Single Electrons in PIXeY

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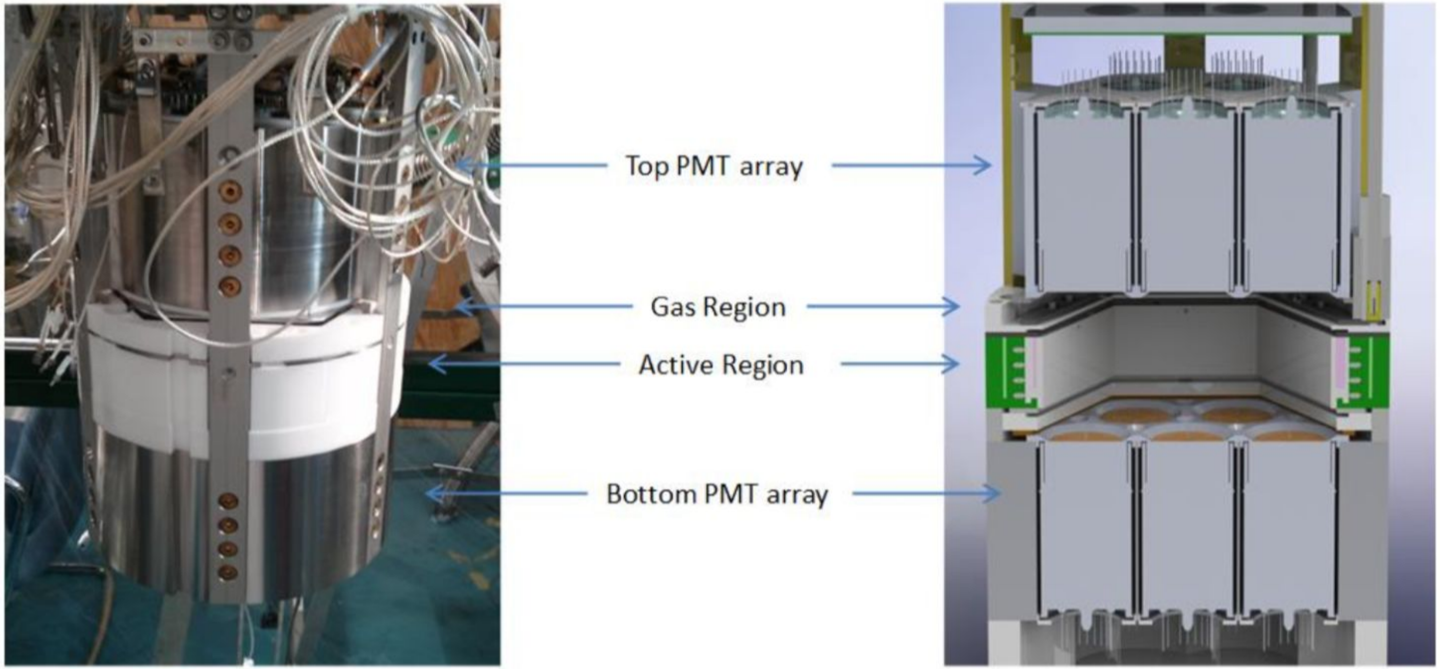
PIXeY Detector

1. The hexagonal volume has **side lengths of 9.2 cm** and a **cathode-to-gate drift length of 5.1 cm**.
2. Two arrays of seven **Hamamatsu R8778** photomultiplier tubes (PMTs) with a 33% quantum efficiency
3. The PMT signal undergoes an eightfold amplification and is digitized with a 12-bit ADC (CAEN V1720) at 250 MHz



Kr-85 Events Selection: Overview

Find the pulses (using RT algorithm)

