The Structure of the Proton

Aug 21, 2022

The Proton is Not a Point-like Particle

- Quark model says p consists of 3 quarks
  - Are they real?
- Gyromagnetic moment $g_p = 5.586$ is far from the Dirac value of 2 that holds for pointlike spin-$\frac{1}{2}$ particles
  - Pattern of baryon magnetic moments can be explained using quark model with fraction charges, fitting for quark masses
- Size of nucleus consistent with nucleons of size ~ 0.8 fm

To study structure of the proton, will use scattering techniques

Similar idea to Rutherford's initial discover of the nucleus

- Rutherford Scattering (spinless electron scattering from a static point charge) in lab frame:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4\left(\frac{1}{2}\theta\right)}$$

here E is energy of incident electron and θ is scattering angle in the lab frame

- Mott Scattering: Taking into account statistics of identical spinless particles

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \cos^2\left(\frac{1}{2}\theta\right)}{4E^2 \sin^4\left(\frac{1}{2}\theta\right)}$$

Scattering of Spin-$\frac{1}{2}$ Pointlike Particles

- Elastic Scattering of a spin-$\frac{1}{2}$ electron from a pointlike spin-$\frac{1}{2}$ particle of mass M:
  - Elastic scattering of electron from infinite mass target changes angle but not energy
  - For target of finite mass M, final electron energy is

$$E' = \frac{E}{1 + \frac{2E}{M}\sin^2\left(\frac{1}{2}\theta\right)}$$

    and the four-momentum transfer is

$$q^2 = -4EE'\sin^2\left(\frac{1}{2}\theta\right)$$

The elastic scattering cross section is:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2\cos^2\left(\frac{1}{2}\theta\right)}{4E^2\sin^4\left(\frac{1}{2}\theta\right)}\frac{E'}{E}\left[1 - \frac{q^2}{2M^2}\tan^2\left(\frac{1}{2}\theta\right)\right]$$

What Happens if the Target Particles Have Finite Size?

- Charge distribution $\rho(r)$: $\int \rho(r) d^3r = 1$
- Scattering amplitude modified by a "Form Factor"

$$F(q^2) = \int d^3r e^{i\vec{q}\cdot\vec{r}} \rho(r)$$

  So that the cross section is modified by a factor of $|F(q^2)|^2$

- Note: As $q^2 \to 0$, $F(q^2) \to 1$
- We therefore can Taylor expand

$$F(q^2) = \int d^3r \left(1 + i\vec{q}\cdot\vec{r} - \frac{1}{2}(\vec{q}\cdot\vec{r})^2 + ...\right) \rho(r)$$

Form Factors

- The first $\vec{q} \cdot \vec{r}$ term vanishes when we integrate

$$\begin{aligned}
F(q^2) &= 1 - \frac{1}{2} \int r^2 dr d\cos\theta d\phi \; \rho(r) (qr)^2 \cos^2\theta \\
&= 1 - \frac{2\pi}{2} \int dr d\cos\theta \; q^2 r^4 \cos^2\theta \\
&= 1 - \frac{< r^2 >}{4} q^2 \int \cos^2\theta \; d\cos\theta \\
&= 1 - \frac{< r^2 >}{4} q^2 \left[\frac{\cos^3\theta}{3} \right]_{-1}^{1} \\
&= 1 - \frac{< r^2 >}{6} q^2
\end{aligned}$$

- For elastic scattering, can relate q to the outgoing angle

$$q = \frac{2p \sin(\theta/2)}{\left[1 + (2E/M_p)\sin^2(\theta/2)\right]^{\frac{1}{2}}}$$

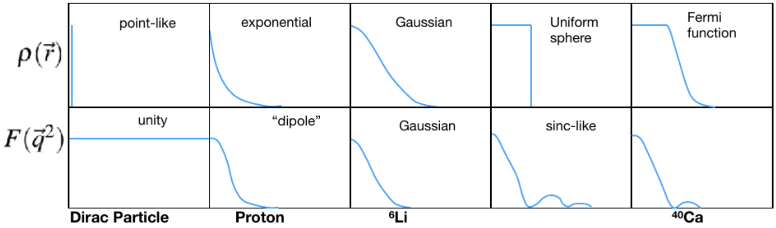
where p and E are the momentum and energy of the incident electron in the lab frame

Interpreting Form Factors

- If proton is not pointlike cross section modified

$$\frac{d\sigma}{d\Omega} \longrightarrow \left(\frac{d\sigma}{d\Omega}\right)_{pointlike} |F(q^2)|^2$$

- Finite size of scattering center introduces a phase difference between plane waves scattered from different points in space

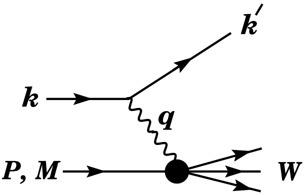


From Thomson

What is the proton made of?

