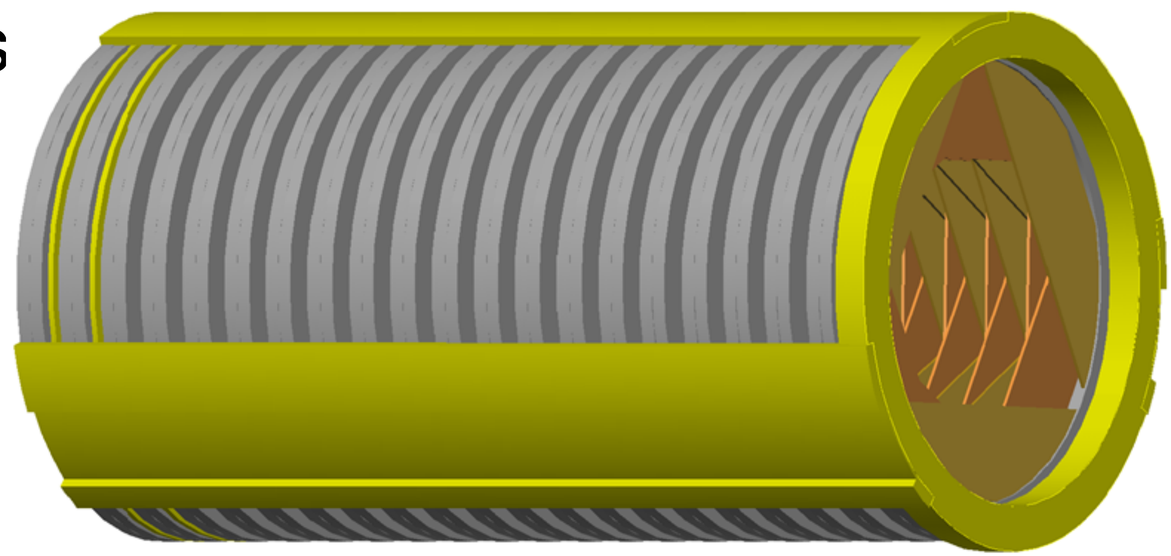
Low Momentum Track Reconstruction (for Mu2e)

David Brown, LBNL

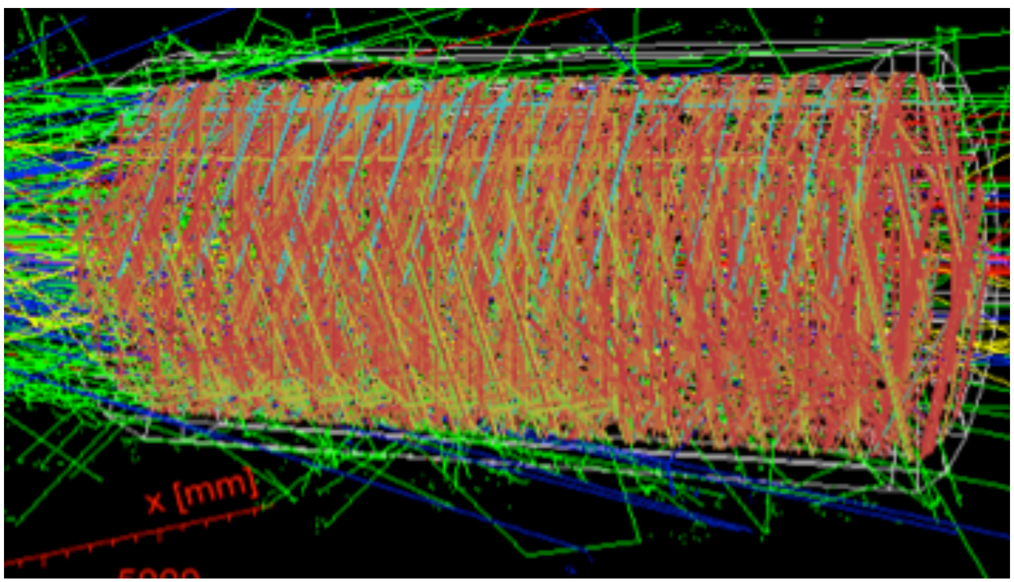
The Mu2e Experiment

- Search for Charged Lepton Flavor Violation in μ^- capture on (Al) nuclei
- Target Sensitivity: $\Gamma_{\mu\rightarrow e}/\Gamma_{\mu\ \mathrm{capture}} \sim 10^{-16}$
- First beam in 2020
- Principal detector: 20-plane straw tracker
  - 0.8 m radius × 3 m length, 1T axial field, in vacuum
  - 5 mm diameter straws
  - Wires ⊥ to Z axis
  - Large angle stereo
  - ~1% X_0 total mass
  - ~20,000 channels

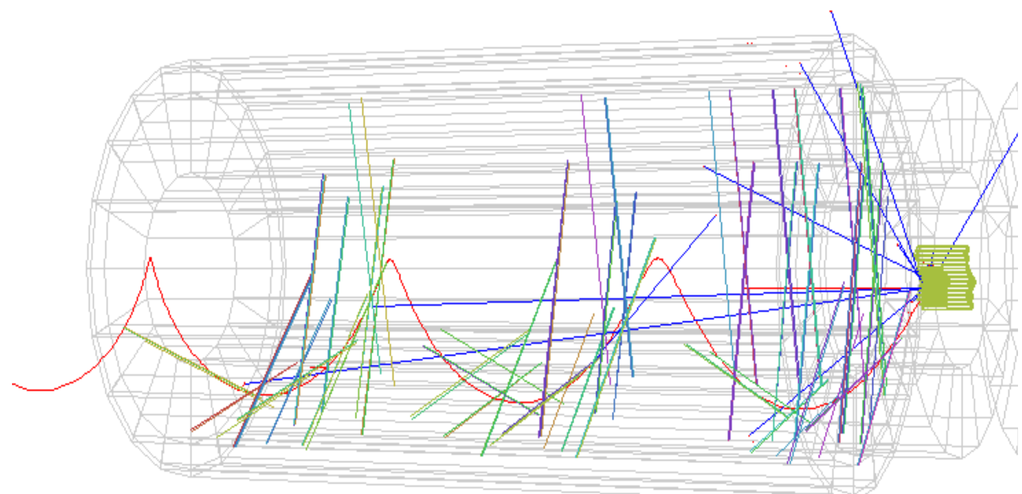


Mu2e Challenges

- Single track signal
- Low-momentum (105 MeV/c) e^- signal track
  - multi-turn helix
- High background rate
  - Most hits are from neutral particles and rays



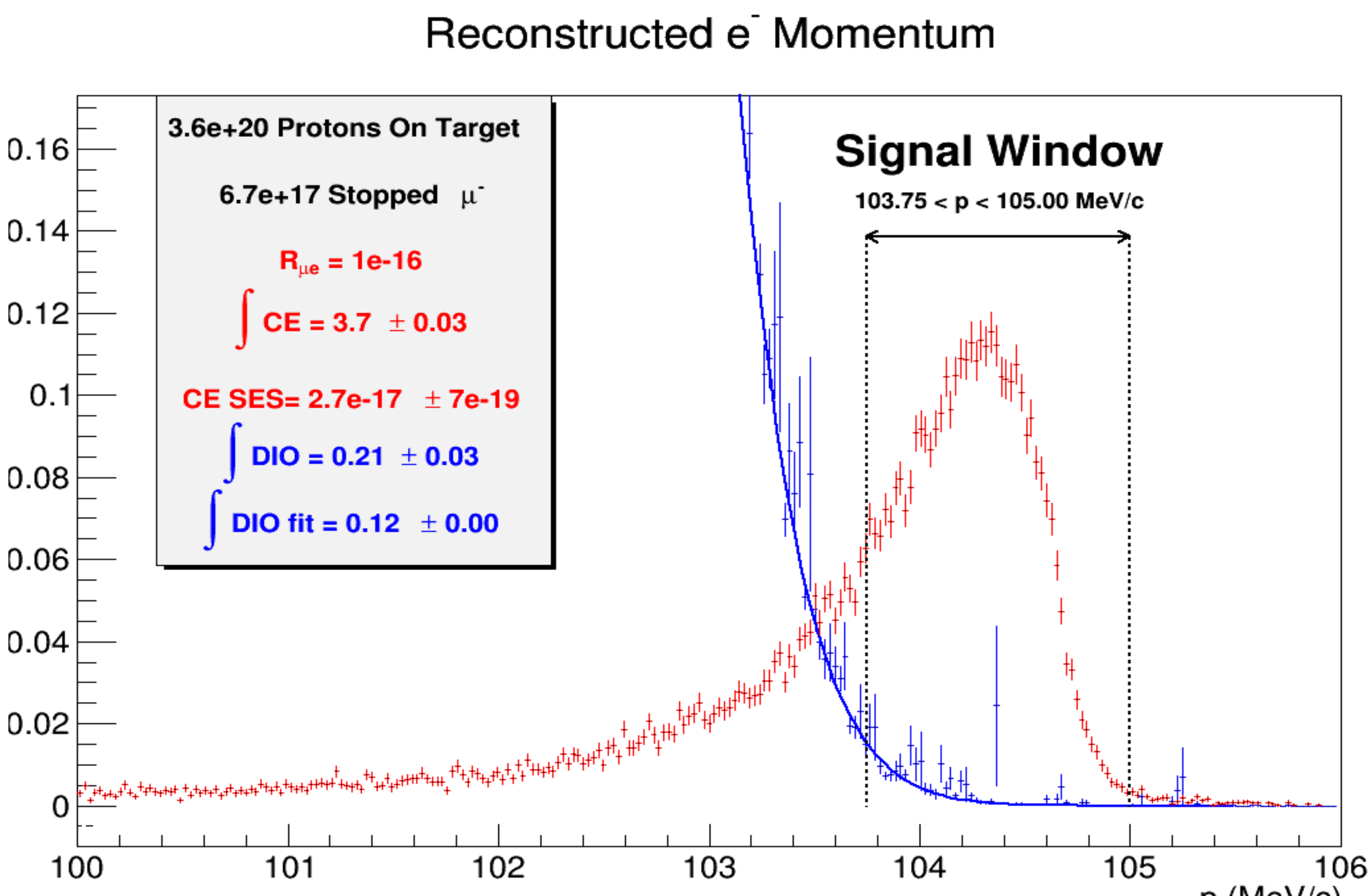
~4K background hits/1 μsec



~40 signal hits

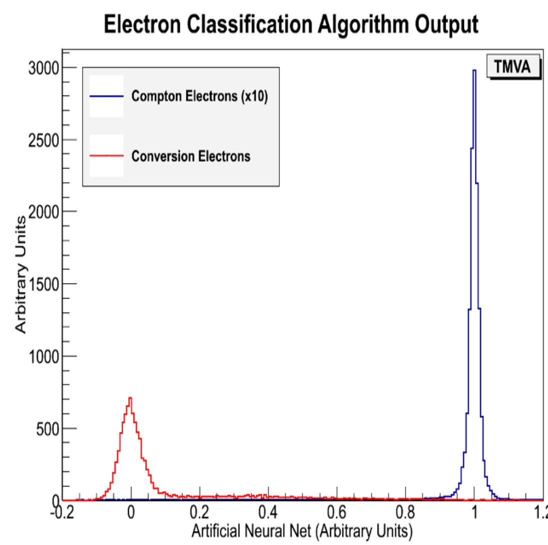
Mu2e Challenges

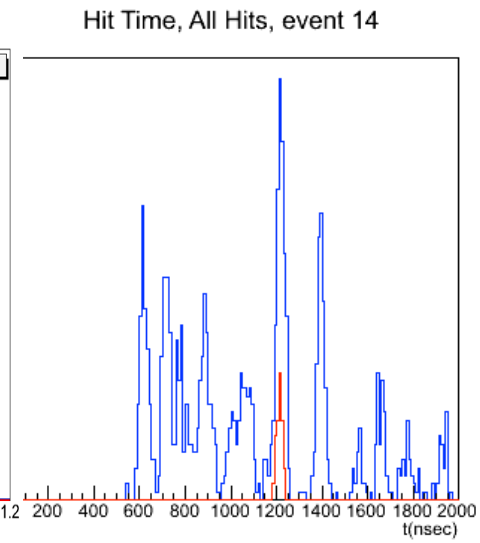
- Extremely sensitive to momentum resolution
  - σ_{core} < 250 KeV/c (0.25% @ 105 MeV/c)
  - resolution tails < 1%

