopy

- Pohl et al 2010 result — **first measurement muonic Lamb shift**, at Paul Scherrer Institute (PSI)
  - Predicted energy difference

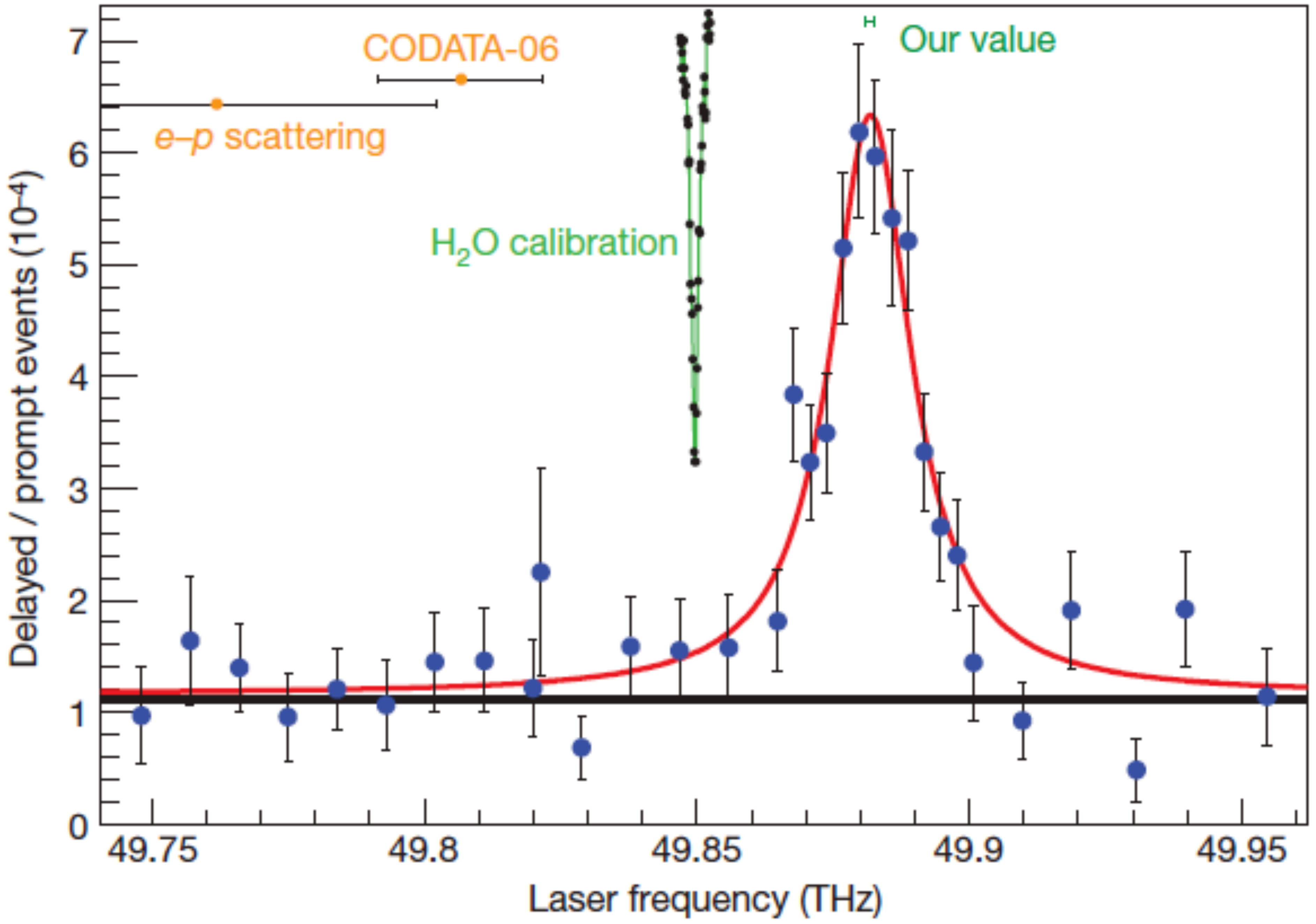
Radiative, recoil, fine/hyperfine splittings corrections

$$\Delta\tilde{E} = 209.9779(49) \boxed{-5.2262\, r_p^2 + 0.0347\, r_p^3}\ \text{meV}$$

Finite size corrections

  - Measured $2S_{1/2}^{F=1} - 2P_{3/2}^{F=2}$ energy difference of **206.2949(32) meV**
  - Determine $\boxed{r_E = 0.84184(67)\ \text{fm}}$ (**0.08% precision**)
    - **10x more precise**, 5.0σ **smaller than CODATA 2006**

