Physics 290e: lectroweak Interactions
The CKM Matrix and CP Violation

M. Shapiro

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Matter-Antimatter Asymmetry of the Universe

- The universe is made largely of matter with very little antimatter

$$\frac{n_B - n_{\overline{B}}}{n_\gamma} \sim 10^{-9}$$

Why is this the case?

- Matter dominance occured during early evolution of the Universe
- Assume Big Bang produces equal numbers of B and $\overline{B}$
- At high temperature, baryons in thermal equilibrium with photons

$$\gamma + \gamma \leftrightarrow p + \overline{p}$$

- Temperature and mean energy of photons decrease as Universe expands
  - Forward reaction ceases
  - Baryon density becomes low and thus backward reaction rare
  - Number of B and $\overline{B}$ becomes fixed

    "Big-Bang" baryogenesis
- Need a mechanism to explain the observed matter-antimatter asymmetry

The Sakharov Conditions

- Sakharov (1967) showed that 3 conditions needed for a baryon dominated Universe
  1. A least one B-number violating process so $N_B - N_{\overline{B}}$ is not constant
  2. C and CP violation (otherwise, for every reaction giving more B there would be one giving more $\overline{B}$)
  3. Deviation from thermal equilibrium (otherwise, each reaction would be balanced by inverse reaction)
- Is this possible?
  - Options exist for $\#1$
  - $\#3$ will occur during phase transitions as temperature falls below mass of relevant particles (bubbles)
  - $\#2$ is the subject of today's lecture:
    - Studies of CP violation in the neutral kaon system
    - Observation of CP violation in B decays (2001) and searches for CP violation outside the SM using B decays

Reminder: Neutal Kaons: Strong Basis vs Mass Basis

- Flavor (K^0, $\overline{K}^0$) and mass eigenstates (K_S, K_L) not the same
- If CP were a good symmetry, mass eigenstates would be

$$|K_1\rangle = \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle + \left|\overline{K}^0\right\rangle\right) \qquad CP\,|K_1\rangle = |K_2\rangle$$
$$|K_2\rangle = \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle - \left|\overline{K}^0\right\rangle\right) \qquad CP\,|K_2\rangle = -\,|K_1\rangle$$

- Associating the CP states with the decays:

$$|K_1\rangle \to 2\pi$$
$$|K_2\rangle \to 3\pi$$

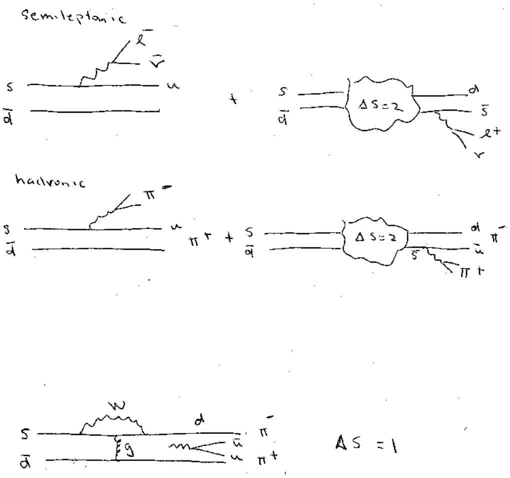
- However, very little phase space for 3π decay: Lifetime of $|K_2\rangle$ much longer than of $|K_1\rangle$
- Physical states (after including CP violations) called "K long" and "K short":

$$\tau(K_S) = 0.9 \times 10^{-10}\ \text{sec}$$
$$\tau(K_L) = 0.5 \times 10^{-7}\ \text{sec}$$

The Kaon Decay Observables

- CP Violation first observed in Kaon system in 1964
- Because Kaon mass low, only 3 observables
  - $|\eta_{+-}| \equiv \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)}$
  - $|\eta_{00}| \equiv \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)}$
  - $\delta_\ell = \frac{\Gamma(K_L \to \pi^-\ell^+\nu_\ell) - \Gamma(K_L \to \pi^+\ell^-\overline{\nu}_\ell)}{\Gamma(K_L \to \pi^-\ell^+\nu_\ell) + \Gamma(K_L \to \pi^+\ell^-\overline{\nu}_\ell)}$
- Initial discussions of CK violation in kaon system preceeded CKM model
  - Language often arcane
- Observables depend on strong interaction dynamics ("strong phase") which makes interpretation far from straightforward
  - Significant theory uncertainties
- Never the less, this is where the field started and it where we'll begin our story

Characterizing CP Violation in the Kaon system



- CP violation requires there be at least 2 amplitudes: need interference term to see difference in rate)
- Mixing diagrams may contain CP-violating terms. [They do in the SM (CKM)]

  These diagrams have $\Delta S = 2$
- Both semi-leptonic and hadronic decays can have $\Delta S = 2$
- There may also be WI diagrams with CP violating terms that have nothing to do with mixing
  - $\Delta S = 1$ (Example shown to left)
  - Only hadronic decays can have $\Delta S = 1$