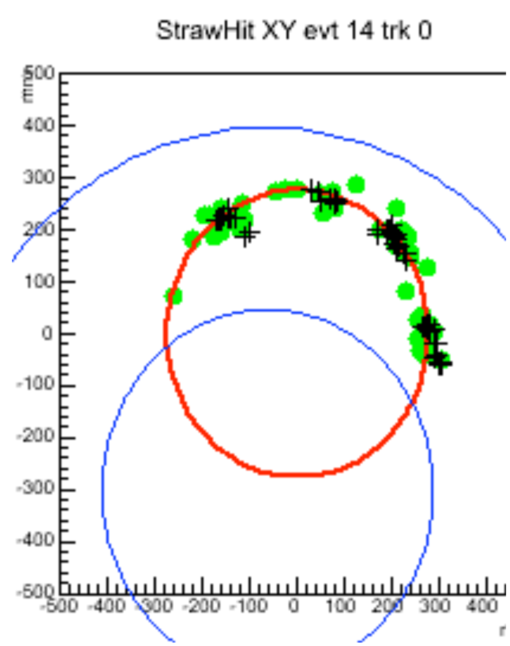


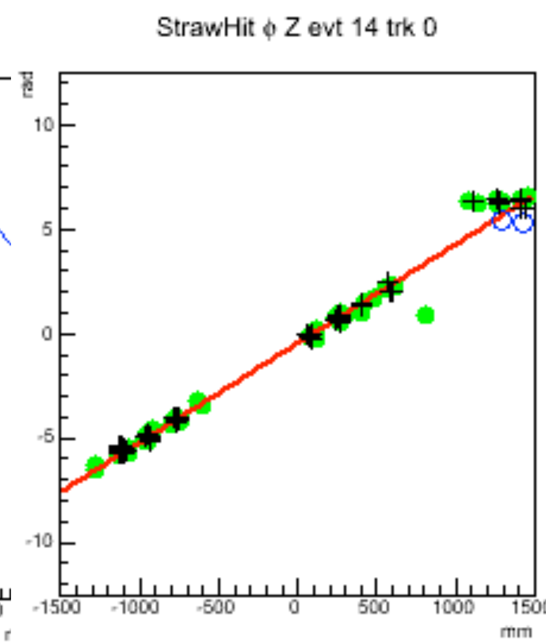
Mu2e Track Reco Strategy

- Neural net filter of ‘obvious’ backgrounds
  - δ-rays, proton hits, …
- Time clustering
  - 50 nsec maximum drift
  - ~8/1 S/N for Pat. Rec.
- Robust helix fit using (stereo) space points
  - ~3cm resolution Time Division along wires to resolve stereo ambiguities
- KF final fit
  - SA outlier filter



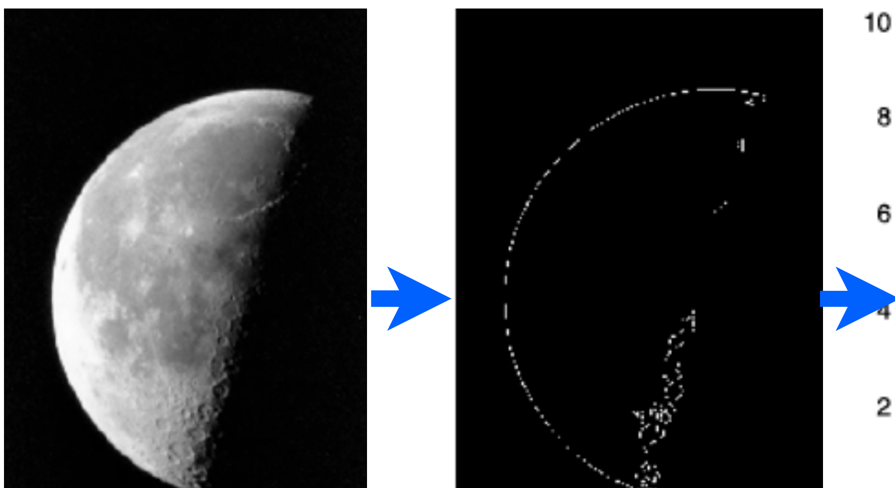


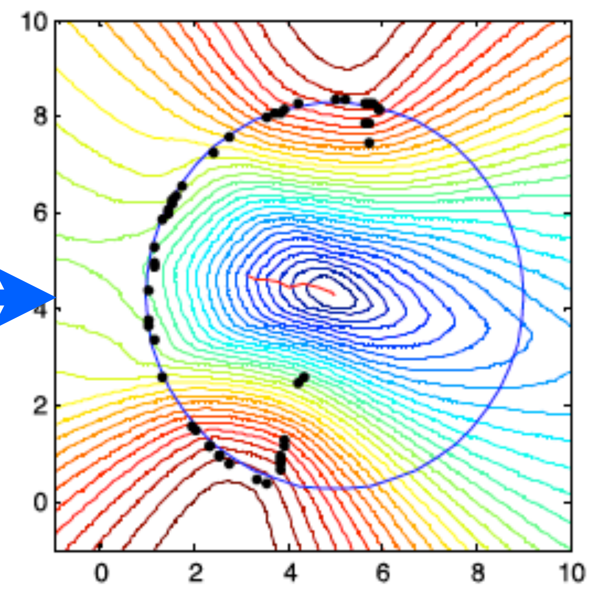


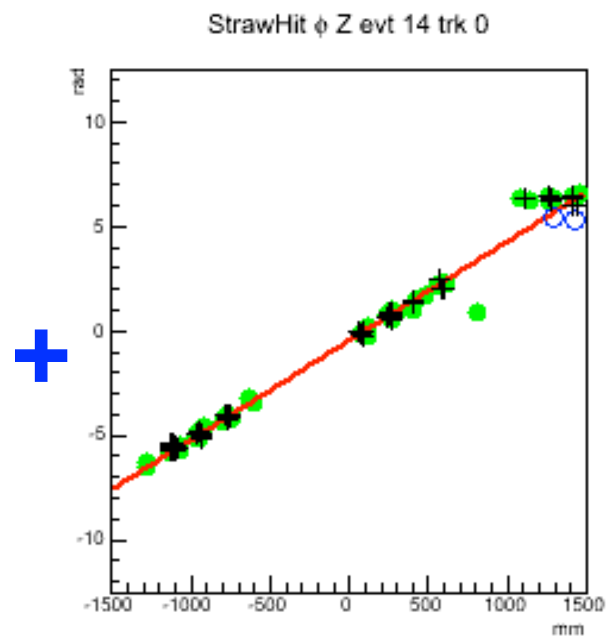


Robust Helix Finding

- Absolute Gradient Error (AGE) circle fit
  - resistant to outliers
  - weak constraint against center drift
- Median-based linear fit to ϕ vs z
  - principle difficulty: resolving 2π ambiguity







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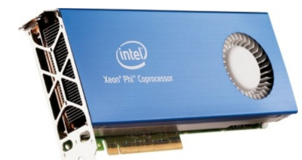
Robust Fitting of Circle Arcs

Computing Performance

- ‘Triggerless’ DAQ
  - data streamed to a 36-server processor farm
  - @200 KHz raw event rate
- Need factor ~200 filtering to meet storage limits
- Select using full track reconstruction
  - ~4 msec/event
  - meets requirements