- Is the proton a soft mush or does it have hard composite objects inside?
- Need a high energy probe to resolve distances well below proton size
- Elastic cross section falls rapidly with q^2
- Inelastic cross section where proton breaks dominates rate at large q^2
  - "Deep inelastic scattering" (DIS)
- Study energy and angle of outgoing electron
  - For inelastic scattering these are independent variables (subject to kinematic bounds of energy and momentum conservation)

Deep Inelastic Scattering: Kinematics



- W is the invariant mass of the hadronic system
- In lab frame: $P = (M, 0)$
- In any frame, $k = k' + q$, $W = p + q$
- Invariants of the problem:

$$\begin{aligned} Q^2 &= -q^2 = -(k - k')^2 \\ &= 2EE'(1 - \cos\theta) \ \ \text{[in lab]} \\ P \cdot q &= P \cdot (k - k') \\ &= M(E - E') \ \ \text{[in lab]} \end{aligned}$$

- Define $\nu \equiv E - E'$ (in lab frame) so $P \cdot q = m\nu$ and

$$\begin{aligned} W^2 &= (P + q)^2 \\ &= (P - Q)^2 \\ &= M^2 + 2P \cdot q - Q^2 \\ &= M^2 + 2M\nu - Q^2 \end{aligned}$$

where $Q^2 = -q^2$

- Elastic scattering corresponds to $W^2 = P^2 = M^2$
  - $Q^2 = 2M\nu$ elastic scattering
- We can define 2 indep dimensionless parameters

$$\begin{aligned} x &\equiv Q^2/2M\nu; \quad (0 < x \leq 1) \\ y &\equiv \frac{P \cdot q}{P \cdot k} = 1 - E'/E; \quad (0 < y \leq 1) \end{aligned}$$

The Most General Form of the Interaction

- Express cross section
$$d\sigma = L^{e}_{\mu\nu} W^{\mu\nu}$$
where W describes the proton current (allowing substructure)
- Most general Lorentz invarient form of $W^{\mu\nu}$
  - Constructed from $g^{\mu\nu}$, p^{μ} and q^{μ}
  - Symmetric under interchange of μ and ν (otherwise vanishes when contracted with $L_{\mu\nu}$)

$$W^{\mu\nu} = -W_1 g^{\mu\nu} + \frac{W_2}{M^2} p^{\mu} p^{\nu} + \frac{W_4}{M^2} q^{\mu} q^{\nu} + \frac{W_5}{M^2} (p^{\mu} q^{\nu} + p^{\nu} q^{\mu})$$

- W_3 reserved for parity violating term (needed for ν scattering)
- Not all 4 terms are independent. Using $\partial_{\mu} J^{\mu} = 0$ can show

$$\begin{aligned}
W_5 &= -\frac{p \cdot q}{q^2} W_2 \\
W_4 &= \frac{p \cdot q}{q^2} W_2 + \frac{M^2}{q^2} W_1 \\
W^{\mu\nu} &= W_1 \left(-g^{\mu\nu} + \frac{q^{\mu} q^{\nu}}{q^2}\right) + W_2 \frac{1}{M^2} \left(p^{\mu} - \frac{p \cdot q}{q^2} q^{\mu}\right)\left(p^{\nu} - \frac{p \cdot q}{q^2} q^{\nu}\right)
\end{aligned}$$

Structure Functions

- Using notation from previous page, we can express the x-section for DIS

$$\frac{d\sigma}{d\Omega dE'} = \frac{\alpha^2}{4E^2}\frac{\cos^2(\frac{1}{2}\theta)}{\sin^4(\frac{1}{2}\theta)}\left[W_2(q^2,W) + 2W_1(q^2,W)\tan^2(\frac{1}{2}\theta)\right]$$