Characterizing CP Violation: ϵ

- $\Delta S = 2$ required for semi-leptonic decays but both $\Delta S = 2$ and $\Delta S = 1$ possible for hadronic decays
- δ, η_{00} and η_{+-} all have similar size: indicates that $\Delta S = 2$ dominates
- CP violation in the mixing can be described by saying K_L has a bit of $|K_1\rangle$ and K_S has a bit of $|K_2\rangle$

$$\begin{aligned} |K_S\rangle &= \frac{(|K_1\rangle + \epsilon\,|K_2\rangle)}{\sqrt{1+|\epsilon|^2}} \\ |K_L\rangle &= \frac{(|K_2\rangle + \epsilon\,|K_1\rangle)}{\sqrt{1+|\epsilon|^2}} \end{aligned}$$

- Note: $|K_S\rangle$ and $|K_L\rangle$ are NOT orthoginal
- Expressing above in terms of K^0 and $\overline{K}^0$:

$$\begin{aligned} |K_S\rangle &= \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+|\epsilon|^2}}\left((1+\epsilon)\left|K^0\right\rangle + (1-\epsilon)\left|\overline{K}^0\right\rangle\right) \\ |K_L\rangle &= \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+|\epsilon|^2}}\left((1+\epsilon)\left|K^0\right\rangle - (1-\epsilon)\left|\overline{K}^0\right\rangle\right) \end{aligned}$$

A General Description of CP Violation in K^0s

- Decompose 2π state into $I = 0$ and $I = 2$ (no $I = 1$ since $L = 0$ and Bose Statistics)
- Can define 4 Amplitudes:

$$
\begin{aligned}
\langle 2\pi, I=0 | H_{wk} \left| K^0 \right\rangle &= A_0 \\
\langle 2\pi, I=0 | H_{wk} \left| \overline{K}^0 \right\rangle &= -A^*{}_0 \\
\langle 2\pi, I=2 | H_{wk} \left| K^0 \right\rangle &= A_2 \\
\langle 2\pi, I=2 | H_{wk} \left| \overline{K}^0 \right\rangle &= -A^*{}_2
\end{aligned}
$$

- Three physical measurements

$$
\begin{aligned}
\eta_{+-} &= \frac{\left\langle \pi^+\pi^- \right| H_{wk} \left| K_L \right\rangle}{\left\langle \pi^+\pi^- \right| H_{wk} \left| K_S \right\rangle} \\
\eta_{00} &= \frac{\left\langle \pi^0\pi^0 \right| H_{wk} \left| K_L \right\rangle}{\left\langle \pi^0\pi^0 \right| H_{wk} \left| K_S \right\rangle} \\
\delta_\ell &= \frac{\Gamma(K_L \to \pi^-\ell^+\nu_\ell) - \Gamma(K_L \to \pi^+\ell^-\overline{\nu}_\ell)}{\Gamma(K_L \to \pi^-\ell^+\nu_\ell) + \Gamma(K_L \to \pi^+\ell^-\overline{\nu}_\ell)}
\end{aligned}
$$

- Now break into $I = 0$ and $I = 2$

Using Isopin Clebsh-Gordon Coefficients

- We find:

$$
\begin{aligned}
\left\langle\pi^+\pi^-\right| H_{wk}\left|K_L\right\rangle &= \sqrt{2/3}e^{i\delta_2}(\epsilon Re\ A_2 + i\mathrm{Im}(A_2)) + 2\sqrt{1/3}e^{i\delta_0}(\epsilon Re\ A_0 + i\mathrm{Im}(A_0)) \\
\left\langle\pi^0\pi^0\right| H_{wk}\left|K_L\right\rangle &= 2\sqrt{1/3}e^{i\delta_2}(\epsilon Re\ A_2 + i\mathrm{Im}(A_2)) - \sqrt{2/3}e^{i\delta_0}(\epsilon Re\ A_0 + i\mathrm{Im}(A_0)) \\
\left\langle\pi^+\pi^-\right| H_{wk}\left|K_S\right\rangle &= \sqrt{2/3}(e^{i\delta_2} Re\ A_2 + \sqrt{2}e^{i\delta_0} Re\ A_0) \\
\left\langle\pi^0\pi^0\right| H_{wk}\left|K_S\right\rangle &= \sqrt{2/3}(\sqrt{2}e^{i\delta_2} Re\ A_2 - \sqrt{2}e^{i\delta_0} Re\ A_0)
\end{aligned}
$$

- By convention A_0 is real so

$$
\begin{aligned}
\eta_{+-} &= \epsilon + \epsilon' \\
\eta_{00} &= \epsilon - 2\epsilon' \\
\epsilon' &= \frac{1}{\sqrt{2}}\frac{\mathrm{Im}(A_2)}{A_0}\exp(i\pi/2 - i\delta_0 + i\delta_2)
\end{aligned}
$$

CP Violation From Mixing Vs Direct CP Violation

- Write as coupled equations for the evolution:

$$i\frac{d\psi}{dt} = \begin{pmatrix} M - i\frac{i}{2}\Gamma/2 & M_{12} - i\frac{i}{2}\Gamma_{12}/2 \\ M^*{}_{12} - i\frac{i}{2}\Gamma^*{}_{12}/2 & M - i\frac{i}{2}\Gamma/2 \end{pmatrix}\psi$$