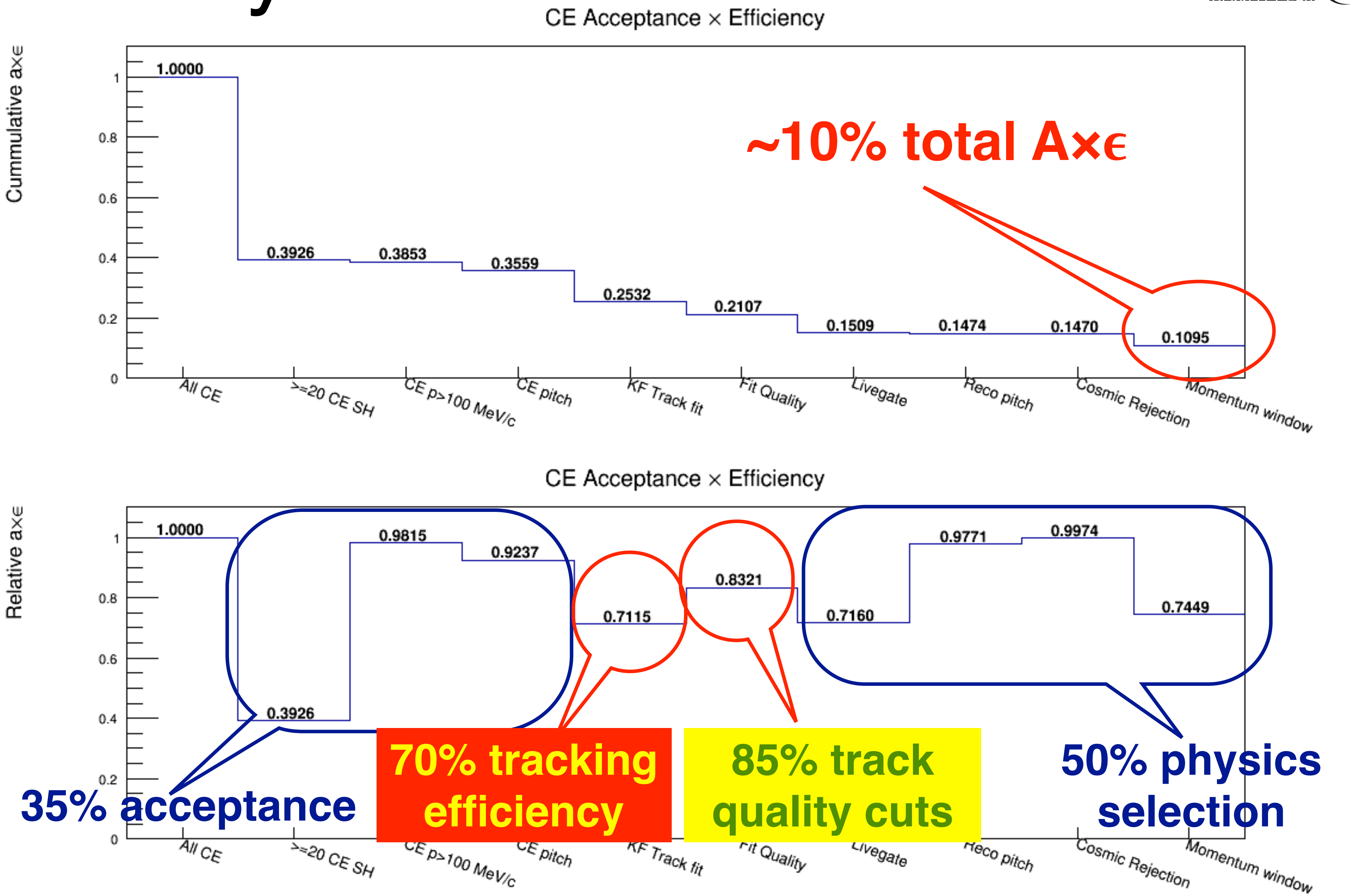
16 cores | 120 cores



	XEON E5-2687v2	XEON PHI 5510P
Stereo Hits		
0) reference code (gcc compiler)	83.6 msec	-
1) algorithmic improvements (gcc compiler)	4.3 msec	-
2) Intel compiler, loop vectorization	**1.4 msec**	**4.8 msec**
Background Hits		
0) reference code (gcc compiler)	9.0 msec	-
1) Intel compiler	5.1 msec	123.0 msec
2) refactoring	3.4 msec	38.1 msec
3) double → single precision	**2.1 msec**	**23.9 msec**
Overhead		
0) reference code (gcc compiler)	0.9 msec	-
1) Intel compiler (estimated)	**0.3 msec**	**2.0 msec**
total processing time	3.8 msec	30.7 msec
events/sec (single core)	260	32
number of cores (36 servers)	720	4,320
events/sec (36 servers)	**187,000**	**138,000**

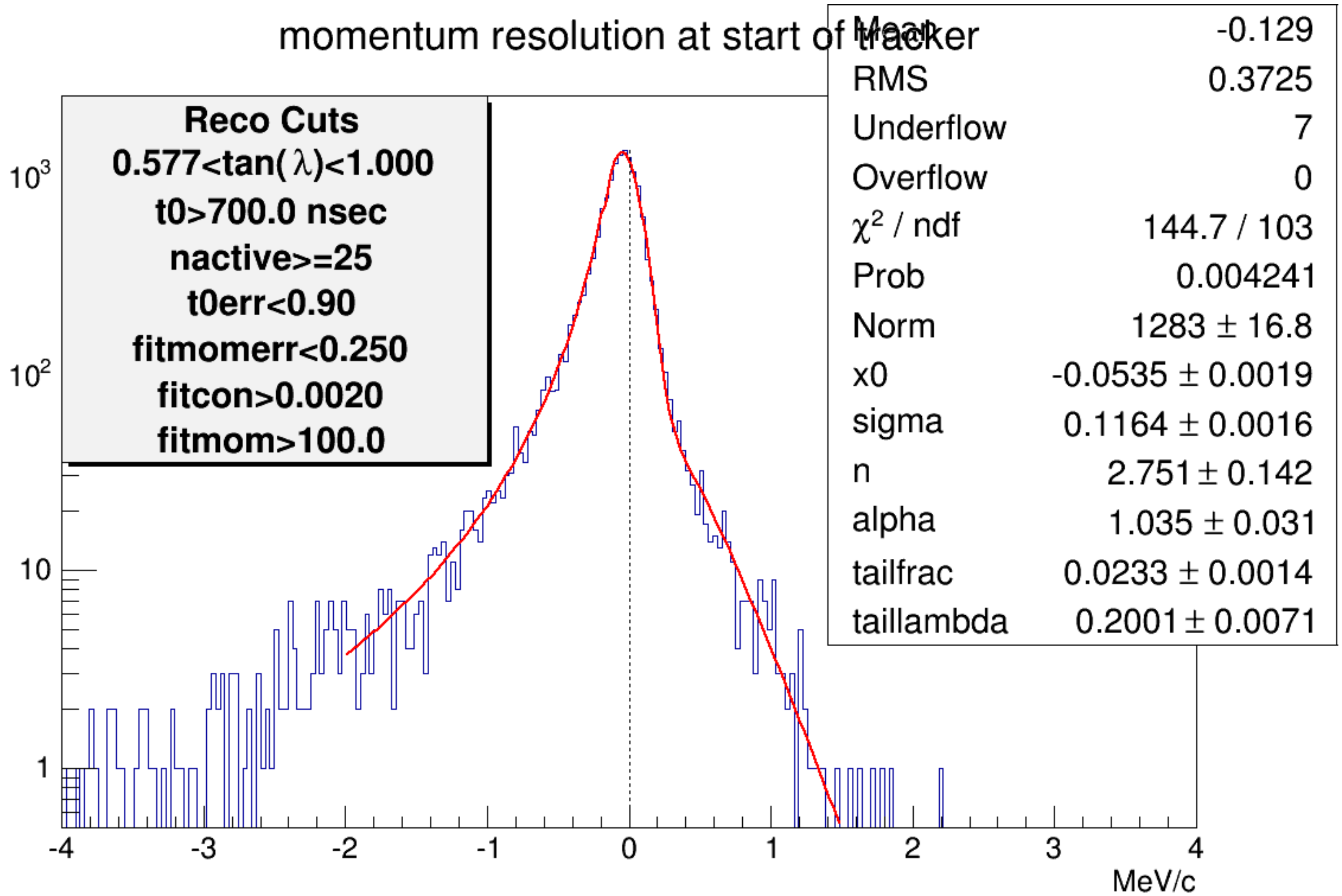
190KHz | **140KHz**

Physics Performance



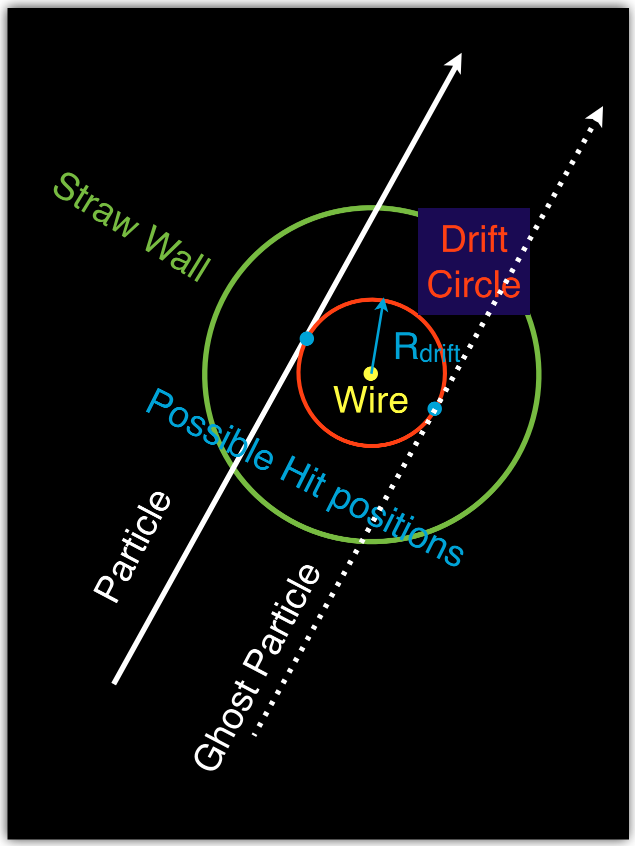
Physics Performance

- 110 KeV/c core resolution
- 2% exponential tails



Discrete Ambiguity Resolution

- Residual sign assigned using extrapolation (iterative)
  - Miss-assignment probability increases at low radius
- Mu2e ‘solution’: use wire position for small drift times
  - Hit errors assigned accordingly