Peak at 49881.88(76) GHz
Stat uncertainty: 0.7 GHz, syst uncertainty: 0.3 GHz

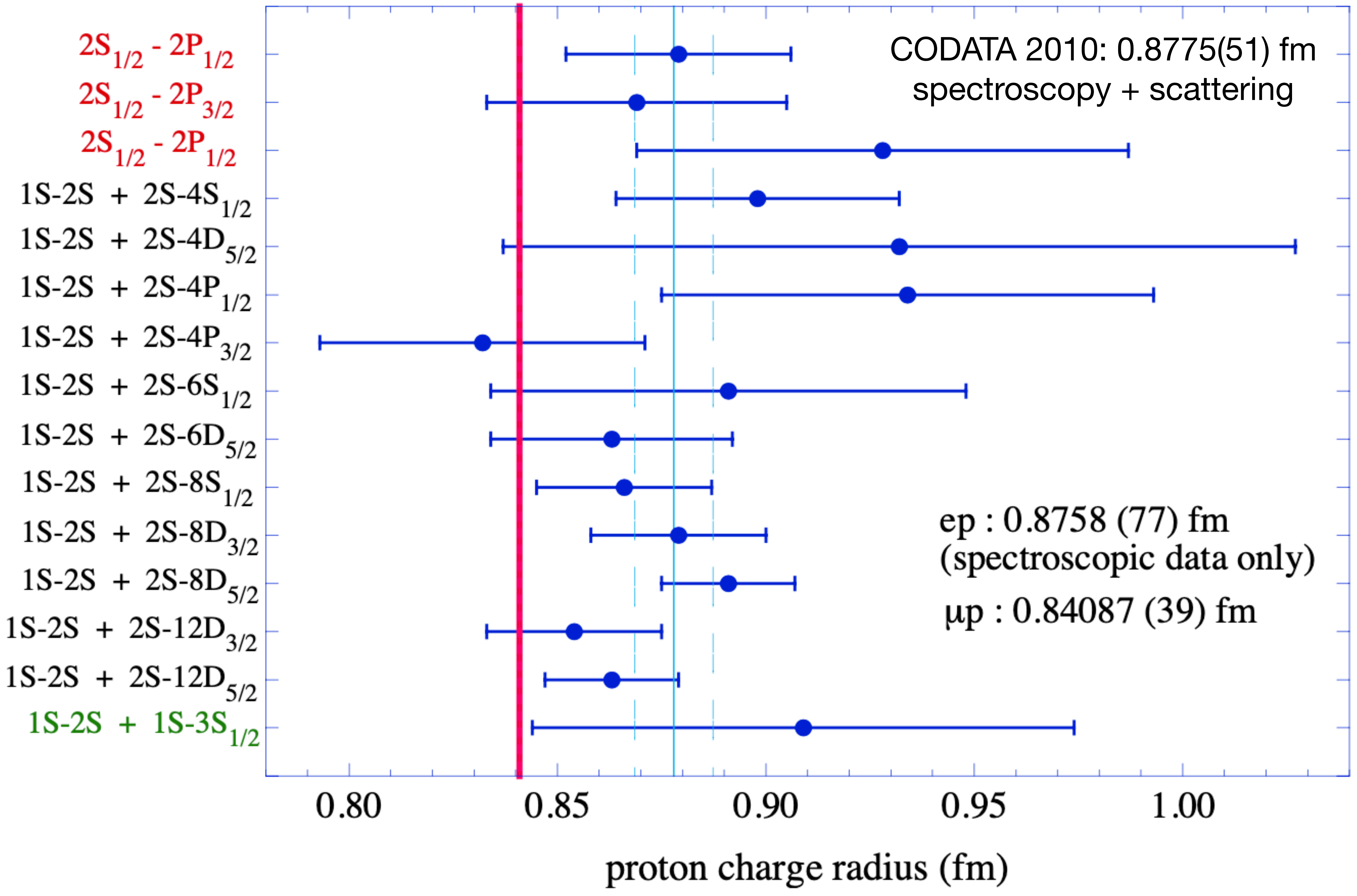


Pohl et al 2010

Proton radius "puzzle"

- "Definition" ~2010: **Electron-proton scattering** and **ordinary H spectroscopy** experiments measure **"large" proton radius**, while **muonic H spectroscopy** experiments measure a **"small" proton radius**

- E.g. r_E measured from **muonic hydrogen** (improved measurement in 2013 from same collaboration as 2010) is **4% smaller than CODATA 2010 value** (spectroscopy + e-p scattering)
    - 7σ **discrepancy** indicates a profound problem
    - The three types of measurements cannot be averaged correctly for the CODATA recommended value



Carlson 2015

Proton radius “puzzle”

- The **discrepancy** in the values of the proton radius as **extracted by the 3 different methods** (e-p scattering, ordinary H spectroscopy, muonic H spectroscopy) seems to “suggest” that electrons and muons might behave differently
- **But** need to keep in mind the following:
    - **Systematics errors**
        - Particularly problematic for early e-p scattering experiments (e.g. normalization accounting for detection efficiency, target purity, luminosity, radiation effects)
    - **Choice of fitting function** for $G_E(Q^2)$
        - Different fitting functions for the same data can lead to different values of the radius
        - Some parameterizations of form factors don’t respect physical constraints (e.g. that the charge density approach 0 at large r)
- What can **more recent results** tell us?