- If we write $\delta m = \delta m_R + i\delta m_I$ can show

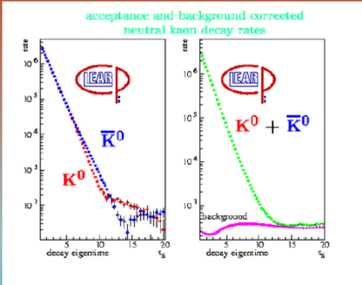
$$\epsilon = \frac{i\delta m_I}{m_L - m_S + i\Gamma_S/2}$$

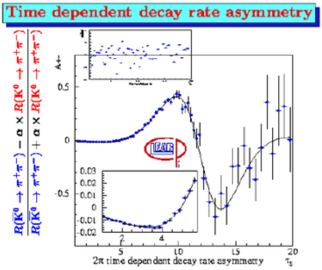
- It can be shown that

$$\delta_\ell = 2\mathrm{Re}\ \epsilon$$

- If direct CP violation ($\Delta S = 1$) will need one additional parameter (called ϵ').
  - In K system, this is small, even when compared to ϵ

CPLear Measurement of η_{+-}





$$A_{+-}(\tau) = -\frac{2|\eta_{+-}|e^{\frac{1}{2}(1/\tau_S - 1/\tau_L)\tau}\cos(\Delta m \cdot \tau - \varphi_{+-})}{1 + |\eta_{+-}|^2 e^{(1/\tau_S - 1/\tau_L)\tau}}$$

$$|\eta_{+-}| = (2.264 \pm 0.023_{sta.} \pm 0.026_{sys.} \pm 0.007_{\tau_S}) \times 10^{-3}$$
$$\varphi_{+-} = 43.19^\circ \pm 0.53^\circ_{sta.} \pm 0.28^\circ_{sys.} \pm 0.42^\circ_{\Delta m}$$

with $\Delta m = (530.1 \pm 1.4) \times 10^7 \hbar s^{-1}$ PDG '98

published in Phys. Lett. B 458 (1999) 545

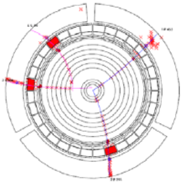
- **α is a free parameter in the fit, $\alpha = \frac{\varepsilon(K^+)}{\varepsilon(K^-)}[1 + 4\mathbb{R}(\varepsilon_T + \delta)]$ used as rate normalization in other decay channels**

$$A_{2\pi} = \frac{R(\overline{K}^0 \to \pi\pi)(\tau) - \alpha \times R(K^0 \to \pi\pi)(\tau)}{R(\overline{K}^0 \to \pi\pi)(\tau) + \alpha \times R(K^0 \to \pi\pi)(\tau)}$$
$$= -2|\eta_{\pi\pi}|\cos(\Delta m\tau - \varphi_{\pi\pi})\frac{e^{\frac{1}{2}(\Gamma_S - \Gamma_L)\tau}}{1 + |\eta_{\pi\pi}|^2 e^{(\Gamma_S - \Gamma_L)\tau}}$$

With Δm free in the fit, not assuming CPT,
$\Delta m = (524.0 \pm 4.4 \pm 3.3) \times 10^7 \hbar s^{-1}$

CPLear Measurement of δ

Analysis of $K^0 \to \pi^{\mp} e^{\pm} \nu$

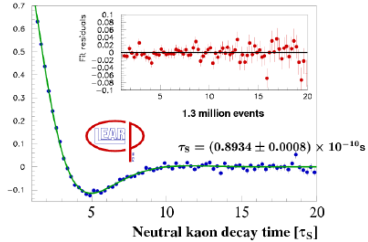


- kinematical constraints
- electron identification based on:
- dE/dx in the scintillators,
- number of photo-electrons in the Čerenkov,
- number of hits in the calorimeter

$K_L - K_S$ Mass Difference

$$A_{\Delta m} = \frac{N_{K^0 \leftarrow K^0, \bar{K}^0 \leftarrow \bar{K}^0} - N_{\bar{K}^0 \leftarrow K^0, K^0 \leftarrow \bar{K}^0}}{N_{K^0 \leftarrow K^0, \bar{K}^0 \leftarrow \bar{K}^0} + N_{\bar{K}^0 \leftarrow K^0, K^0 \leftarrow \bar{K}^0}}$$

$$= 2\frac{e^{-\bar{\Gamma}\tau}\cos\Delta m\tau + 2\Im(x_-)e^{-\bar{\Gamma}\tau}\sin\Delta m\tau}{[1 + 2\Re(x_+)]\,e^{-\Gamma_S\tau} + [1 - 2\Re(x_+)]\,e^{-\Gamma_L\tau}}$$



$$\Delta m = (529.5 \pm 2.0_{stat.} \pm 0.3_{syst.}) \times 10^7 \hbar s^{-1}$$

$$\Delta m = (348.5 \pm 1.3) \times 10^{-9}\ \mathrm{eV/c^2}$$

$\Delta S = \Delta Q$ violating decays or wrong tagging:

$$\Re e\, x_+ = (-1.8 \pm 4.1_{stat.} \pm 4.5_{syst.}) \times 10^{-3}$$

Best single measurements: Phys.Lett. B444 (1998) 38

Precise measurement of the oscillation frequency Δm (setting $\Im(x_-)$=0) :