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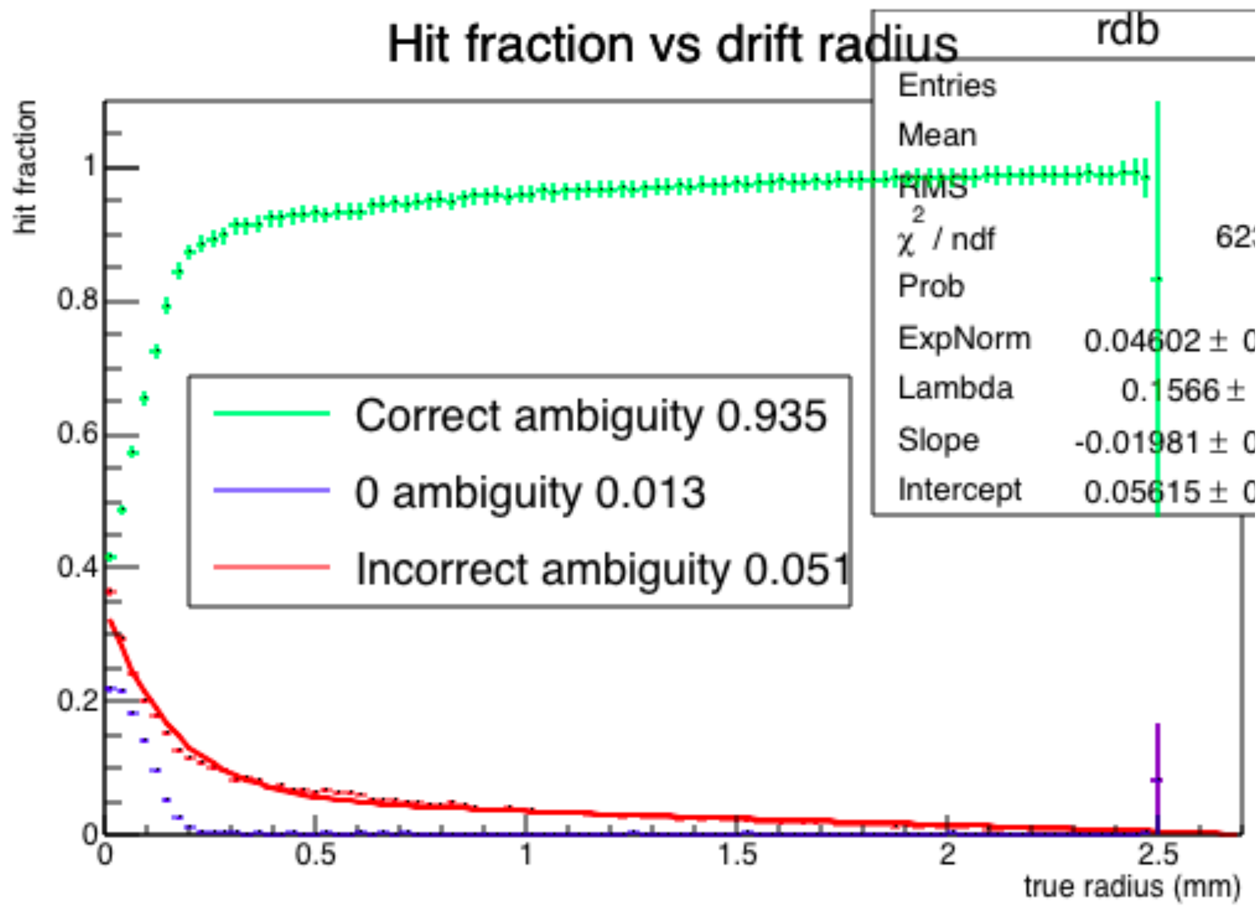
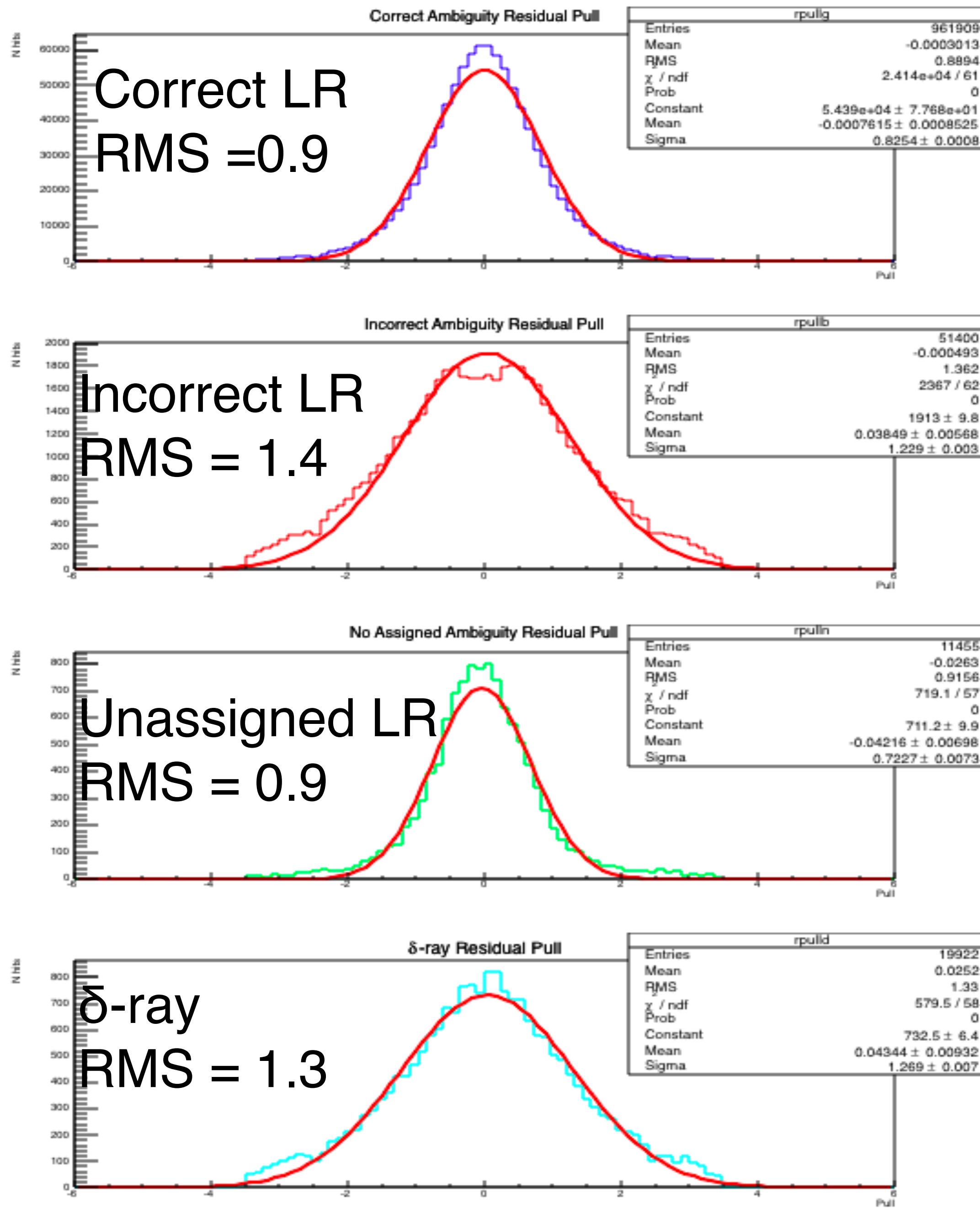


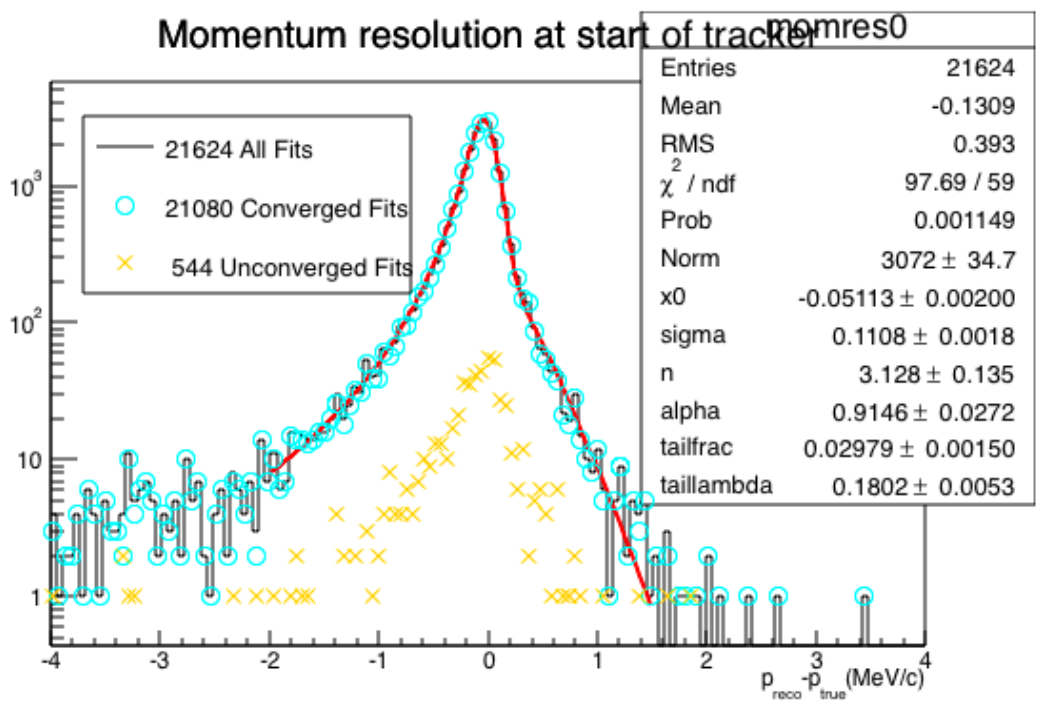
Hit Residual Pulls

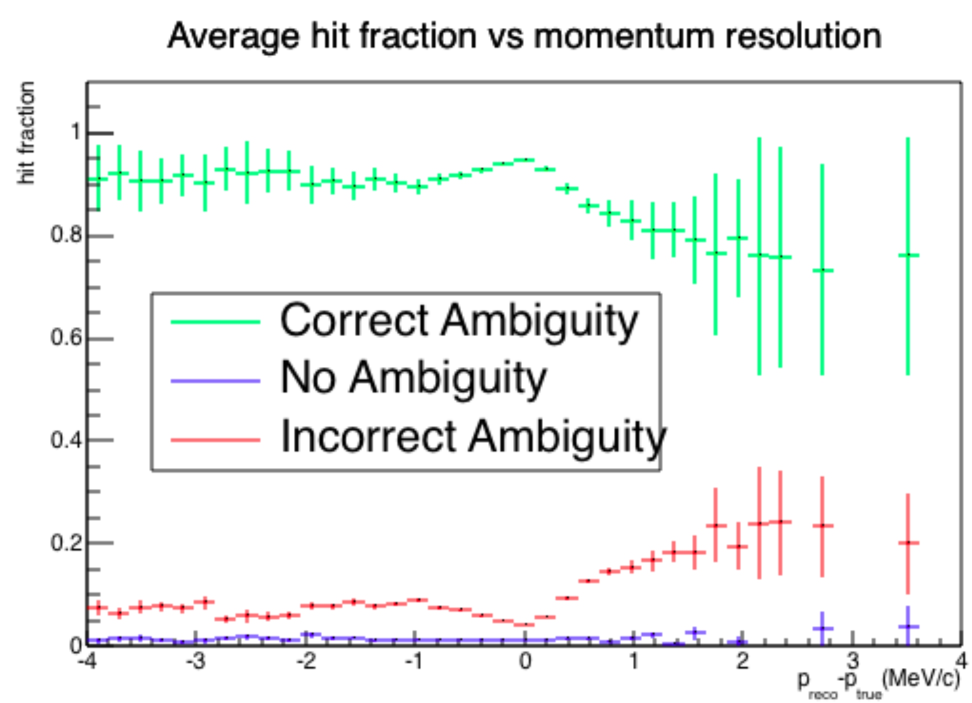
- Pull = $\Delta/\sigma_{estimated}$
- Assigned hit errors tuned to give ~uniform track fit probability
- Outlier filtering sculpts LR ambiguity hit pulls
- LR mis-assignment comparable to intrinsic δ-ray background



Discrete Ambiguity Error Effect

- Momentum resolution high-side tails are correlated with discrete ambiguity mis-assignment fraction
  - ~2% mis-assignment in core
  - ~20% mis-assignment in (high-side) tail

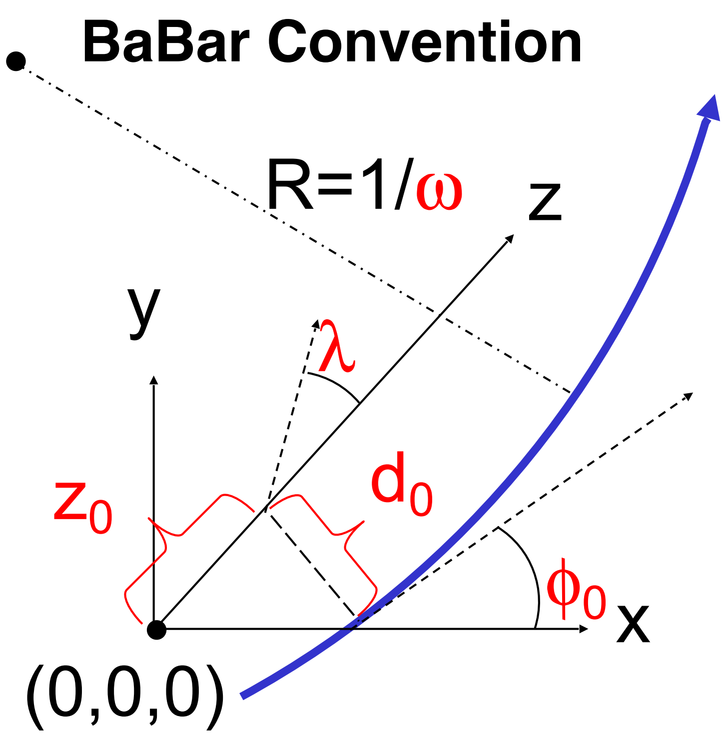




Track Fit Parameterization (Helix)