Results from post-2010 measurements

Bezginov et al 2019

- Measurement of the Lamb shift in ordinary hydrogen

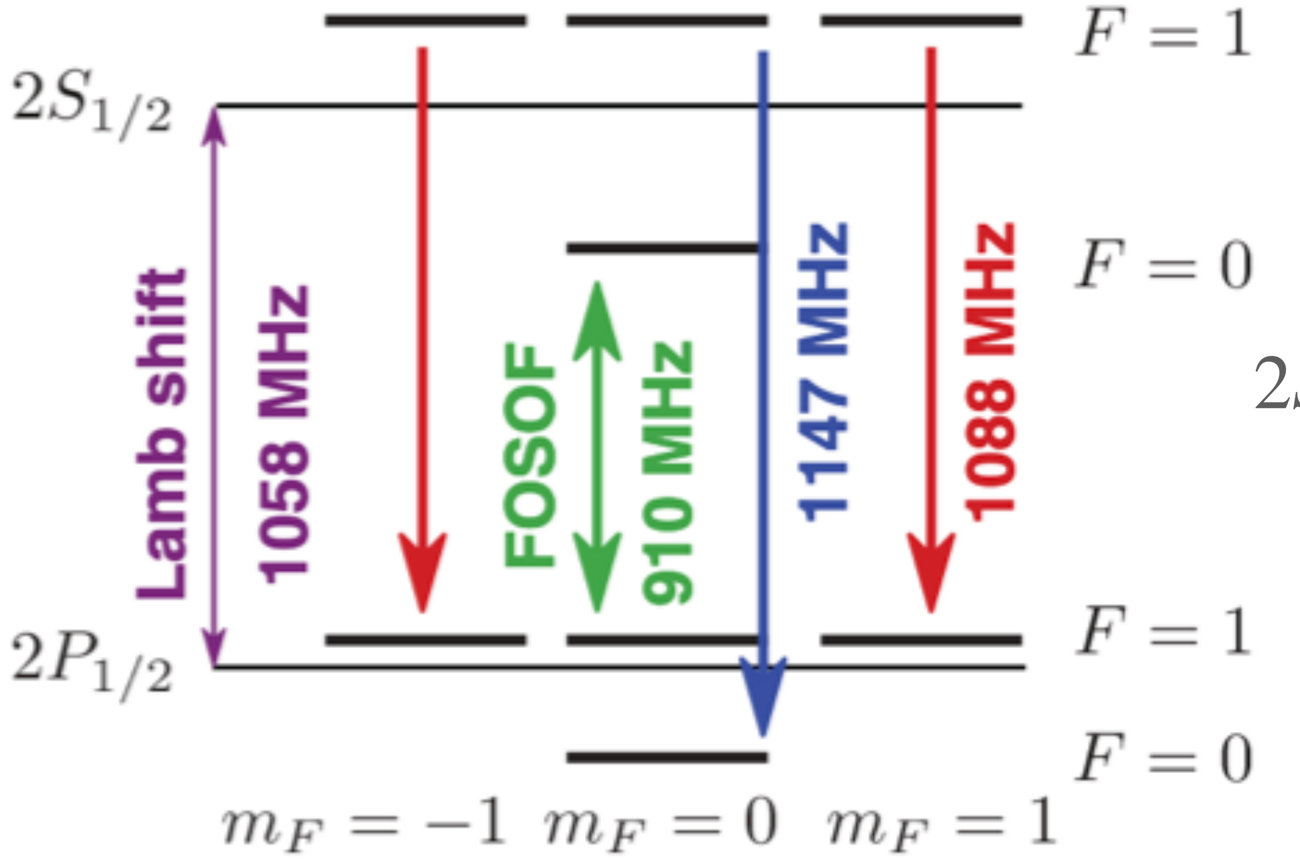
RF cavities (red and blue) remove >99.9% of F=1 atoms

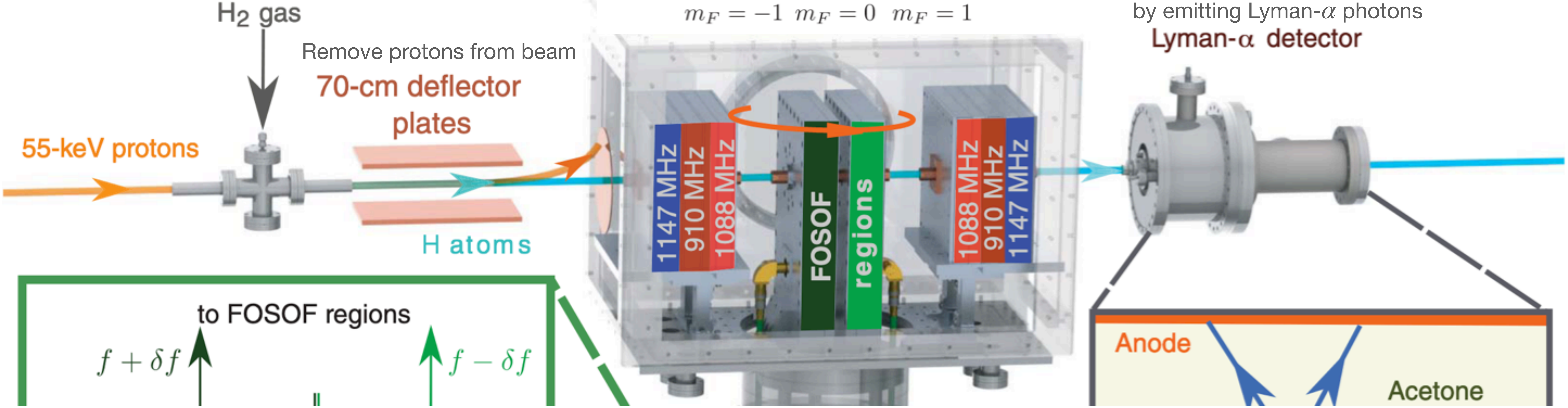
$2S_{1/2}(F=0)$ atoms are driven to $2P_{1/2}(F=1)$ state (green)