- These are the same two terms as for the elastic scattering
- W_1 and W_2 care called the *structure functions*
  - Angular dependence here comes from expressing covariant form on last page in lab frame variables
  - Two structure functions that each depend on Q^2 and W
  - Alternatively, can parameterize wrt dimensionless variables:

$$\begin{aligned} x &\equiv Q^2/2M\nu \\ y &\equiv \frac{P\cdot q}{P\cdot k} = 1 - E'/E \end{aligned}$$

Structure Functions in terms of Lorentz Invariants

- Change variables

$$\begin{aligned} F_1(x,Q^2) &\equiv MW_1(\nu,Q^2) \\ F_2(x,Q^2) &\equiv \nu W_2(\nu,Q^2) \end{aligned}$$

- Rewrite cross section in terms of x, y, Q^2

$$\frac{d^2\sigma^{ep}}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1-y-\frac{M^2y^2}{Q^2}\right)\frac{F_2(x,Q^2)}{x} + y^2F_1(x,Q^2)\right]$$

- In DIS limit, $Q^2 >> M^2y^2$:

$$\frac{d^2\sigma^{ep}}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2(x,Q^2)}{x} + y^2F_1(x,Q^2)\right]$$

- Can event-by-event determine x, y and Q^2 from lab frame variables

$$Q^2 = 4EE'\sin^2\frac{\theta}{2}, \quad x = \frac{Q^2}{2M(E-E')} = \frac{Q^2}{2M\nu} =, \quad y = 1-\frac{E'}{E}$$

The Parton Model

- Supposed there are pointlike partons inside the nucleon
- Work in an "infinite momentum" frame: ignore mass effects
- Proton 4-momentum: $P = (P, 0, 0, P)$
- Visualize stream of parallel partons each with 4-momentum xP where $0 < x < 1$; neglect transverse motion of the partons
  - x is the fraction of the proton's momentum that the parton carries
- If electron elastically scatters from a parton

$$
\begin{aligned}
(xP + q)^2 = m^2 &\simeq 0 \\
x^2 P^2 + 2xP \cdot q + q^2 &= 0
\end{aligned}
$$

Since $P^2 = M^2$, if $x^2 M^2 << q^2$ then

$$
\begin{aligned}
2xP \cdot q &= -q^2 = Q^2 \\
x &= \frac{Q^2}{2P \cdot q} = \frac{q^2}{2M\nu}
\end{aligned}
$$

Deep inelastic scattering can be described as elastic scattering of the lepton with a parton with momentum xP

Electron Quark Scattering

- Quarks are Dirac particles, so can just calculate the scattering in QED
- We won't do the calculation here (see Thomson p. 191). Answer is

$$\frac{d\sigma^{eq}}{dQ^2} = \frac{4\pi e_i^2}{Q^4}\left[(1-y) + \frac{y^2}{2}\right]$$

- Looks pretty similar to the previous page

$$\frac{d^2\sigma^{ep}}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\,\frac{F_2(x,Q^2)}{x} + y^2 F_1(x,Q^2)\right]$$

- The $F_1(x,Q^2)$ and $F_2(x,Q^2)$ carry the information about the distribution of the quarks inside the proton
- Note: $F_1(x,Q^2)$ is due to the magnetic (spin-spin) interaction while $F_2(x,Q^2)$ is the electric (Coulomb) interaction
    - If partons are Dirac particles, we expect a well defined relationship between these two terms

Convolution of PDF with scattering cross section

- Cross section is incoherent sum over elastic scattering with partons

$$
\begin{aligned}
\frac{d\sigma^{eq}}{dQ^2} &= \frac{4\pi\alpha^2}{Q^4}\left[(1-y)+\frac{y}{2}\right] \\
\frac{d\sigma^{ep}}{dxdQ^2} &= \int_0^1 dx \sum_i e_i^2 f_i(x,Q^2)\frac{4\pi\alpha^2}{Q^4}\left[(1-y)+\frac{y^2}{2}\right]\delta(x-\frac{Q^2}{2M\nu})
\end{aligned}
$$