L ≡ transverse flight
P ≡ $\{d_0, \phi_0, \omega, z_0, \tan\lambda\}$

- Based on seeing a small segment of a helix arc
- Geometric description
  - with kinematic interpretation
- Arbitrary parametric variable



$$x(L) = 1/\omega \cdot \sin(\phi_0+\omega L)-(1/\omega+d_0)\sin\phi_0$$

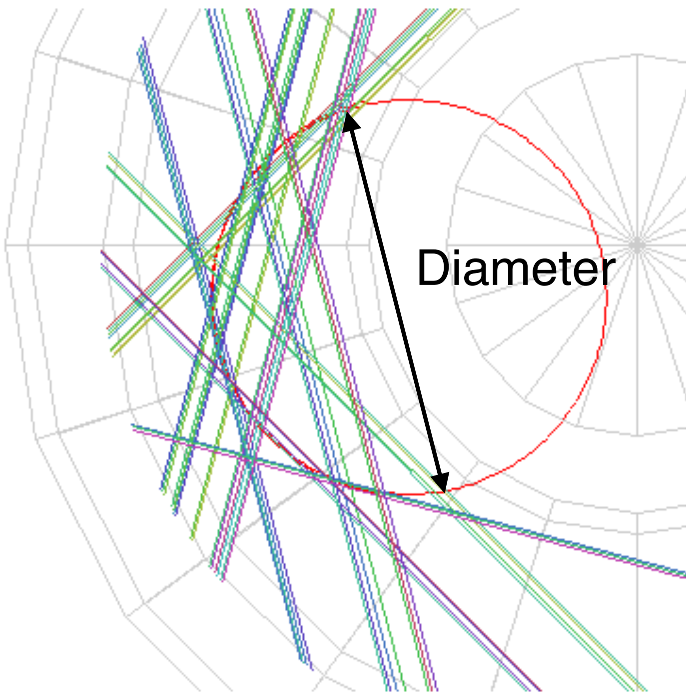
$$y(L) = -1/\omega \cdot \cos(\phi_0+\omega L)+(1/\omega+d_0)\cos\phi_0$$

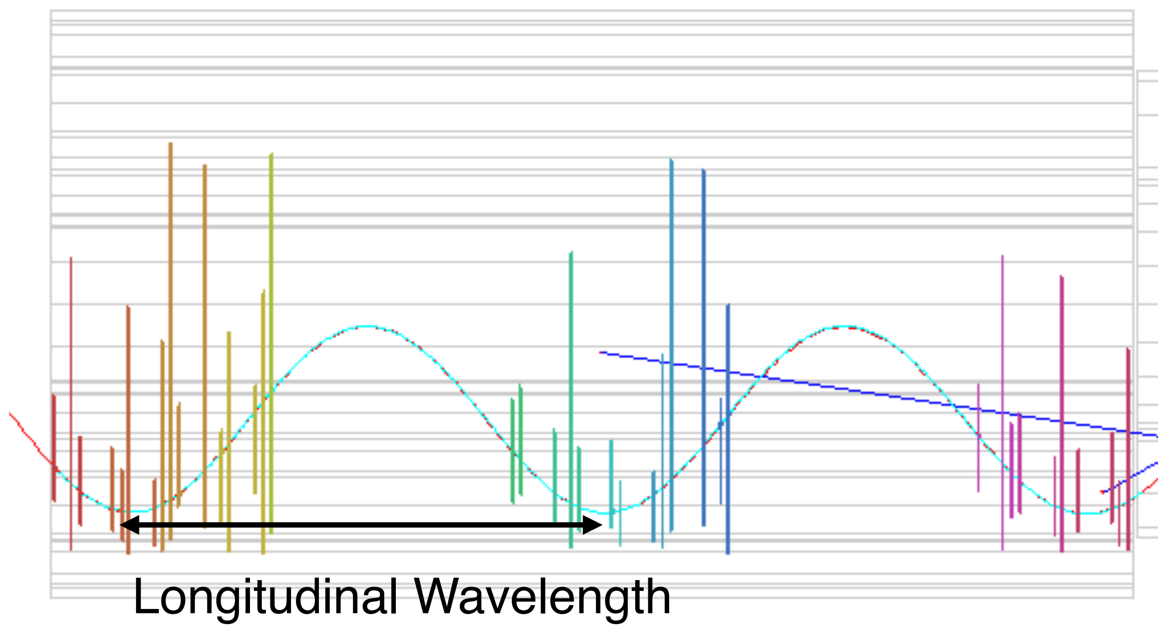
$$z(L) = z_0+L \cdot \tan\lambda$$

Natural description of low-curvature tracks coming from a known point

Natural Mu2e Track Description

- Mu2e tracks make ~3 turns
- No origin point within tracker
- Tracker measures circle **Diameter**, not sagitta
- Tracker measures **longitudinal wavelength,** not angle
- **Time**, not position, is directly measured quantity



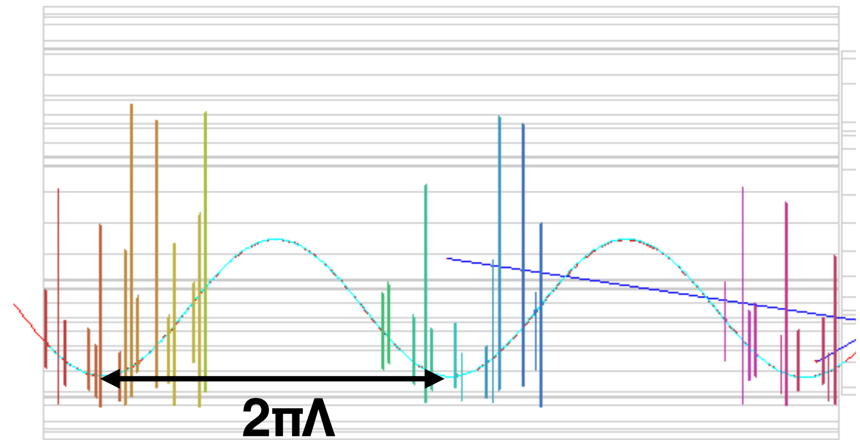


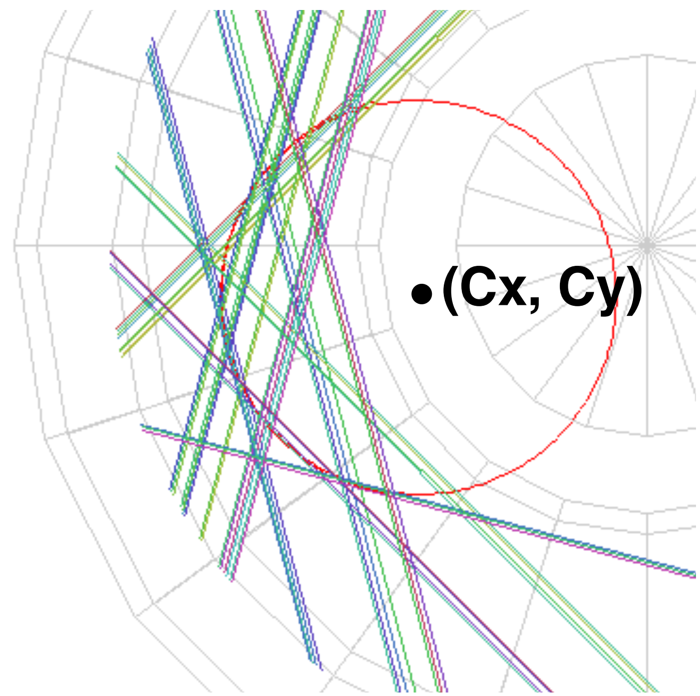
Parameterization Issues

- Purely geometric description isn't helpful
  - Scattering, energy loss depend on momentum, β
- Helix uses (arbitrary) parametric variable
  - Time is the natural physical parametric variable
- ⇒ Put time + momentum in helix description
  - Direct use of experimental observable (time)
  - Correct correlations between kinematics and geometry
  - Physical parametric variable
  - Intrinsic fit for time origin (t_0)
- 6-parameter helix

R, Λ, t Helix Parameterization

- **R** = transverse radius
  - sign(**R**) ≡ sign(dΦ/dt) = -sign(qB_z)
- **Λ** = longitudinal wavelength
  - **Λ** ≡ dz/dϕ
  - sign(**Λ**) = helicity
- **C_x,C_y** = helix axis transverse position
- **t_0** = time when particle passes z=0
- **ϕ_0** = momentum azimuth when z=0
  - **ϕ_0** ≡ atan2(P_y,P_x), t=**t_0**
- **Q** = $cqB_z(0,0,0)$
  - c = speed of light
  - B(x) = magnetic field
- **m** = particle mass





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Momentum Parameters

- **C_x,C_y** = helix axis transverse position

Position Parameters

- **t_0** = time when particle passes z=0
- **ϕ_0** = momentum azimuth when z=0
  - **ϕ_0** ≡ atan2(P_y,P_x), t=**t_0**

Convention for Mu2e

- **Q** = $cqB_z(0,0,0)$
  - c = speed of light
  - B(x) = magnetic field
- **m** = particle mass

Kinematical Parameters, not dynamic

R, Λ, t Helix Parameterization

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- **Q** = $cqB_z(0,0,0)$
  - c = speed of light
  - B(x) = magnetic field
- **m** = particle mass

Requires $P_t \neq 0$

Requires $P_z \neq 0$

Helix Equations

- Angular Velocity
- $\Omega \equiv d\Phi/dt = -c\mathbf{Q}/\mathrm{sqrt}((\mathbf{R}^2 + \mathbf{\Lambda}^2)\mathbf{Q}^2 + \mathbf{m}^2)$
- Position
- $x(t) = \mathbf{C_x} - \mathbf{R} \cdot \sin(\Omega(t-\mathbf{t_0}) + \boldsymbol{\phi_0})$
- $y(t) = \mathbf{C_y} + \mathbf{R} \cdot \cos(\Omega(t-\mathbf{t_0}) + \boldsymbol{\phi_0})$
- $z(t) = \mathbf{\Lambda}\Omega(t-\mathbf{t_0})$
- Momentum
- $P_x(t) = \mathbf{QR} \cdot \cos(\Omega(t-\mathbf{t_0}) + \boldsymbol{\phi_0})$
- $P_y(t) = \mathbf{QR} \cdot \sin(\Omega(t-\mathbf{t_0}) + \boldsymbol{\phi_0})$
- $P_z(t) = -\mathbf{Q\Lambda}$
- $|P| = \mathbf{Q} \cdot \mathrm{sqrt}(\mathbf{R}^2+\mathbf{\Lambda}^2)$