~1/2 of protons become H atoms, ~4% of which created in $2S_{1/2}$ state

Remove protons from beam

Surviving $2S_{1/2}$ atoms decay to $1S_{1/2}$ by emitting Lyman-α photons



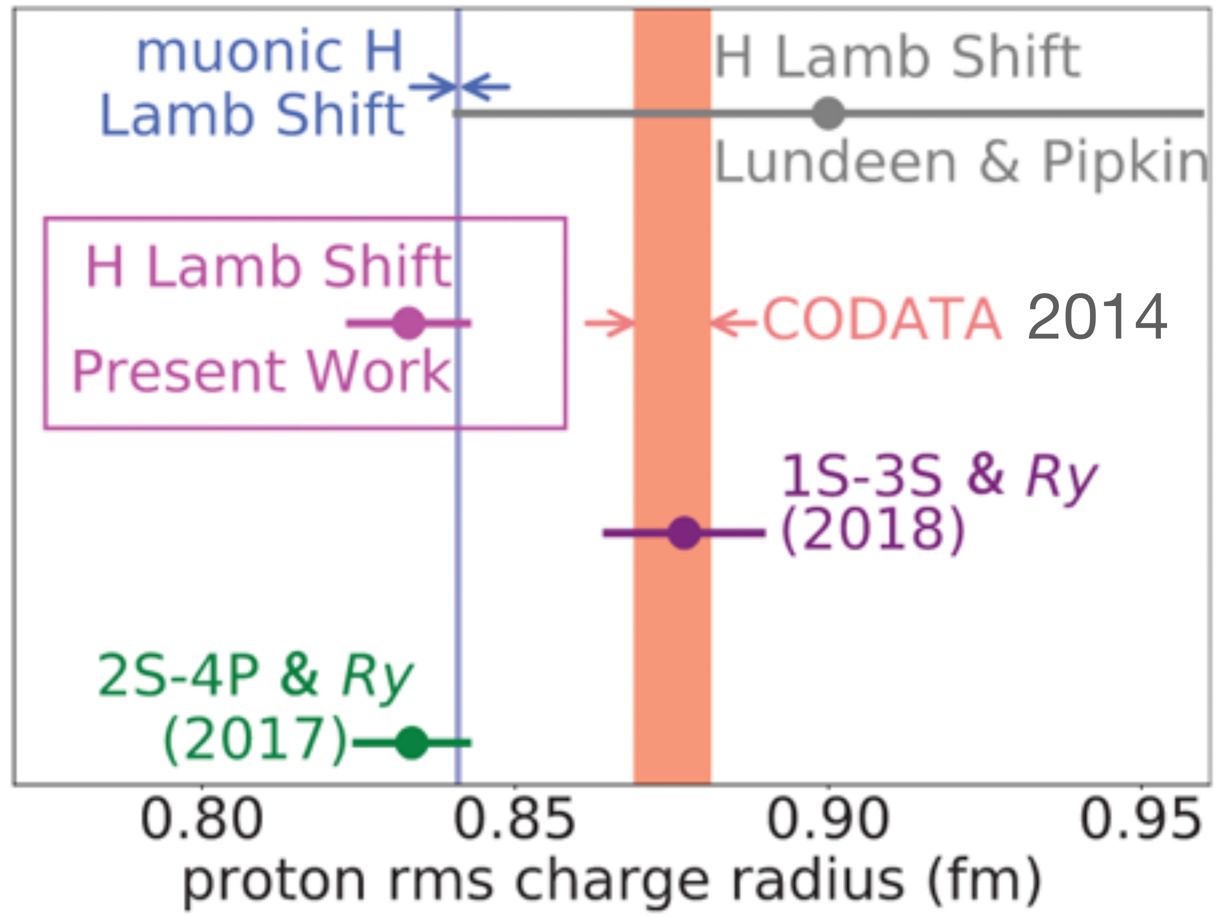


Adapted from Bezginov et al 2019

Results from post-2010 measurements

Bezginov et al 2019 — H spectroscopy

- Measurement of the **Lamb shift in ordinary hydrogen**
- Measure energy difference between $2S_{1/2}$ and $2P_{1/2}$ of **909.8717 MHz** with a **3.2 kHz uncertainty** (1.4 kHZ statistical uncertainty)
  - Lamb shift (including hyperfine contribution) of 1057.8298(32) MHz
- $r_E = 0.833(10)$ fm
  - In **agreement** with **muonic hydrogen Lamb shift** measurement of $r_E = 0.84184(67)$ fm