Δm and $\Im(x_-)$ are strongly correlated, >0.99.
With $\Delta m = (530.1 \pm 1.4) \times 10^7 \hbar s^{-1}$ obtain
$\Im(x_-) = (-0.8 \pm 3.5) \times 10^{-3}$

A Modern Treatment of CP Violation

- The CKM Matrix:

$$\begin{aligned} V_{CKM} &= \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{ds} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \\ &\approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \end{aligned}$$

- Note, from the explicit form, you can prove:

$$\rho + i\eta = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$

- Unitarity insures $VV^\dagger = V^\dagger V = 1$. Thus

$$\begin{aligned} \sum_i V_{ij}V_{ik}^* &= \delta_{jk} \text{ column orthogonality} \\ \sum_j V_{ij}V_{kj}^* &= \delta_{ik} \text{ row orthogonality} \end{aligned}$$

- Eg:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

The Unitarity Triangle (I)

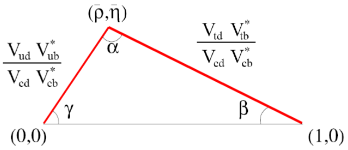
- From previous page

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

- Divide by $|V_{cd}^* V_{cb}|$:

$$\frac{V_{ud}V_{ub}^*}{|V_{cd}^* V_{cb}|} - 1 + \frac{V_{td}V_{tb}^*}{|V_{cd}^* V_{cb}|} = 0$$

- Think of this as a vector equation in the complex plane
- Orient so that base is along x-axis



- Reminder from previous page:

$$\rho + i\eta = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$

The Unitarity Triangle (II)

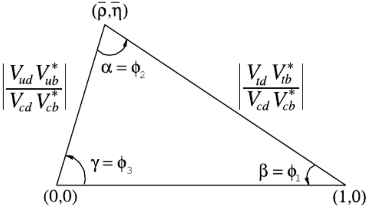


Figure 12.1: Sketch of the unitarity triangle.

$$\beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$
$$\alpha = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$
$$\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

- CP violating phase in V_{ub} and V_{td}
  - By convention: can do rotations to move the phase to other elements
- $|A|^2$ is real for any single amplitude
  - Need at least 2 amplitudes to see CP violating effects
- Only cases where all 3 generations are involved exhibt CP violation

Classifying CP Violating Effects

- CP Violation in Decays

$$\Gamma(P \to f) \neq \Gamma(\overline{P} \to \overline{f})$$

or (even better) if $f = \overline{f}$

$$\Gamma(P^0 \to f) \neq \Gamma(\overline{P}^0 \to f)$$

- CP Violation in Mixing

$$Prob(P^0 \to \overline{P}^0) \neq Prob(\overline{P}^0 \to P^0)$$

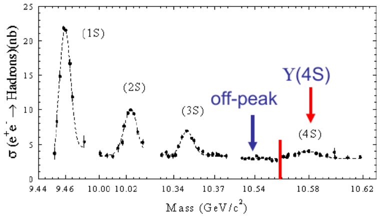
- CP Violation in Interference
  - Time dependent asymetry dependent on fraction of P^0 at time t

B-decays will provide a rich laboratory for studying all three of these

Sources of B-hadrons

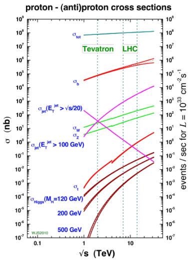
- CP violating effects small
  - Need large number of B mesons to study decay rates with high accuracy
- Two strategies:

$$e^+e^- \to \Upsilon(4s) \to B\overline{B}$$



- Just above threshold
- Only B^+ and B^0
- Coherent stats with no additional particles

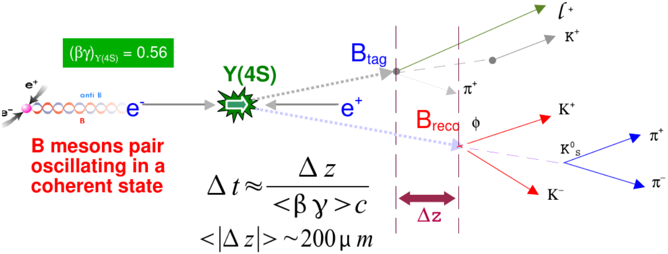
pp or $p\overline{p} \to b\overline{b} + X$



- Very large cross section, but less friendly environment
- Allows studies of B_s and B baryons, was well as $B^{\pm}$ amd B^0

$e^+e^- \to \Upsilon(4s)$: How do the $B\overline{B}$ pairs behave?