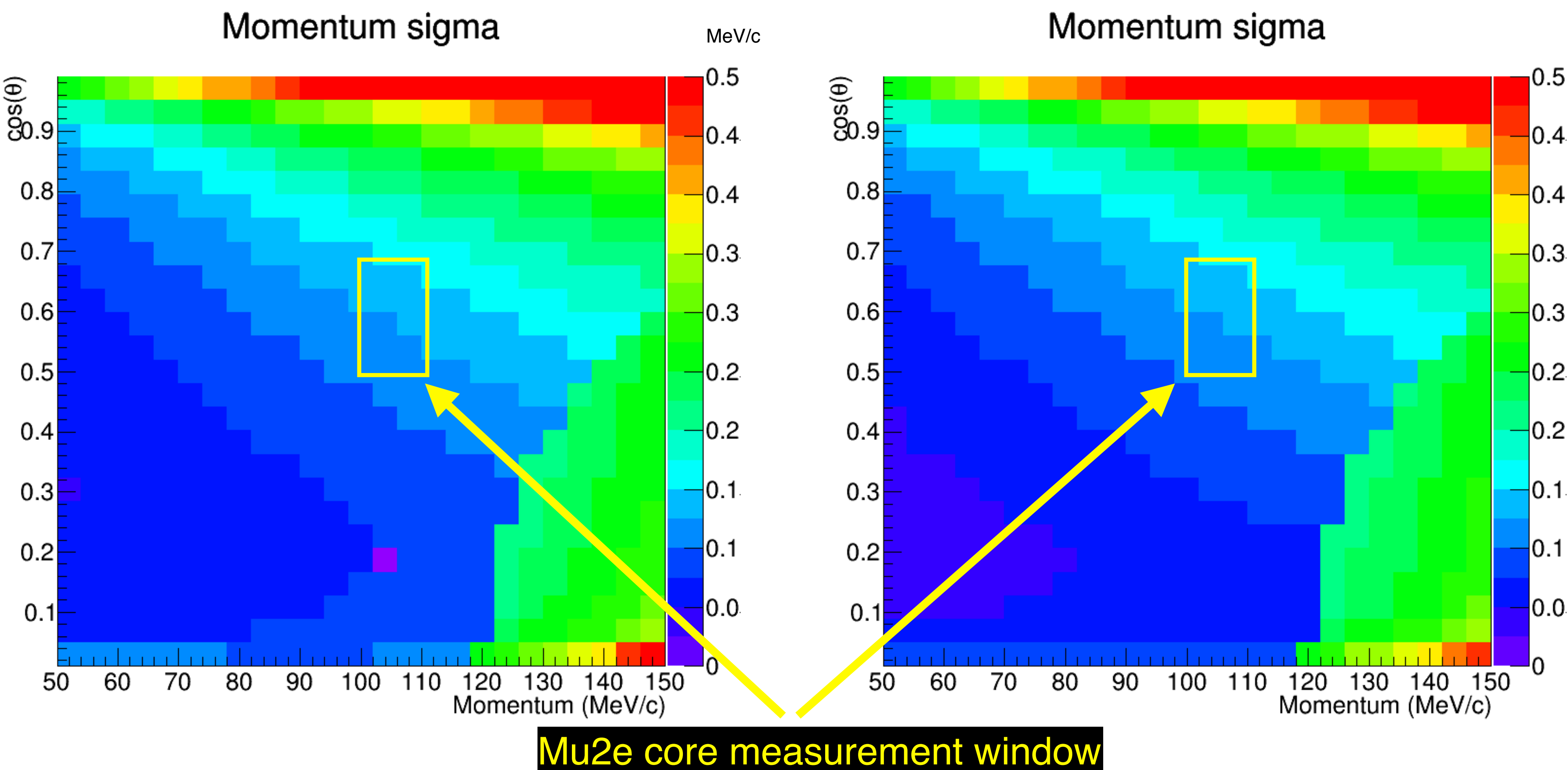
Parameterization Comparison

- Toy MC of Mu2e detector
  - 0.8 m radius × 3 m length cylindrical chamber
  - 100 µm resolution measurements ⊥ to z axis
  - 0.0002 X_0 material (scattering, ΔE) per measurement
- Study non-linearities, parameter errors and correlations in pseudo Kalman filter fit
  - Covariance computation only
  - As a function of momentum and polar angle

Estimated Momentum Error

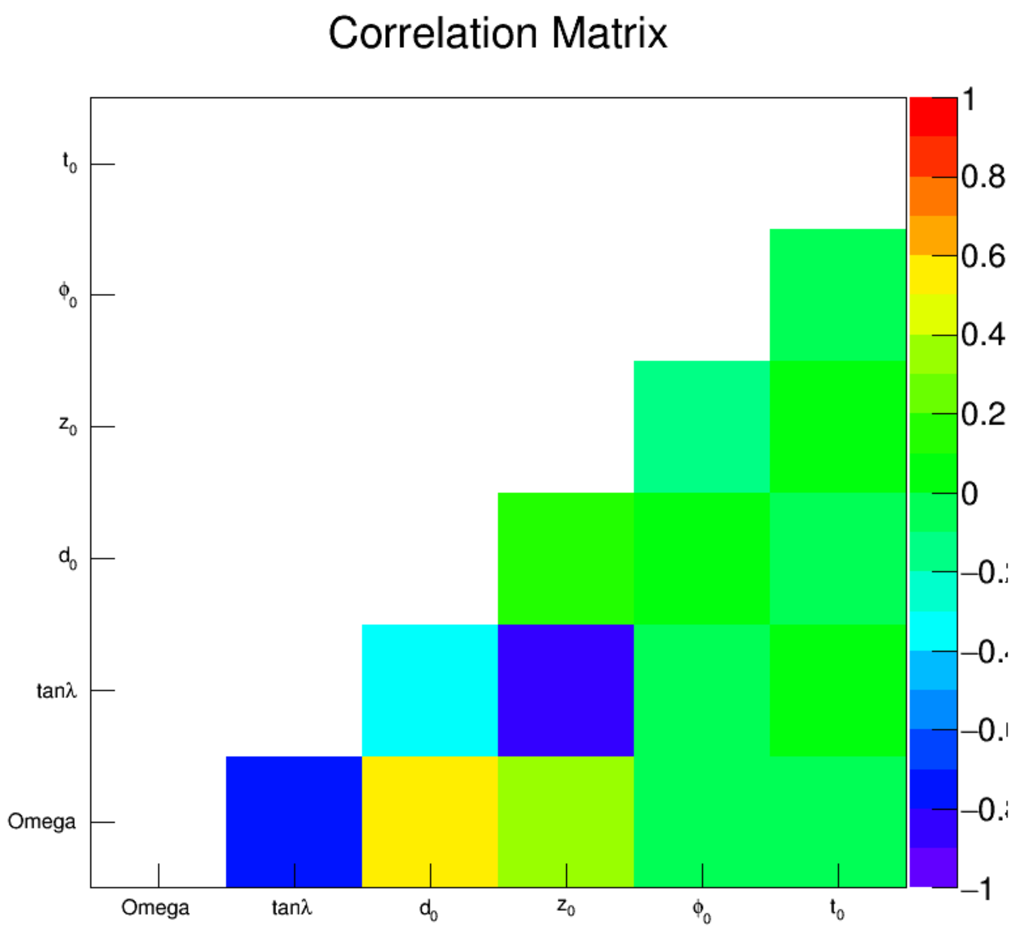
- Equivalent for both parameterizations

(ω,tanλ,L) (R,Λ,t)

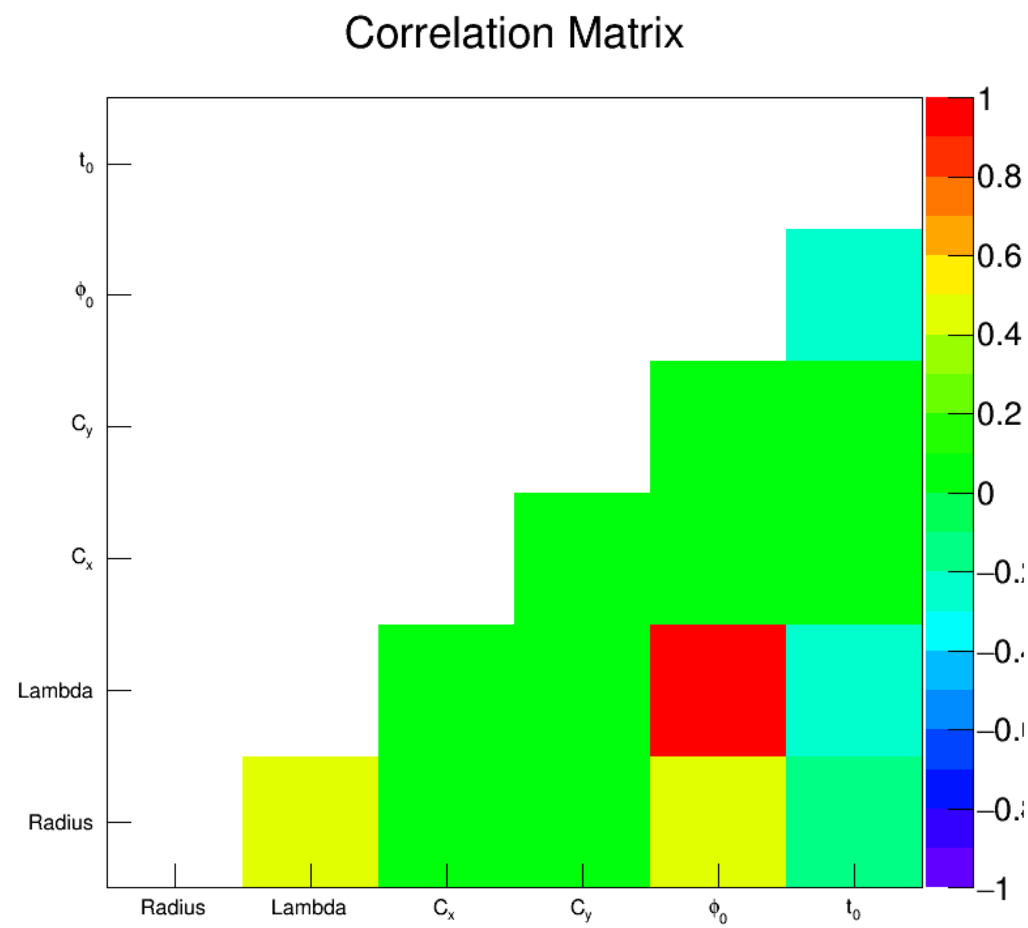


Covariance Correlations

(ω,tanλ,L)



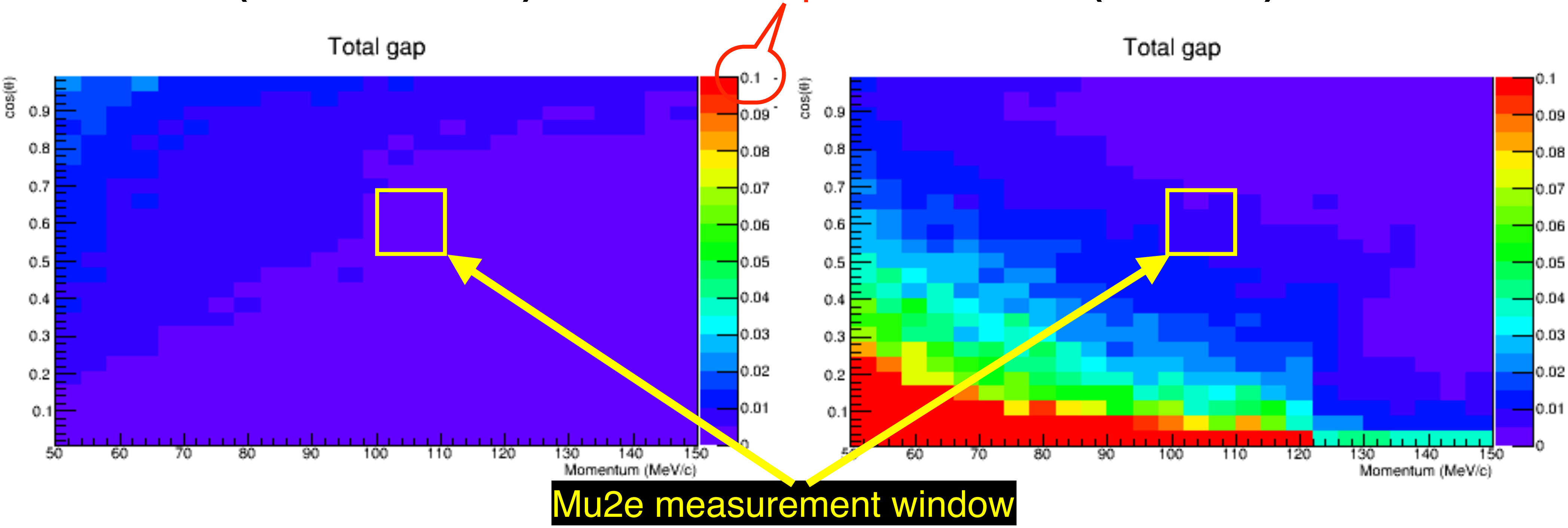
(R,Λ,t)