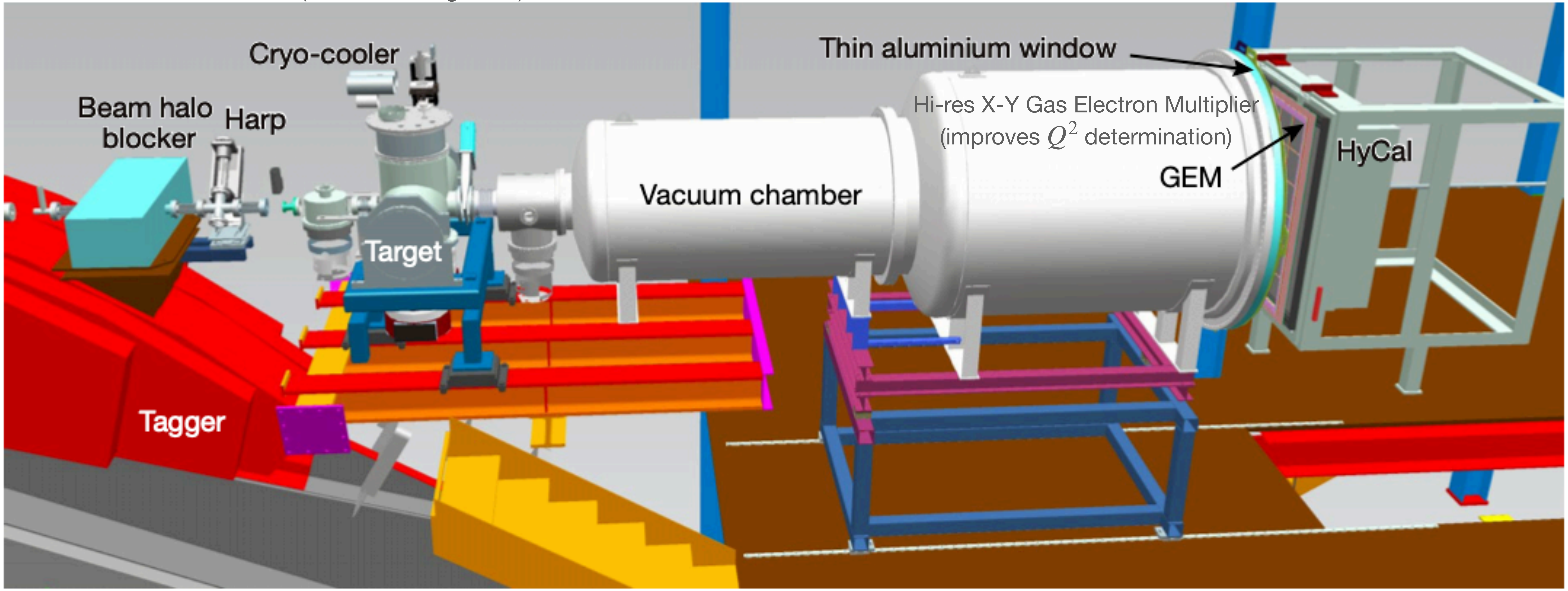
Bezginov et al 2019

Results from post-2010 measurements

Xiong et al 2019 — e-p scattering

Hydrogen gas target
inside cryo-cooler
(reduces background)

Hybrid EM calorimeter
High-resolution, large acceptance ($0.7° - 7.0°$)
Simultaneously measure e-e scattering (luminosity)