- Comparing to the previous expression for ep scattering

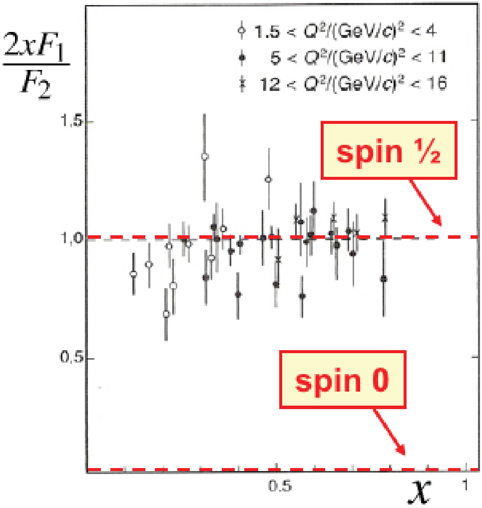
$$
\frac{d^2\sigma^{ep}}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\,\frac{F_2(x,Q^2)}{x} + y^2 F_1(x,Q^2)\right]
$$

We find

$$
\begin{aligned}
F_2^{ep}(x,Q^2) &= x\sum_i e_i^2 f_i(x,Q^2) \\
F_1^{ep}(x,Q^2) &= \frac{1}{2}\sum_i e_i^2 f_i(x,Q^2) \\
\therefore F_2^{ep}(x,Q^2) &= 2xF_1^{ep}(x,Q^2)
\end{aligned}
$$

- Last equation is called the Callan-Gross relation
- If partons had spin-0 rather than spin-$\frac{1}{2}$, we would have found $F_1 = 0$

What does the data look like?



The partons act like spin-1/2 Dirac particles!

Some Observations (I)

- $f_i(x)$ is the prob of finding a parton of species i with mom fraction between x and $x + dx$ in the proton.
- If the partons together carry all the momentum of the proton

$$\int dx \, x f(x) = \int dx \, x \sum_i f_i(x) = 1$$

  where $\sum_i$ is a sum over *all* species of partons in the proton
- We call $f(x)$ the parton distribution function since it tells us the momentum distribution of the parton within the proton
- This is the first example of a "sum rule"

Some Observations (II)

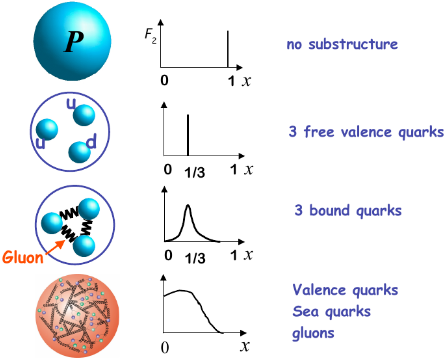
- It's natural to associate the partons with quarks, but that's not the whole story
- Because ep scattering occurs through the electromagnetic interaction, it only occurs via scattering with charged partons.
- If the proton also contains neutral partons, the EM scattering won't "see" them
  - For example: EM scattering blind to gluons
- Let's assume that the ep scattering occurs through the scattering of the e off a quark or antiquark
  - We saw that the SU(3) description of the proton consists of 2 u and 1 d quark.
  - However we can in addition have any number of $q\overline{q}$ pairs without changing the proton's quantum numbers
  - The 3 quarks (uud) are called *valence quarks*. The additional $q\overline{q}$ pairs are called *sea* or *ocean* quarks.
    - Pair production of $q\overline{q}$ pairs within the proton

Another Sum Rule

- To get the right quark content for the proton:

$$\begin{aligned}
\int u(x) - \overline{u}(x)dx &= 2 \\
\int d(x) - \overline{d}(x)dx &= 1 \\
\int s(x) - \overline{s}(x)dx &= 0
\end{aligned}$$

If partons are quarks, what do we expect?