Trajectory Gaps

- Scattering, energy loss modeled as discrete effects
  - Track is modeled as a sequence of separate helices
- Extended KF (1st order approximation) creates small gaps between helix segments
- Size indicates (parameterization-dependent) non-linearities

(ω,tanλ,L) 100 µm (R,Λ,t)



Conclusions + Future Work

- Low-momentum, high-precision tracking imposes special requirements
  - Low mass detectors
  - 3-d measurements
  - high speed, high efficiency, high resolution reconstruction
- Mu2e has a working solution
- Outstanding issues:
  - Helix finding efficiency
  - Discrete ambiguity resolution
  - Optimal parameterization

Backup

What is Special About Low Momentum?

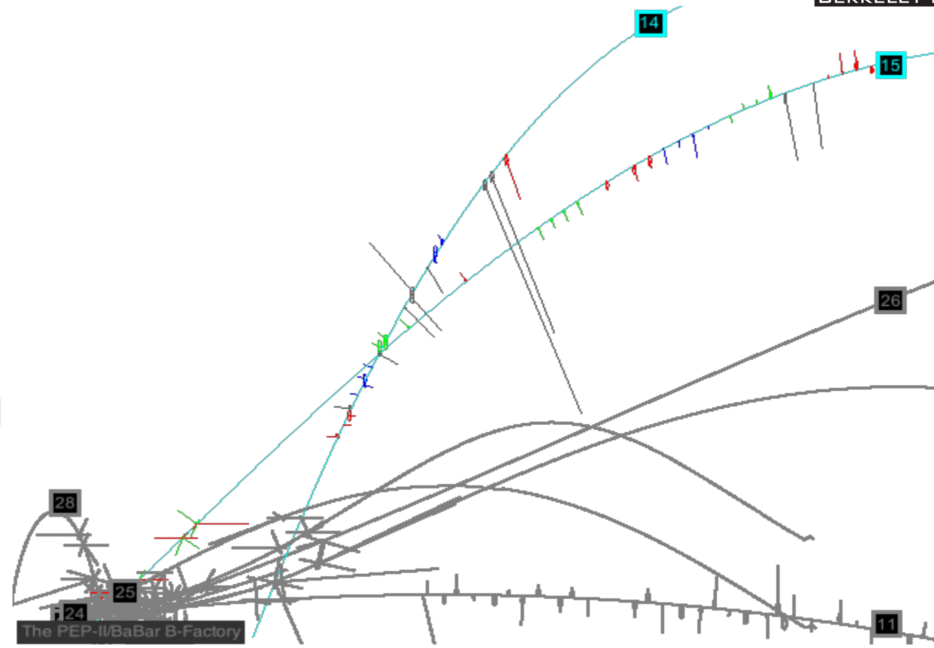
- Increased material effects
  - multiple Coulomb scattering ~$1/P\beta$
  - energy loss straggling RMS ~$1/P\beta^2$
- Looping
  - multi-turn helices overlap in x-y and time
- Overlapping intrinsic background processes
  - delta-rays, hadronic interactions, albedo, decay, …

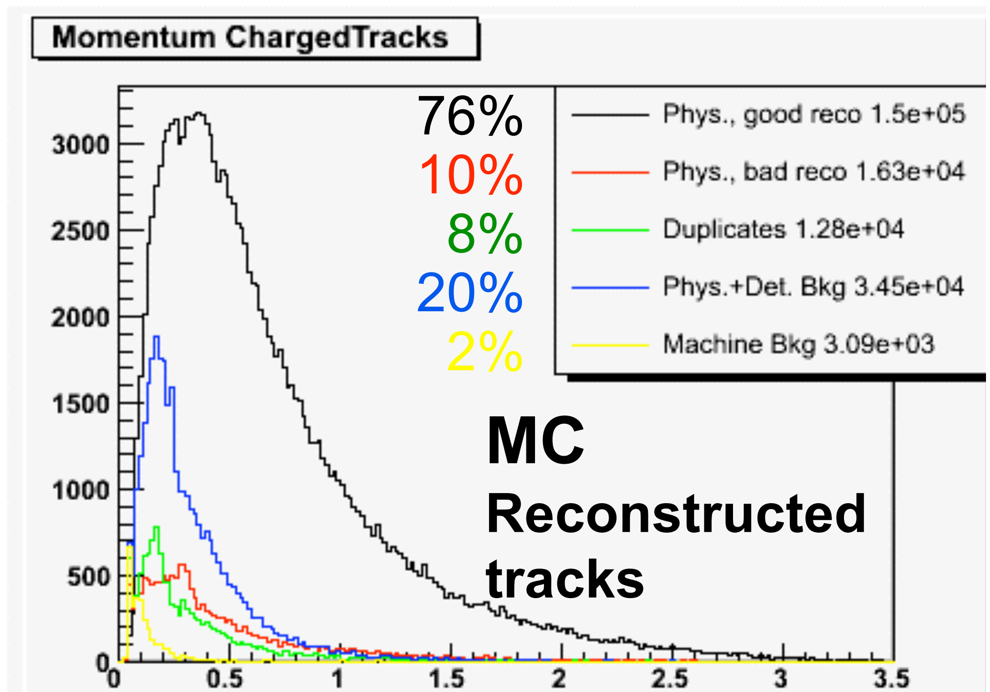
Material Effects

- Model using Lynch-Dahl formula (NIM B58 (1991))
  - Screened Rutherford cross-section
  - Parameterized by tail truncation factor
    - Can be tuned to model reconstruction truncation
- Most probable value for energy loss, straggling
  - Mean is biased towards tail
  - Landau tails are ‘self-truncated’ by pat. rec.

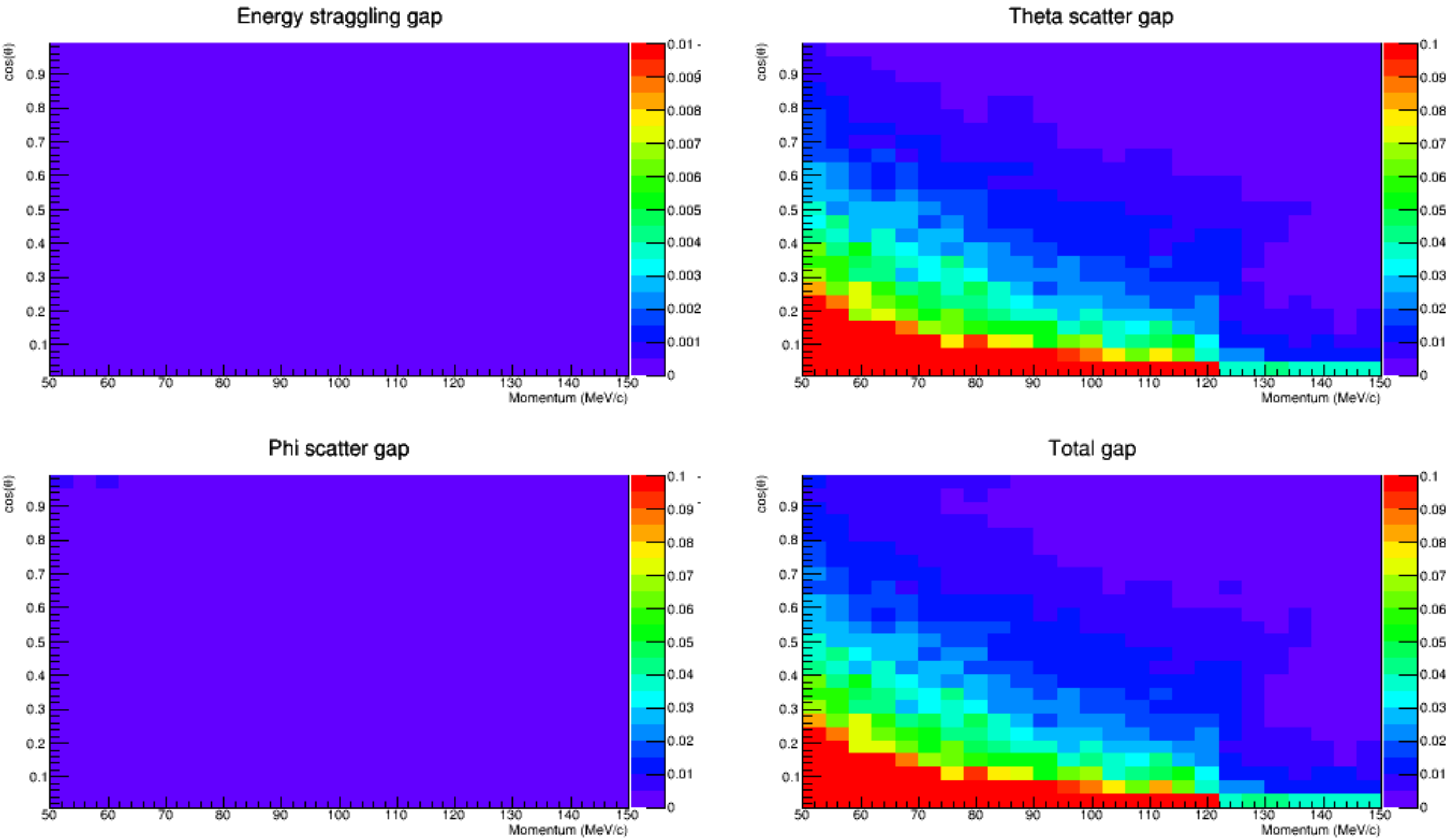
BaBar Experience

- Low momentum tracks loop in the drift chamber
  - transverse hit overlaps
- Two kinds of reconstruction errors:
  - Loop branch found as primary particle
  - branch hits mis-assigned to primary
- BaBar ‘solution’: open track finding, filter at physics level
  - ~10% fake tracks
  - ~10% compromised tracks