First pass: Identify all physical pulses

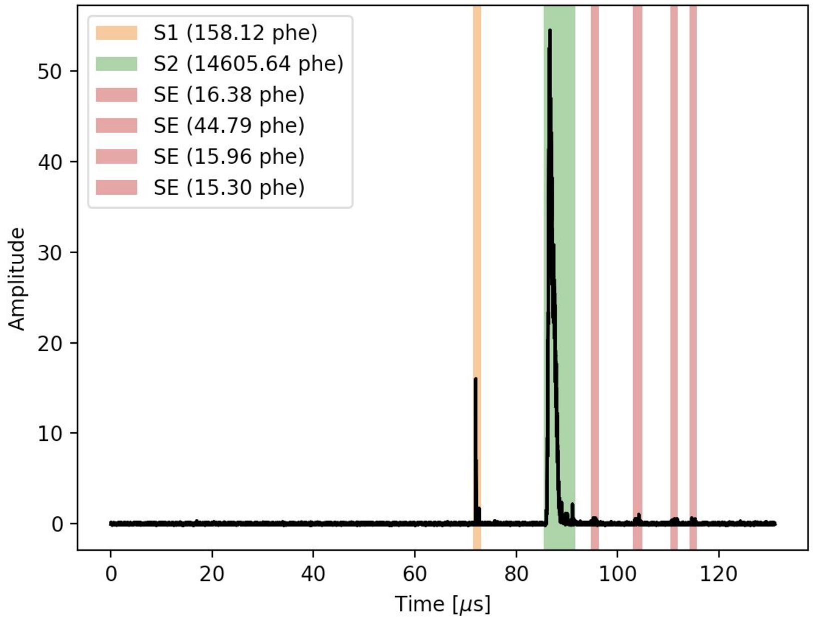
Classify “s1”, “s2”

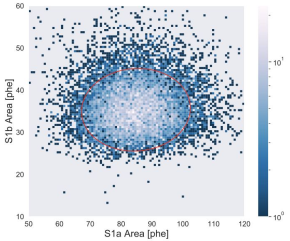
(risetime 10-90% -area)

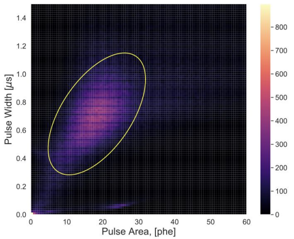
Feed the waveform ignoring s1 and s2 into second pass