Max Klein, CTEQ School Rhodos 2006

- Elastic scattering from proton has $x = 1$
- If 3 quarks carry all the proton's momentum each has $x = 0.3$
- Interactions among quarks smears $f(x)$
- Radiation of gluons softens distribution and adds $q\bar{q}$ pairs
  - Describe the 3 original quarks as "valence quarks"
  - $q\bar{q}$ pairs as sea or ocean
- Some of proton's momentum carried by gluons and not quarks or antiquarks

Using Isospin: Comparing the Proton and Neutron

- Ignore heavy quark content in the proton: consider only u, d, s
- Write the proton Structure Function

$$\frac{F_2^p(x)}{x} = \sum_i f_i^p(x) e_i^2 = \frac{4}{9}(u^p(x) + \overline{u}^p(x)) + \frac{1}{9}(d^p(x) + \overline{d}^p(x)) + \frac{1}{9}(s^p(x) + \overline{s}^p(x))$$

- Similarly, for the neutron

$$\frac{F_2^n(x)}{x} = \sum_i f_i^n(x) e_i^2 = \frac{4}{9}(u^n(x) + \overline{u}^n(x)) + \frac{1}{9}(d^n(x) + \overline{d}^n(x)) + \frac{1}{9}(s^n(x) + \overline{s}^n(x))$$

- But isospin invariance tells us that $u^p(x) = d^n(x)$ and $d^p(x) = u^n(x)$
- Write F_2 for the neutron in terms of the proton pdf's (assuming same strange content for the proton and neutron)

$$\frac{F_2^n(x)}{x} = \frac{4}{9}(d^p(x) + \overline{d}^p(x)) + \frac{1}{9}(u^p(x) + \overline{u}^p(x)) + \frac{1}{9}(s^p(x) + \overline{s}^p(x))$$

- Assuming sea q and $\overline{q}$ distributions are the same:

$$u(x) - \overline{u}(x) = u_v(x), \quad d(x) - \overline{d}(x) = d_v(x), \quad s(x) - \overline{s}(x) = 0$$

- Taking the difference in F_2 for protons and neutrons:

$$\frac{1}{x}[F_2^p(x) - F_2^n(x)] = \frac{1}{3}[u_v(x) - d_v(x)]$$

which gives us a feel for the valence quark distribution

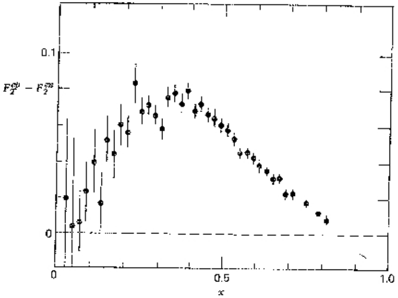
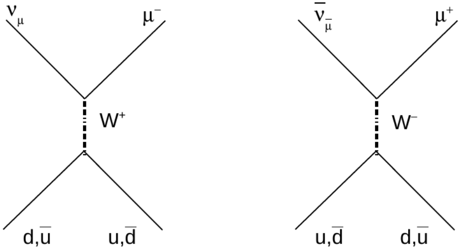


Fig. 9.8 The difference $F_2^{ep} - F_2^{en}$ as a function of x, as measured in deep inelastic scattering. Data are from the Stanford Linear Accelerator.

From Halzen and Martin

- Looks the way we expect from the cartoon on page 27
- Next question: How to measure the partons' charge
  - To do this, must compare e and ν scattering!

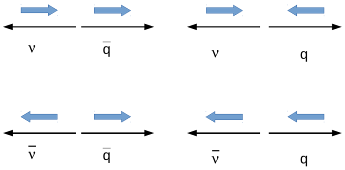
Neutrino-(anti)quark Charged Current Scattering (I)



- Start with ν_μ or $\overline{\nu}_\mu$ beam
  - Distribution of ingoing ν 4-momenta determined from beam design
  - Outgoing $\mu^\pm$ momentum measured in spectrometer
- Exchange via $W^\pm$ ("charged current interaction")
  - ν scatter against d and $\overline{u}$
  - $\overline{\nu}$ scatter against u and $\overline{d}$

Neutrino-(anti)quark Scattering (II)

- Neutrinos left handed, anti-neutrinos right handed
- Left handed $W^{\pm}$ couples to left-handed quarks and right-handed anti-quarks



- νq and $\overline{\nu}\,\overline{q}$ scattering allowed for all angles, but $\overline{\nu} q$ and $\nu\overline{q}$ vanish in backward direction

$$\frac{d\sigma^{\nu q}}{d\cos\theta} \propto \text{constant} \qquad \frac{d\sigma^{\overline{\nu} q}}{d\cos\theta} \propto (1+\cos\theta^*)^2$$

where θ^* is scattering angle in νq center of mass

Neutrino-(anti)quark Scattering (III)