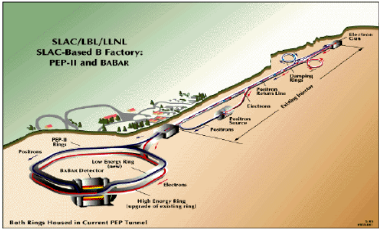
- B and $\overline{B}$ come from $\Upsilon(4s)$ in a coherent $L = 1$ state
  - $\Upsilon(4s)$ is $J^{PC} = 1^{--}$
  - B mesons are scalars
  - Thus, $L = 1$
- $\Upsilon(4s)$ decays strongly so B and $\overline{B}$ produced as flavor eigenstates
  - After production, each meson oscillates in time, but *in phase* so that at any time there is only one B and one $\overline{B}$ until one particle decays
  - Once one B decays, the other contines to oscillate, but coherence is broken
  - Possible to have events with two B or two $\overline{B}$ decays
- This common evolution is important for CP studies
  - Time integrate asymmetries vanish for cases where CP violation comes from mixing diagrams
- In addition, in center-of-mass, B hadrons have almost no momentum
  - Difficult to distinguish which tracks come from B and which from $\overline{B}$

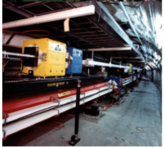


- e^+ amd e^- beams with different energies
  - $\Upsilon(4s)$ boosted along beamline
  - B mesons travel finite distance before decaying
  - Typical distance between decay of the two B mesons: $\sim 200\ \mu$m
- Two B-factories built:
  - SLAC (1999-2008)
  - KEK (1999-2010, upgraded SuperKEKB 2018 onward)

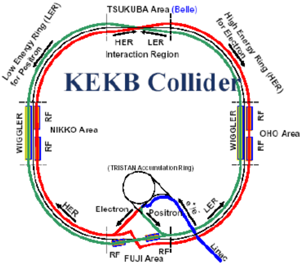
PEP-II and KEKB

PEP-II
- 9 GeV e^- on 3.1 GeV e^+
- $\Upsilon(4S)$ boost: $\beta\gamma = 0.56$

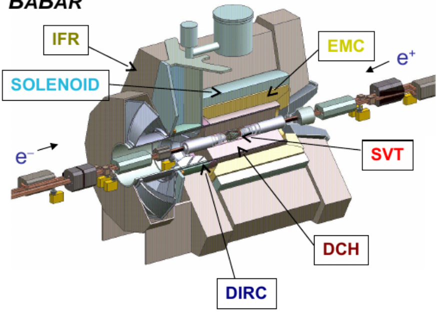




KEKB
- 8 GeV e^- on 3.5 GeV e^+
- $\Upsilon(4S)$ boost: $\beta\gamma = 0.425$

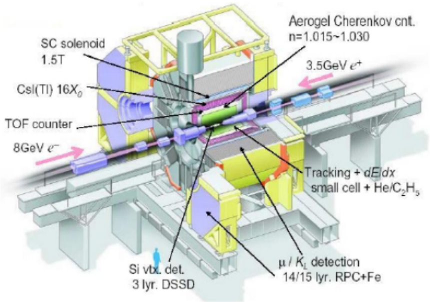


BABAR and Belle



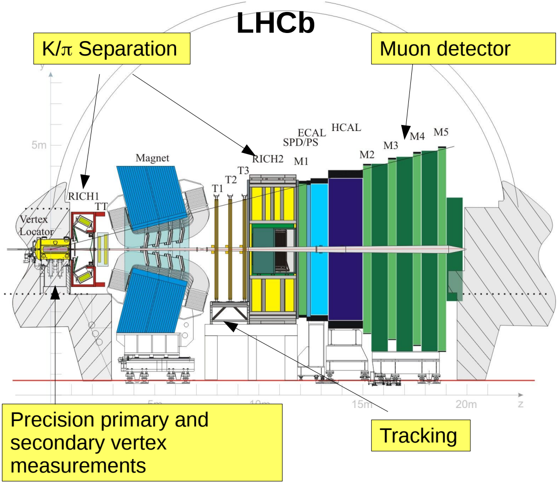
The differences between the two detectors are small. Both have:

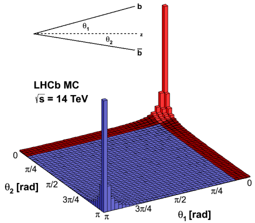
- **Asymmetric design.**
- **Central tracking system**
- **Particle Identification System**
- **Electromagnetic Calorimeter**
- **Solenoid Magnet**
- **Muon/K^0_L Detection System**
- **High operation efficiency**



B Physics at Hadron Colliders

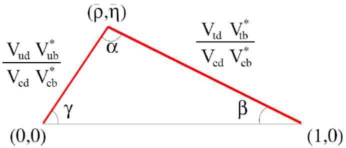
- Tevatron 2001-2010 (including first observation of B_s mixing)
  - Importance of secondary vertex trigger
- Dedicated LHCb detector at LHC
  - Forward detector with fixed target-like geometry