Second Pass: Spectrogram

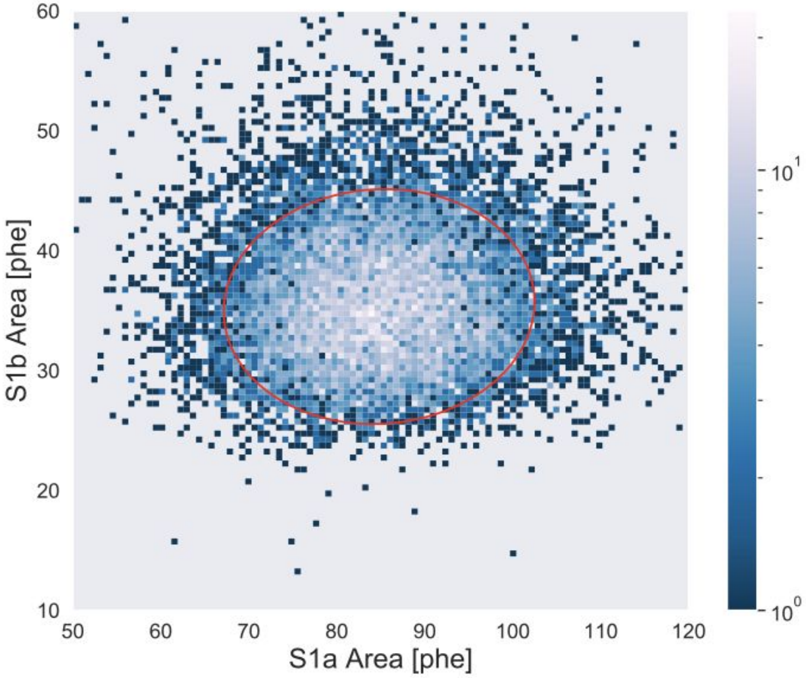
Single Electrons

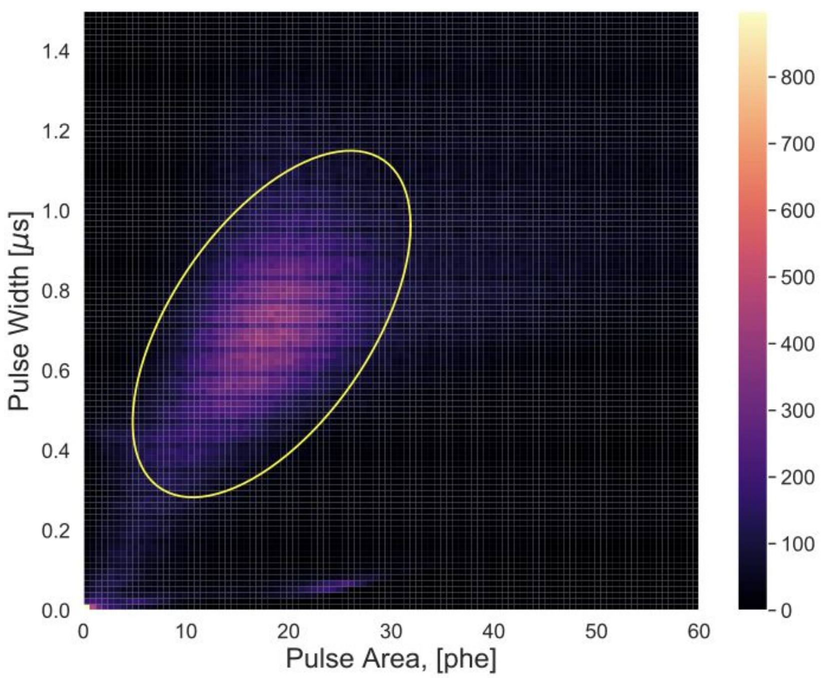






Kr-85 Events Selection: Cuts



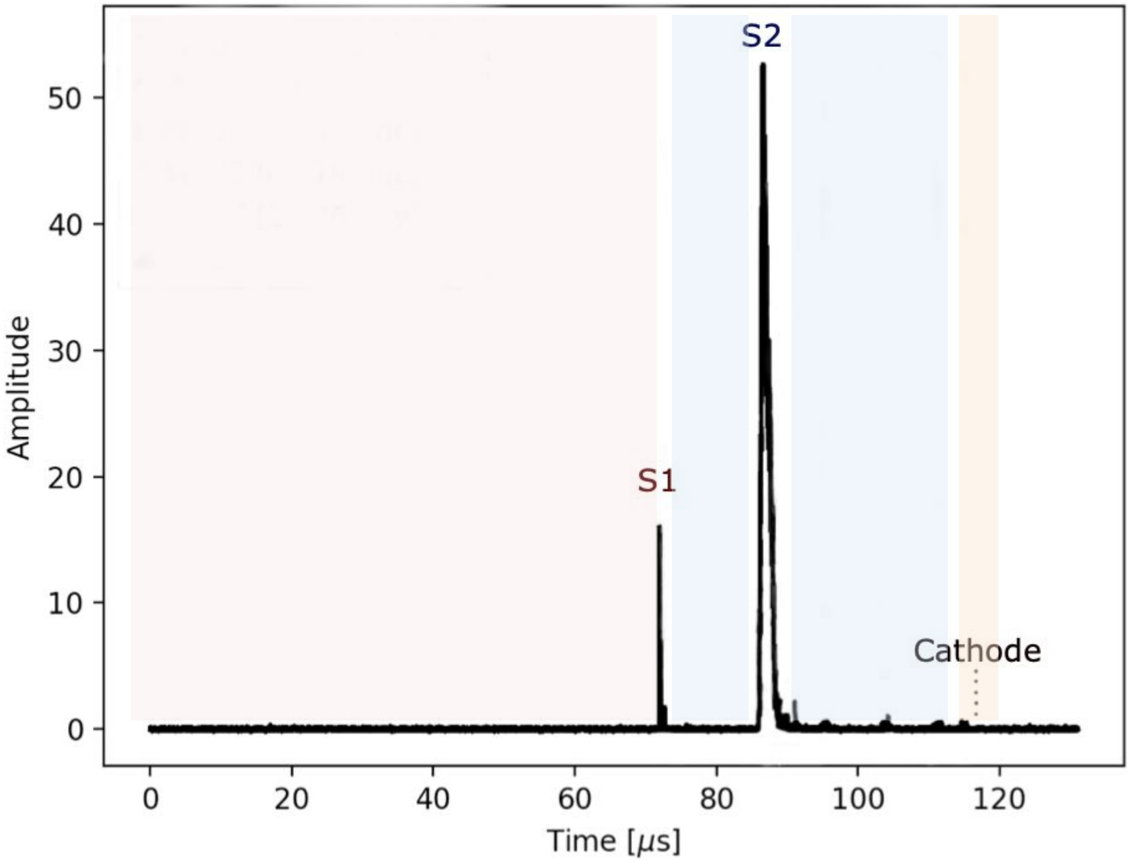


Kr event rate: ~ 20 Hz

Extraction Field (kV/cm)	Single Electron Size (phe)	Single Electron Width (μs)
2.7	11.15 ± 3.18	2.34 ± 0.37
3.6	15.26 ± 3.17	2.15 ± 0.26
4.4	18.91 ± 3.14	1.91 ± 0.18
5.3	23.75 ± 3.20	1.84 ± 0.18
5.6	25.62 ± 4.20	1.82 ± 0.19
6.1	27.65 ± 4.87	1.77 ± 0.19
6.6	28.32 ± 5.44	1.73 ± 0.19
7.1	29.14 ± 5.76	1.69 ± 0.18

Table 1. The signal magnitudes and widths for single electrons observed in ^{83m}Kr events.

Regions for the Single Electrons



1. Long timescale (~ s):

a. General background: before the S1 pulse

2. Short time scale (~ us):

a. Between S1 and S2
b. After the S2

3. Cathode spike:

a. Quantum efficiency
b. Extended Fowler-Nordheim theory

The Big Picture

Single Electron Sources

TPC grids (cathode, gate)