- The charged current cross sections are:

$$\begin{aligned}
\frac{d\sigma(\nu_\mu\ d \to \mu^- u)}{d\Omega} &= \frac{G_F^2 s}{4\pi^2} \\
\frac{d\sigma(\overline{\nu_\mu}\ u \to \mu^+ d)}{d\Omega} &= \frac{G_F^2 s}{4\pi^2}\,\frac{(1+\cos\theta)^2}{4} \\
\frac{d\sigma(\nu_\mu\ \overline{u} \to \mu^- \overline{d})}{d\Omega} &= \frac{G_F^2 s}{4\pi^2}\,\frac{(1+\cos\theta)^2}{4} \\
\frac{d\sigma(\overline{\nu_\mu}\ \overline{d} \to \mu^+ \overline{u})}{d\Omega} &= \frac{G_F^2 s}{4\pi^2}
\end{aligned}$$

- Also, one can prove that:

$$1 - y = \frac{p \cdot k'}{p \cdot k} = \frac{1}{2}\left(1 + \cos\theta^*\right)$$

which allows us to rewrite the above expressions in terms of the relativistically invariant variable y

- Since $\frac{1}{4}\int (1+\cos\theta)^2 d\cos\theta = 1/3$,

$$\begin{array}{ccccc}
\sigma^{\nu d} : & \sigma^{\nu\overline{u}} : & \sigma^{\overline{\nu}u} : & \sigma^{\overline{\nu}\overline{d}} & = \\
1 : & \frac{1}{3} : & \frac{1}{3} : & 1 &
\end{array}$$

What is the Advantage of ν Scattering?

- The quarks and antiquarks have different angular dependence, so we can extract their pdf's separately by looking at cross sections as a function of angle
  - Angular dependence can be expressed in terms of dimensionless variable y
  - Parity violation means we have a third structure function F_3 that I won't talk about today
- Weak "charge" of the u and d is the same, so factors of $4/9$ and $1/9$ are not present
- Using previous expressions and integrating over angle:

$$
\begin{aligned}
\frac{d\sigma(\nu\, p)}{dx} &= \frac{G_F^2 xs}{\pi}\left[d(x) + \frac{1}{3}\overline{u}(x)\right] \\
\frac{d\sigma(\nu\, n)}{dx} &= \frac{G_F^2 xs}{\pi}\left[d^n(x) + \frac{1}{3}\overline{u}^n(x)\right] \\
&= \frac{G_F^2 xs}{\pi}\left[u(x) + \frac{1}{3}\overline{d}(x)\right]
\end{aligned}
$$

where we have written everything in terms of the proton PDFs

- If we believe the partons in the proton and neutron are quarks, we can relate the structure functions measured in νN and eN

Comparing eN and νN νN Scattering (I)

- Now, let's take an isoscalar target N (equal number of protons and neutrons)
- In analogy with electron scattering

$$\frac{F_2^{\nu N}}{x} = u(x) + d(x) + \overline{u}(x) + \overline{d}(x)$$

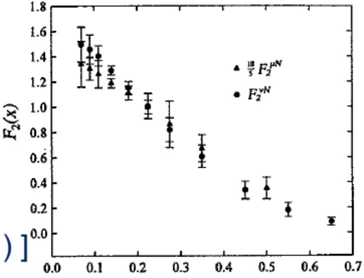
- If we go back to our electron scattering and also require an isoscalar target

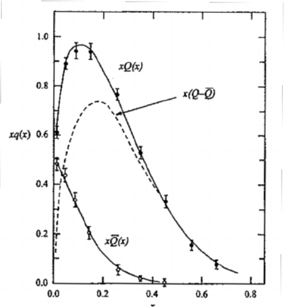
$$\frac{F_2^{e\ N}}{x} = \frac{5}{18}\left(u(x) + d(x) + \overline{u}(x) + \overline{d}(x)\right)$$

- So, if the partons have the charges we expect from the quark model

$$F_2^{e\ N}(x) = \frac{5}{18} F_2^{\nu N}(x)$$

Comparing eN and νN νN Scattering (II)





- The partons we “see” in eN scattering are the same as the ones we “see” in νN scattering
- This confirms our assignment of the quark charges:

  The Quarks Have Fractional Charge!