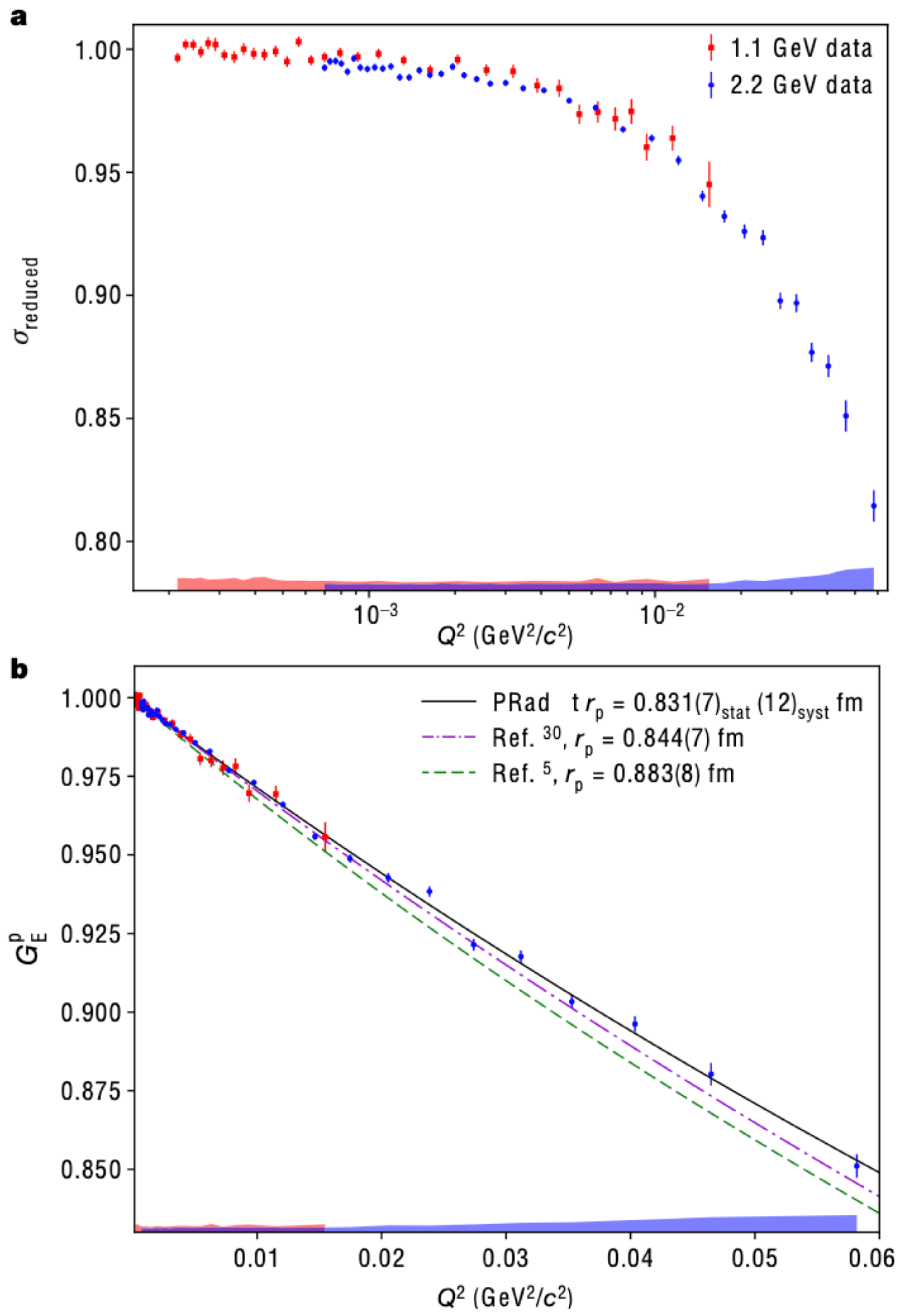


Xiong et al 2019

Results from post-2010 measurements

Xiong et al 2019

- Proton charge radius (**PRad**) experiment at Jefferson Laboratory
- Used **1.1** and **2.2 GeV** electron beams
- Q^2 coverage of 2.1×10^{-4}GeV^2 to 6×10^{-2}GeV^2
- Fit a **Rational(1,1)** to form factor (consistent results with smallest uncertainties) for each data set
  - $f(Q^2) = nG_E^p(Q^2) = n\dfrac{1 + p_1 Q^2}{1 + p_2 Q^2}$
- n is a normalization parameter, which should equal 1
  - Fit values: $n_1 = 1.0002 \pm 0.0002_{stat} \pm 0.0020_{syst}$ and $n_2 = 0.9983 \pm 0.0002_{stat} \pm 0.0013_{syst}$