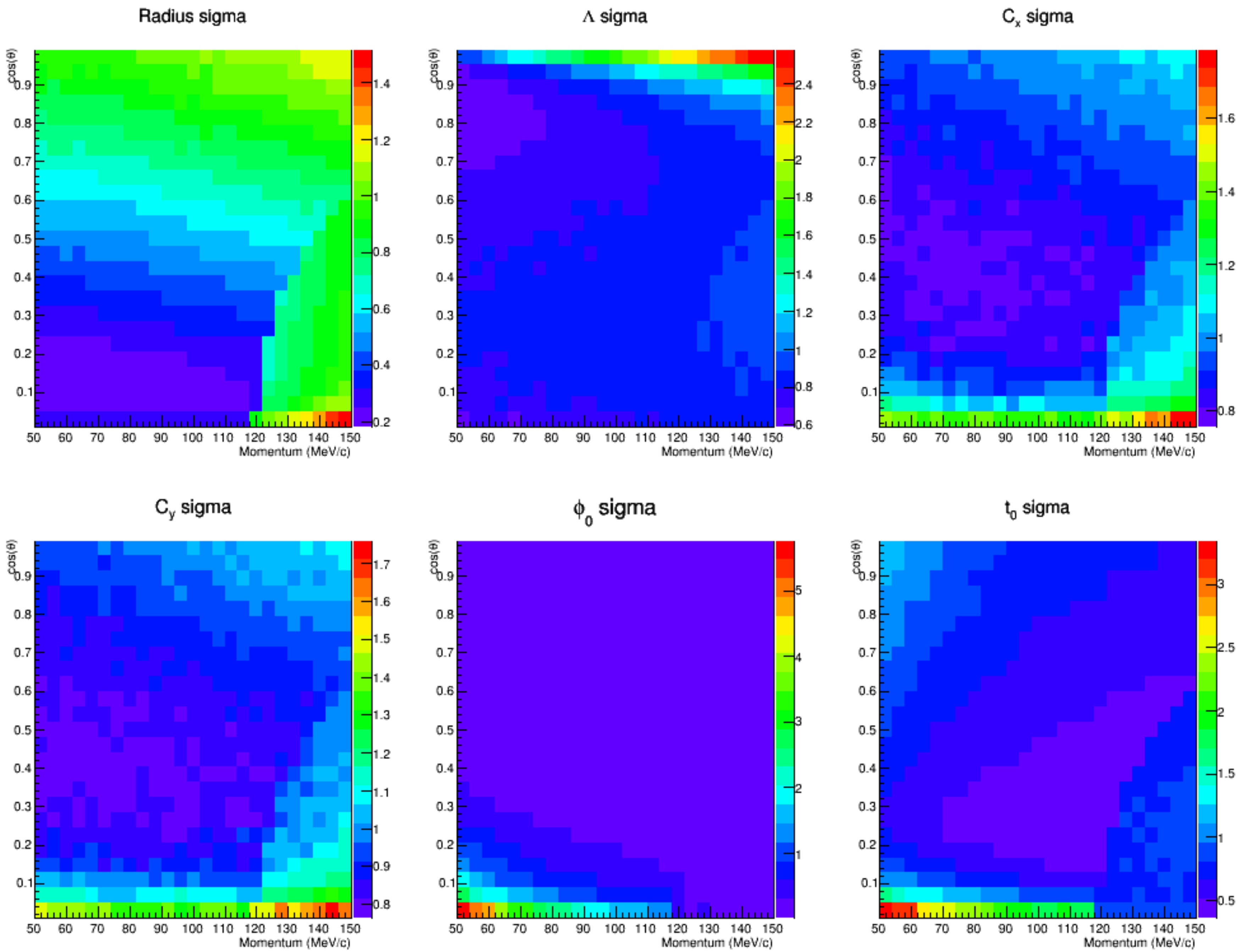




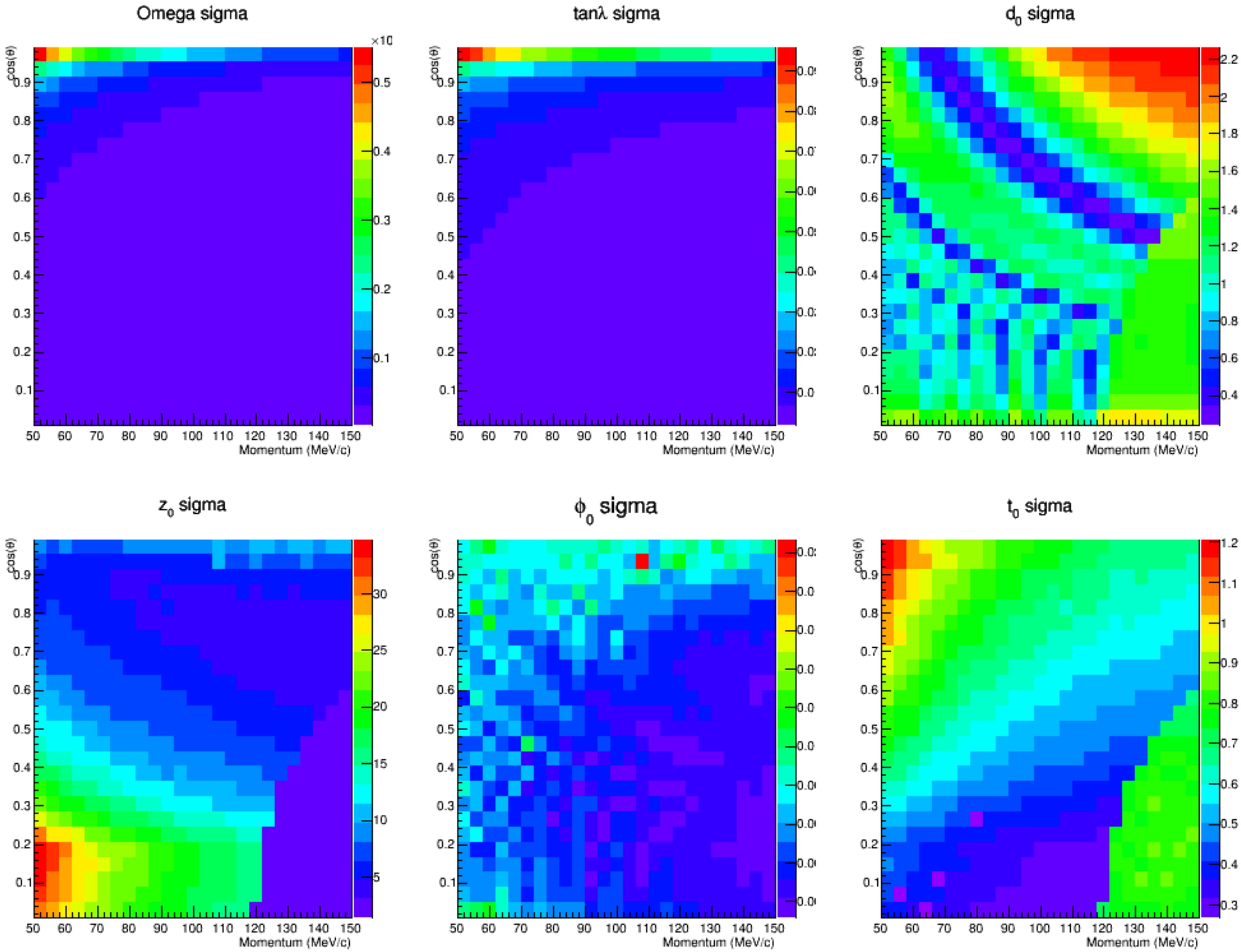
(R,Λ,t) Helix Trajectory Gaps



(R,Λ,t) Parameter Errors

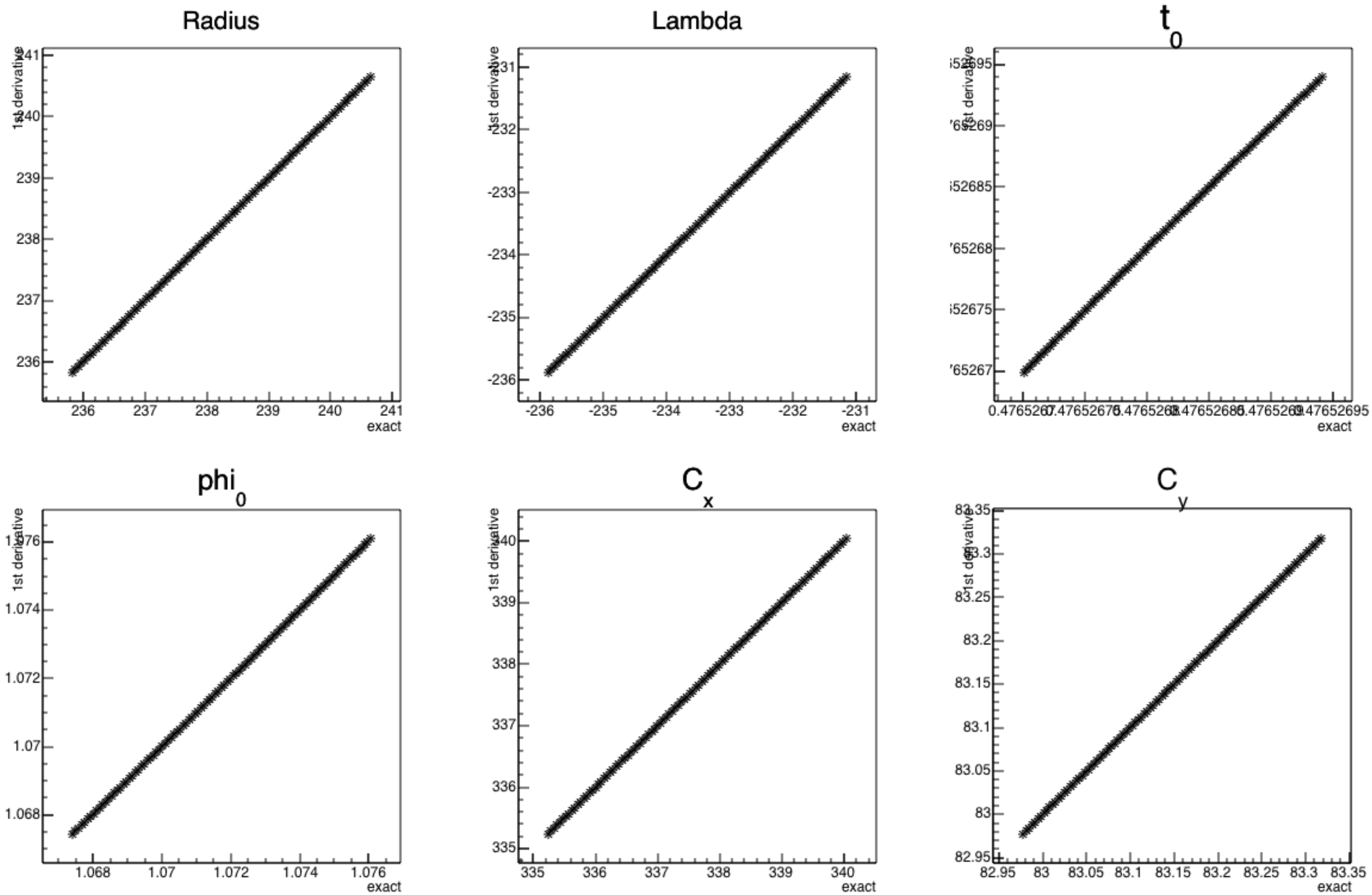


(ω.tanλ.L) Parameter Errors



Helix Derivative Tests

Study change in parameters with ±1% momentum magnitude change



X-axis = exact solution, Y-axis = 1st derivative