Using νN scattering to Count Quarks and Antiquarks

- As we previously did for electron scattering, we can look at an isoscalar target N
- Starting with the cross sections for νq scattering we can go through the same convolution with the PDFs that we did for the eN case
- The result is

$$\sigma^{\nu N} = \frac{G_F 2ME}{2\pi}\left[Q + \frac{1}{3}\overline{Q}\right]$$

$$\sigma^{\overline{n}uN} = \frac{G_F 2ME}{2\pi}\left[\overline{Q} + \frac{1}{3}Q\right]$$

where

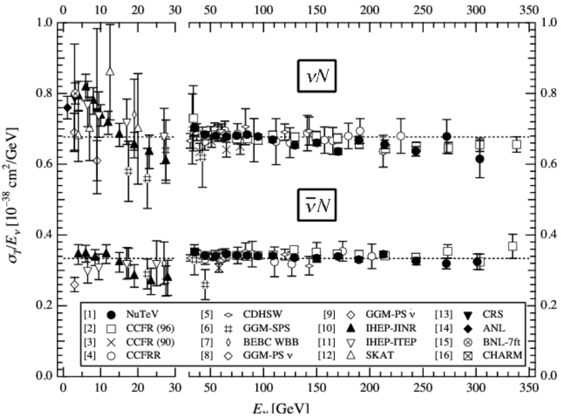
$$\begin{aligned} Q &\equiv \int x[u(x) + d(x)] \\ \overline{Q} &\equiv \int x[\overline{u}(x) + \overline{d}(x)] \end{aligned}$$

and we have ignored the small strange component in the nucleon

- Thus

$$R_{\nu/\overline{\nu}} \equiv \frac{\sigma^{\overline{\nu}N}}{\sigma^{\nu N}} = \frac{\overline{Q} + Q/3}{Q + \overline{Q}/3} = \frac{1 + 3\overline{Q}/Q}{3 + \overline{Q}/Q}$$

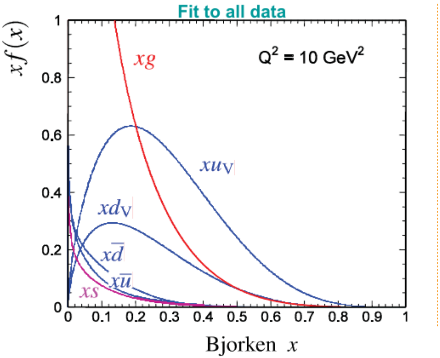
Experimental Measurements of νN Scattering



- Experimentally $R_{\nu/\overline{\nu}} = 0.45 \rightarrow \overline{Q}/Q = 0.5$

There are antiquarks within the proton!

How Much Momentum do the q and $\overline{q}$ Carry?



- Momentum fraction that the q and $\overline{q}$ together carry is

$$\int xF_2^{\nu N}(x)dx = \frac{18}{5}\int xF_2^{eN}(x)dx$$

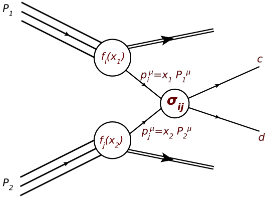
- At $q^2 \sim 10$ GeV^2 that this fraction ~ 0.5

  Only half the momentum of the proton is carried by quarks and antiquarks

- What's Left? The gluon!

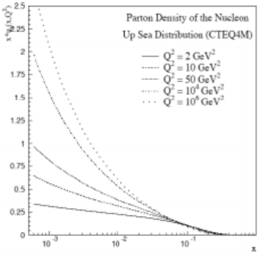
Some Comments

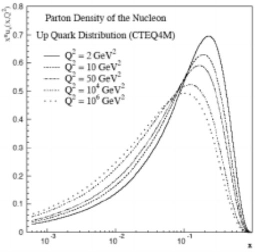
- Charged lepton probes study charged partons
- Neutrinos study all partons with weak charge
  - $\int xF_2^{\nu N}(x)dx = \frac{18}{5}\int xF_2^{eN}(x)dx$ tells us that all the weakly interacting partons are charged
- To study the gluon directly, will need a strong probe
  - No pointlike strong probes
  - Will need to convolute two pdf's