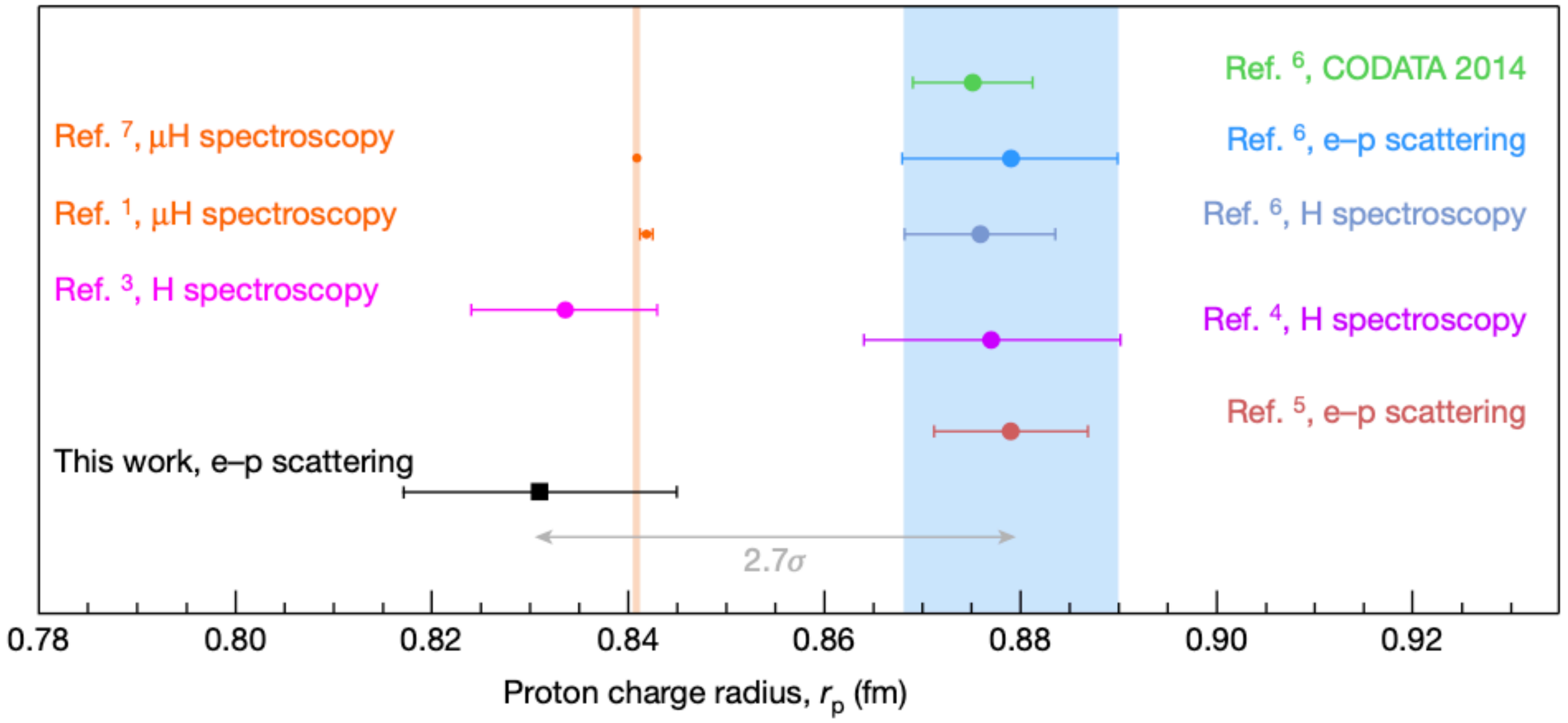


Xiong et al 2019

Results from post-2010 measurements

Xiong et al 2019

- Proton charge radius (**PRad**) experiment at Jefferson Laboratory
- Measure $r_E = 0.831 \pm 0.007_{stat} \pm 0.012_{syst}$ fm
  - 2.7σ **smaller** than average of all previous e-p scattering results
  - **Consistent** with **muonic H spectroscopy** and (some) **newer ordinary H spectroscopy** results