The Measurement Game Plan

- Want to test if CKM is the only source of CP violation
  - All CP violation in SM comes from the one phase η
  - Can relate CP rates in different modes, using appropriate theory calculations and knowledge of CKM matrix elements
- Want to test if matrix is unitary
  - Failure of unitarity means new physics
- Make *many* measurements of sides and angles to over-constrain the triange and test that it closes



$$\alpha \equiv arg[-V_{td}V_{tb}^*/V_{ud}V_{ub}^*]$$
$$\beta \equiv arg[-V_{cd}V_{cb}^*/V_{td}V_{tb}^*]$$
$$\gamma \equiv arg[-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*]$$

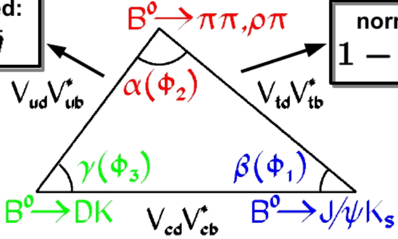
$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

many observables functions of $\bar{\rho}$ and $\bar{\eta}$: overconstraining

$$\alpha = \pi - \beta - \gamma$$

normalized: $\bar{\rho} + i\bar{\eta}$

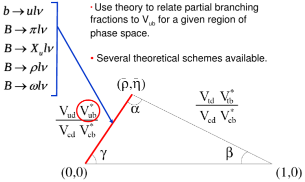
normalized: $1 - \bar{\rho} - i\bar{\eta}$

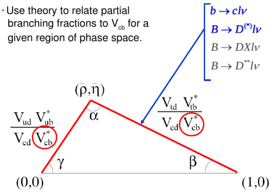


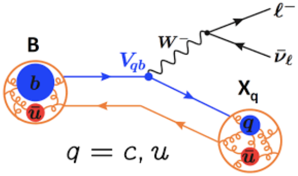
$$\gamma = atan\left(\frac{\bar{\eta}}{\bar{\rho}}\right)$$

$$\beta = atan\left(\frac{\bar{\eta}}{(1-\bar{\rho})}\right)$$

Measuring the Sides: B and D Decays







- Sides are combinations of magnitudes of CKM matrix elements
- Heavy flavor decays can be used to measure these
  - V_{cd} from $D_s \to K\ell\nu$, $D \to \pi\ell\nu$
  - V_{cs} from $D_s^+ \to \mu^+\nu$, $D \to K\ell\nu$
  - V_{cb} from $B \to X_c\ell\nu$ ($X_c \equiv D, D^*$, etc)
  - V_{ub} from $B \to X_d\ell\nu$ ($X_d \equiv \pi, \rho$, etc)
- Requires precise measurement of branching fractions
- Must correct for fact that c or b-quark is bound in a meson
  - Need theory for this (HQET)

Angle Measurements: Types of CP Violation

- Three different categories
  - Direct CP Violation

$$Prob(B \to f) \neq Prob(\overline{B} \to \overline{f})$$

  - Indirect CP Violation (CPV in mixing)

$$Prob(B \to \overline{B}) \neq Prob(\overline{B} \to B)$$

  - CP Violation between mixing and decay
- Third category cleanest theoretically since no issues of final state interations
- Always need more than one amplitude to allow interference

CP Violation and Phases

- CP conserved:

$$\mathcal{H} = \sum_j \mathcal{H}_j + \sum_j \mathcal{H}_j^\dagger$$

where $CP\mathcal{H}_j CP = \mathcal{H}_j^\dagger$.

- CP violated:

$$\mathcal{H} = \sum_j e^{i\phi_j} \mathcal{H}_j + \sum_j e^{-i\phi_j} \mathcal{H}_j^\dagger$$

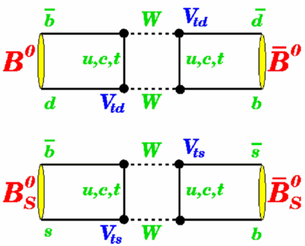
where each piece acquires its phase from a particular combination of CKM matrix elements. The result then is that while $CP\mathcal{H}_j CP = \mathcal{H}_j^\dagger$, in general, $CP\mathcal{H}CP \neq \mathcal{H}$.

The Simplest Case: B^0 and $\overline{B}^0$ decay to same CP eigenstate

- If one single part $\mathcal{H}_j$ of the weak Hamiltonian is responsible for the decay $B^0 \to f$ (where f is a CP eigenstate) then

$$\begin{aligned} \langle f | \mathcal{H} \left| B^0 \right\rangle &= \langle f | e^{i\phi_j} \mathcal{H}_j \left| B^0 \right\rangle &= \langle f | e^{i\phi_j} CP \mathcal{H}_j^\dagger CP \left| B^0 \right\rangle \\ &= \eta_f e^{2i\phi_j} \langle f | e^{-i\phi_j} \mathcal{H}_j^\dagger \left| \overline{B}^0 \right\rangle &= \eta_f e^{2i\phi_j} \langle f | \mathcal{H} \left| \overline{B}^0 \right\rangle , \end{aligned}$$

  where η_f is the value of CP for the state f.
- Interference in the decay of a neutral B depends on the weak phases for the decay ϕ_j, which come from the CKM matrix, and on the phase introduced by the mixing, whichresults from box diagram.