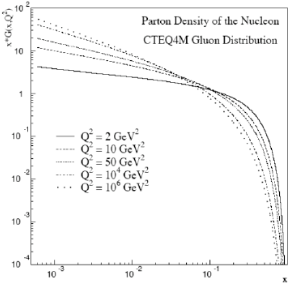


- Can also indirectly study gluons by seeing how they affects the quark distributions

- QCD corrections to DIS come from incorporating gluon brem from the q and $\overline{q}$ and pair production $g \rightarrow q\overline{q}$
- The ability to resolve these QCD corrections are q^2 dependent
  - Higher q^2: can disguish $q + g$ from q
- Effect on observed PDFs:
  - At high x the quark PDFs decrease
  - At low x the quark and antiquark PDFs increase
- Complete treatment in QCD via coupled set of differential equations, the Alterelli-Parisi evolution equations

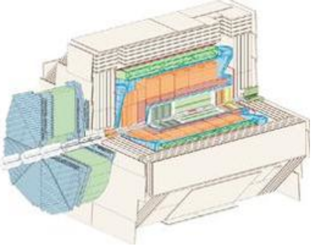




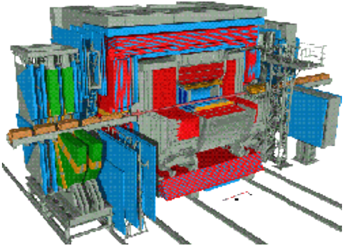


DIS in the Modern Era: The HERA collider

H1

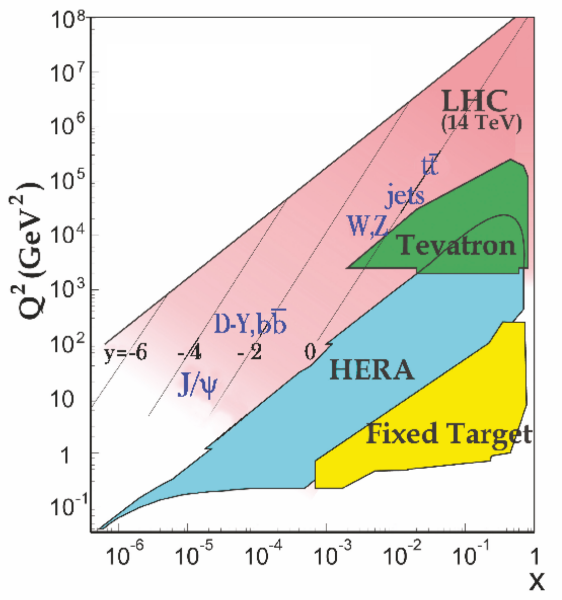


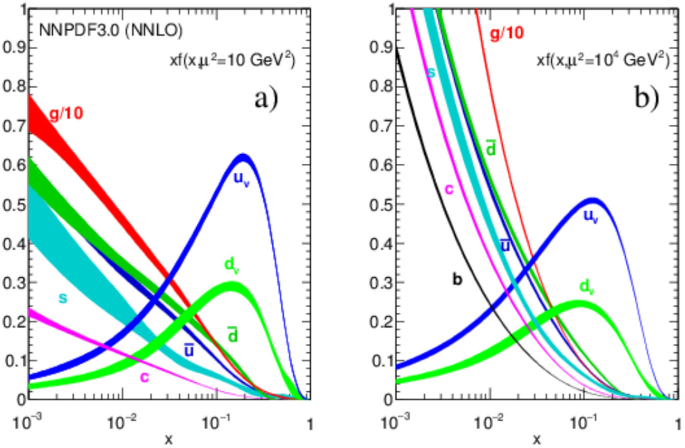
Zeus



- ep collider located at DESY lab in Hamburg
- 27.5 Gev (e) x 920 GeV (p)
- Two general purpose detectors (H1 and Zeus)

What Q^2 and x are relevant?





- Fit experimental data to theoretically motivated parameterizations
- Combine data from many experiments, using Alterelli-Parisi to account for differences in Q^2 (correct to common value)
- Analysis of uncertainties to provide a systematic uncertainty band

Modern $F_2(x, Q^2)$ Measurements