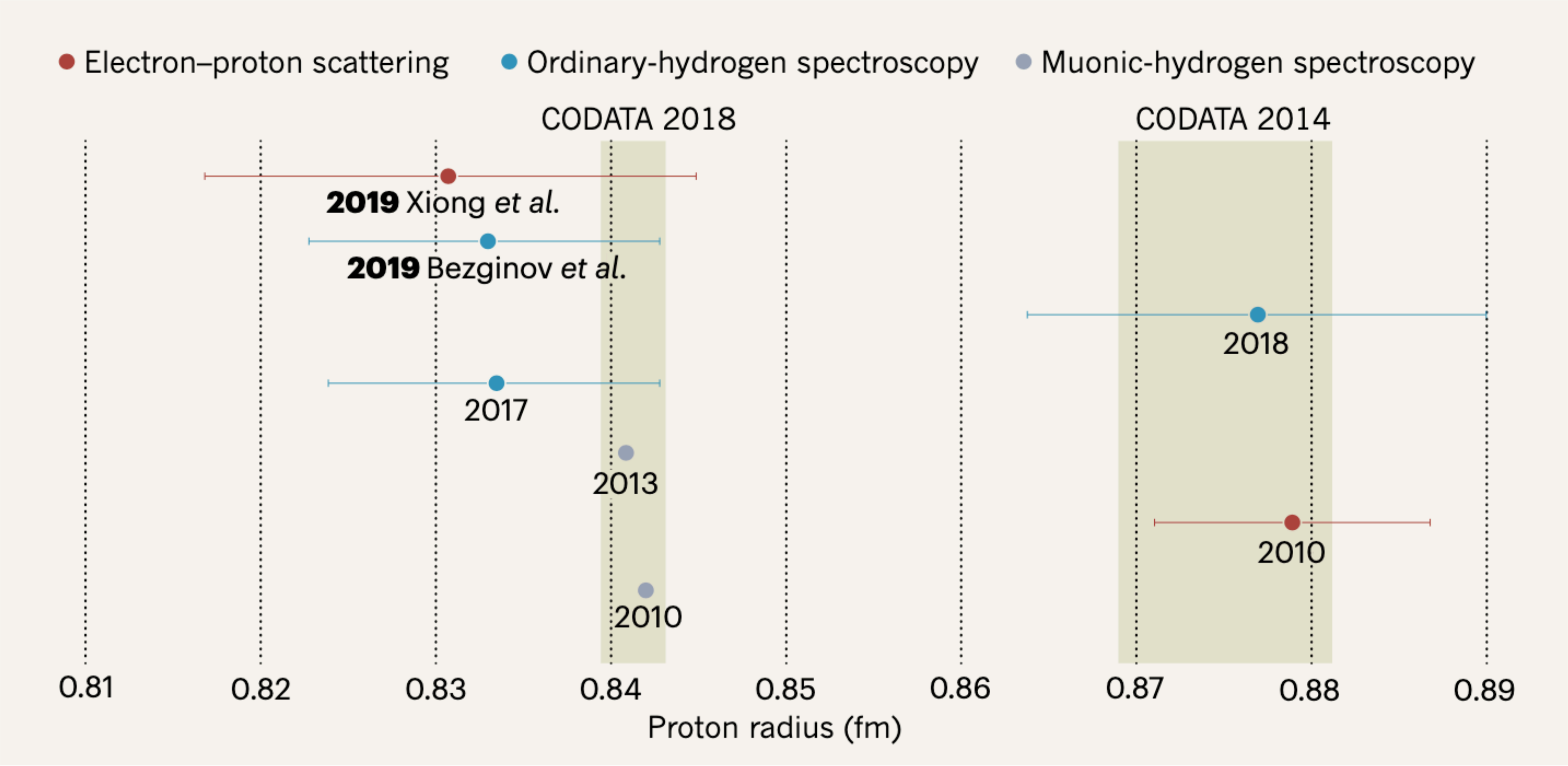


Xiong et al 2019

Proton radius within the last ~10 years

VALUE (fm)	DOCUMENT ID		TECN	COMMENT
0.8409 ± 0.0004 OUR AVERAGE				
0.833 ±0.010	[1] BEZGINOV	2019	LASR	2S-2P transition in H
0.831 ±0.007 ±0.012	[2] XIONG	2019	SPEC	$e\,p \to ep$ form factor
0.84087 ±0.00026 ±0.00029	ANTOGNINI	2013	LASR	μp -atom Lamb shift
• • We do not use the following data for averages, fits, limits, etc. • •				
→ 0.847 ±0.008	[3] CUI	2021	FIT	use existing ep data
0.878 ±0.011 ±0.031	[4] MIHOVILOVIC	2021		ISR $e\,p \to ep$ reanalysis
0.877 ±0.013	[5] FLEURBAEY	2018	LASR	1S-3S transition in H
0.8335 ±0.0095	[6] BEYER	2017	LASR	2S-4P transition in H
0.8751 ±0.0061	MOHR	2016	RVUE	2014 CODATA value
0.895 ±0.014 ±0.014	[7] LEE	2015	SPEC	Just 2010 Mainz data
0.916 ±0.024	LEE	2015	SPEC	World data, no Mainz
0.8775 ±0.0051	MOHR	2012	RVUE	2010 CODATA, ep data
0.875 ±0.008 ±0.006	ZHAN	2011	SPEC	Recoil polarimetry

PDG live website



Karr & Marchand 2019

- Still no clear resolution!
  - **Measurements** (e-p scattering, H spectroscopy) from the last decade **agree with “small” and “large” proton radius values**
  - Need for more measurements (particularly e-p scattering with improved precision)

Future experiments

For comparison, recent e-p scattering result:
Xiong et al 2019 measurement: $r_E = 0.831 \pm 0.014$ fm
(1.7% precision)

- **MUSE** (MUon Scattering Experiment) experiment at PSI
  - Lepton(e/μ) - proton scattering
  - Q^2 coverage from 0.0016 to 0.08 GeV^2
  - Expected **uncertainty on proton radius ~0.01 fm**
- **COMPASS++/AMBER** experiment at CERN
  - Muon-proton scattering, 100 GeV muons
  - Q^2 from 0.001 to 0.04 GeV^2, relative point-to-point precision better than 0.001
  - Proton **radius precision** expected to be **better than 0.01 fm**
- **PRad-II** experiment at Jefferson Lab
  - Electron-proton scattering
  - Q^2 below 10^{-4} GeV^2
  - Projected **uncertainty on radius** smaller than **0.5% (0.0036 fm)**

- Electron scattering experiments at **Mainz**
  - **PRES**
    - Measure recoil protons from e-p scattering
    - Q^2 coverage from 0.001 to 0.04 GeV^2
    - **Radius** measurement with **0.5% statistical precision** and **systematic errors < 0.3%**
  - **MAGIX**
    - Q^2 down to 10^{-4} GeV^2
    - Relative **precision** on electric **form factor** down to **0.05%**
- **Ultra-Low** Q^2 at Tohoku University
  - Electron-proton scattering
  - Q^2 from 0.0003 to 0.008 GeV^2
  - **Precision** of **0.1%** on **cross-section** measurement

Summary

- The **proton radius puzzle** refers to the **disagreement between the value of the proton charge radius extracted from different types of measurements** (elastic electron-proton scattering, ordinary hydrogen spectroscopy, and muonic hydrogen spectroscopy)
    - ~2010: e-p scattering and H spectroscopy results consistent with each other and with a **"large" value of r_E~0.87 fm**, while muonic H spectroscopy experiments measured a **smaller r_E~0.84 fm** (7σ **discrepancy**)
    - More recently, some newer scattering and H spectroscopy results are consistent with "small" radius value
- Opinions on whether the proton radius puzzle can be considered "solved" are split into **two camps**:
    - CODATA 2018 updated the recommended value to r_E^p = 0.8414(19) fm
    - To definitively solve the puzzle, actually need to understand why there are discrepancies between the latest results and data from previous H and e-p experiments
- **Look forward to improved precision results from next-generation experiments!**

References

- **Summary review** papers:
  - Hammer & Meissner, “The proton radius: from a puzzle to precision”, Science Bulletin 65 (2020) 257-258
  - Karr & Marchand, “Progress on the proton-radius puzzle”, Nature 575, 61-62 (2019)
- **Long review** papers:
  - Carlson, “The Proton Radius Puzzle”, arXiv:1502.05314 [hep-ph] (2015)
  - Gao & Vanderhaeghen, “The proton charge radius”, arXiv:2105.00571 [hep-ph] (2021)
  - Khabarova & Kolachevsky, “Proton charge radius”, Physics-Uspekhi 64 1038-1048 (2021)
- **Measurement** papers:
  - Bernauer et al 2010, “High-precision determination of the electric and magnetic form factors of the proton”, Physical Review Letters 105, 242001 (2010)
  - Pohl et al 2010, “The size of the proton”, Nature 466, 213-217 (2010)
  - Bezginov et al 2019, “A measurement of the atomic hydrogen Lamb shift and the proton charge radius”, Science 365, 1007-2023 (2019)
  - Xiong et al 2019, “A small proton charge radius from an electron-proton scattering experiment”, Nature 575, 147-151 (2019)