- Dominant diagram has t-quark intermediates and $\propto (V_{tb}V_{td}^*)^2$ for B^0 and $M_{12} \propto (V_{tb}V_{ts}^*)^2$ for B_s
- For B^0, $|M_{12}|/M_{12} = -e^{-2i\beta}$

What we find

- ϕ_{wk} is the single weak phase in the amplitude for $B^0 \to f$.
- $\Delta\Gamma$ can be ignored for B_d
- The decay rate is then determined by

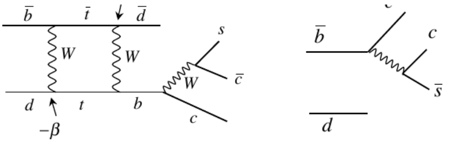
$$|\langle f|\mathcal{H}|B^0_{phys}(t)\rangle|^2 \propto e^{-\Gamma t}\left[1 + \eta_f \sin 2(\beta + \phi_{wk}) \sin \Delta m t\right],$$

.

$$|\langle f|\mathcal{H}|\overline{B}^0_{phys}(t)\rangle|^2 \propto e^{-\Gamma t}\left[1 - \eta_f \sin 2(\beta + \phi_{wk}) \sin \Delta m t\right].$$

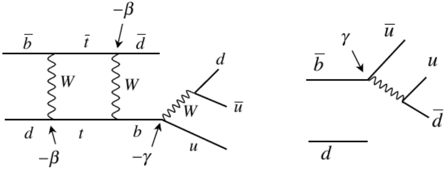
- What is remarkable here is that there are no unknown matrix elements involving hadrons: when just a single weak phase occurs, the hadronic uncertainty disappears.
- The weak phase can be calculated using CKM matrix elements
- Theoretical uncertainties small

Examples of relevant decays



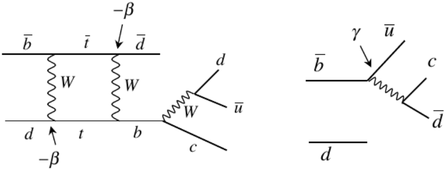
$B^0 \to J/\psi K_S$

B



$B^0 \to \pi\pi$

C



$B^0 \to D\pi$

Tagging

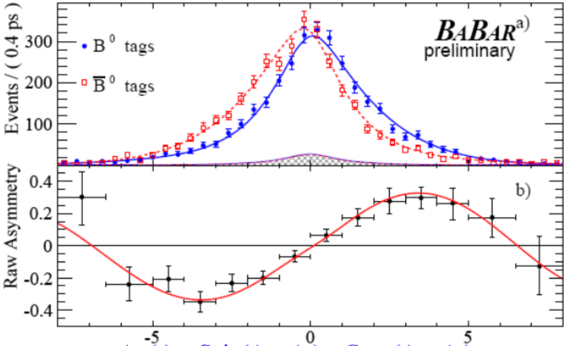
- Need to know how observed B began life.
- Observe other B and determine whether it is B^0 or $\overline{B}^0$.
- Determination will be imperfect.
- If it is wrong a fraction w of the time, $1 - A\sin\Delta mt$ becomes

$$(1-w)(1-A\sin\Delta mt) + w(1+A\sin\Delta mt) = 1 - DA\sin\Delta mt$$

  where the dilution D is just $1-2w$.
- Figure of merit $Q = \sum \epsilon_i D_i^2$, where the ith tagging category captures a fraction ϵ_i of the neutral B events and has a dilution D_i.
- Most effective tagging method: charge of lepton from semileptonic decay
- But can also use charge of kaon or charge of secondary vertex

sin2β in golden b → $\bar{c}$cs modes

- The '*Golden Measurement*' of the B factories. The aims of this measurement were:
  - Measure an angle of the Unitarity Triangle.
  - Discover CP violation in B meson decays.



$A_{CP}(t) = S\sin(\Delta m_d \Delta t) - C\cos(\Delta m_d \Delta t)$

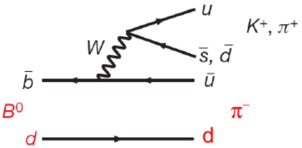
Sine term has a non-zero coefficient

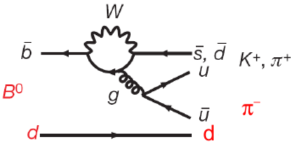
S = sin 2β = 0.671 ± 0.024

This tells us that there is CP violation in the interference between mixing and decay amplitudes in $\bar{c}$cs decays.

Direct CP violation

- $B^0 \to K^{\pm}\pi^{\mp}$: Tree and gluonic penguin contributions

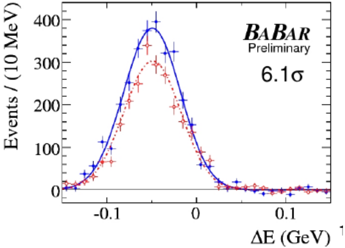




- Compute time integrated asymmetry

$$\mathcal{A}_{K^{\pm}\pi^{\mp}} \equiv \frac{N(\bar{B}^0 \to K^-\pi^+) - N(B^0 - K^+\pi^-)}{N(\bar{B}^0 \to K^-\pi^+) + N(B^0 \to K^+\pi^-)} = -0.098 \pm 0.012$$

- Experimental results from Belle, BaBar, and CDF have significant weight in the world average of this CP violation parameter.
- Direct CP violation present in B decays.
- Unknown strong phase differences between amplitudes, means we can't use this to measure weak phases!



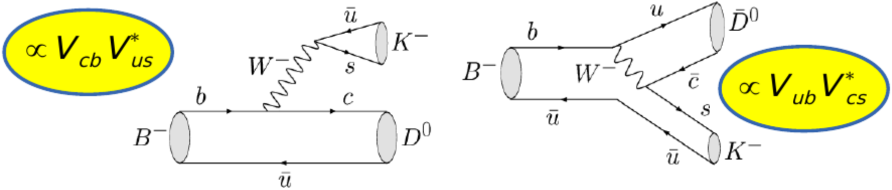
Importance of γ from $B \to DK$

- γ plays a unique role in flavour physics

the only CP violating parameter that can be measured through tree decays (*)

(*) i.e. without uncertainty due to short distance loops

- A benchmark Standard Model reference point
  - doubly important after New Physics is observed



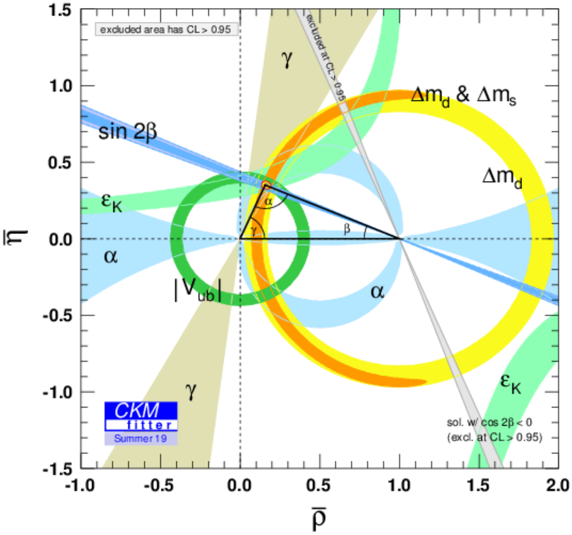
Variants use different B or D decays
require a final state common to both D^0 and $\overline{D}^0$

Where has CP Violation been observed?

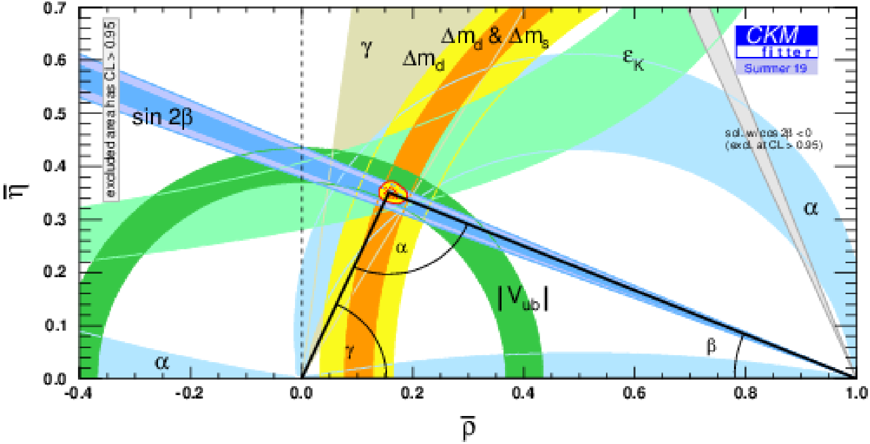
Table 1: Summary of the systems where CP violation effects have been observed. A five standard deviation (σ) significance threshold is required for a ✓; several such observations in different channels are required for a ✓✓. Note that CP violation in decay is the only possible category for particles that do not undergo oscillations.

	K^0	K^+	Λ	D^0	D^+	D_s^+	Λ_c^+	B^0	B^+	B_s^0	Λ_b^0
CP violation in mixing	✓✓	–	–	✗	–	–	–	✗	–	✗	–
CP violation in mixing/decay interference	✓	–	–	✗	–	–	–	✓✓	–	✗	–
CP violation in decay	✓	✗	✗	✗	✗	✗	✗	✓✓	✓✓	✓	✗

from T. Gershon and V.V. Gligorov, arXiv 1607.06746v2


excluded area has CL > 0.95
excluded at CL > 0.95
γ
Δm_d & Δm_s
sin 2β
Δm_d
ε_K
α
β
|V_ub|
α
γ
ε_K
CKM fitter
Summer 19
sol. w/ cos 2β < 0
(excl. at CL > 0.95)
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
η̄
ρ̄

Zooming in



0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-0.4
-0.2
0.0
0.2
0.4
0.6
0.8
1.0
$\bar{\eta}$
$\bar{\rho}$
excluded area has CL > 0.95
γ
Δm_d & Δm_s
Δm_d
ε_K
CKM fitter
Summer 19
sin 2β
sol. w/ cos 2β < 0
(excl. at CL > 0.95)
α
$|V_{ub}|$
β

- X