Neutral impurities

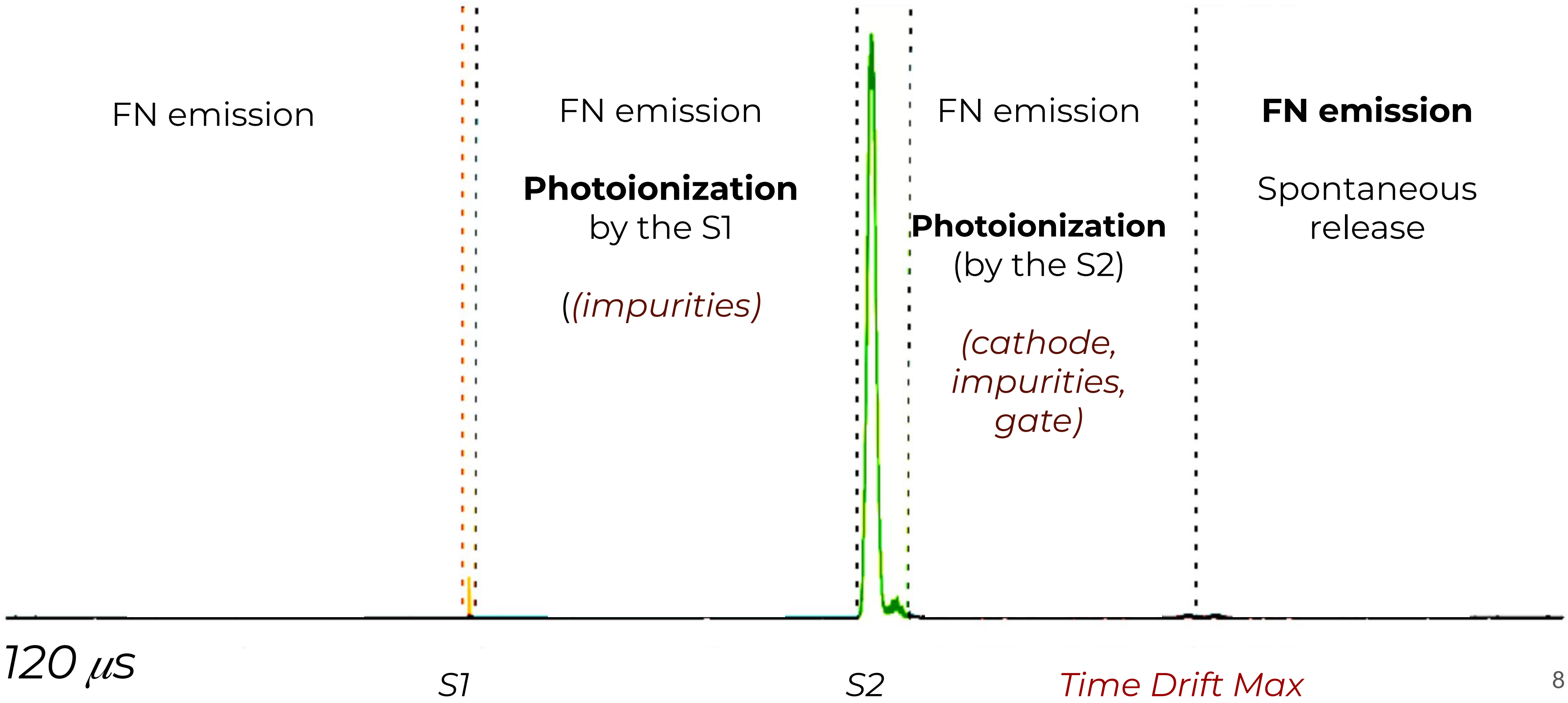
Emission Mechanisms

Spontaneous release

Fowler-Nordheim (FN) tunneling

Photoionization

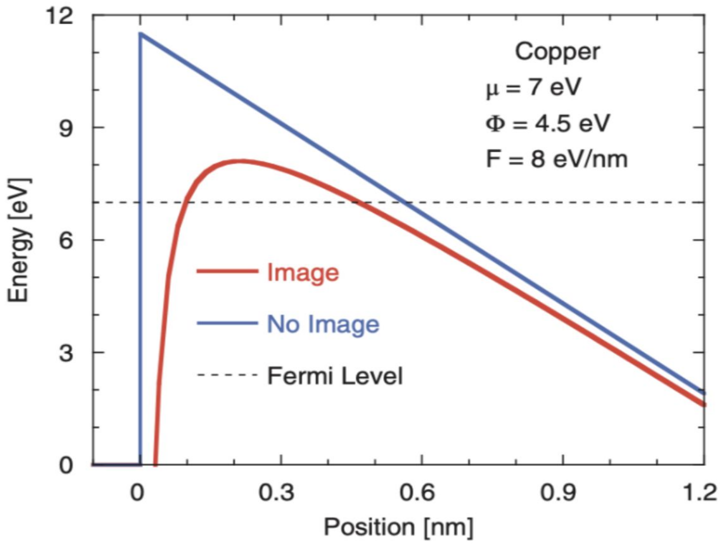
The Big Picture



Fowler-Nordheim: Overview

1. Oppenheimer, Schottky,Millikan and Eyring , Fowler and Nordheim (~ 1920)
2. Pure tunneling effect
3. Typical operation params: T ~ 100-400 K

   For T ~ 1000K: (Richardson–Laue–Dushman theory)
4. Electrons with energies closer to Fermi level are more likely to tunnel through
5. High surface field values resulting emission
6. Triangular barrier approximation gives FN equation as a solution



$$V_{image} = \mu - F \cdot x - \frac{Q}{x} + \Phi$$

$$j(E) = \frac{A_0 E^2}{\phi t^2(y)} \cdot \exp[\frac{-B_0 \cdot \phi^{3/2} \cdot v(y)}{E}]$$

$$R_e(E) = \frac{A}{e} \frac{1.54 \times 10^{-6} E^2}{\phi} \exp\left(\frac{-6.83 \times 10^9 \phi^{3/2}}{E}\right)$$

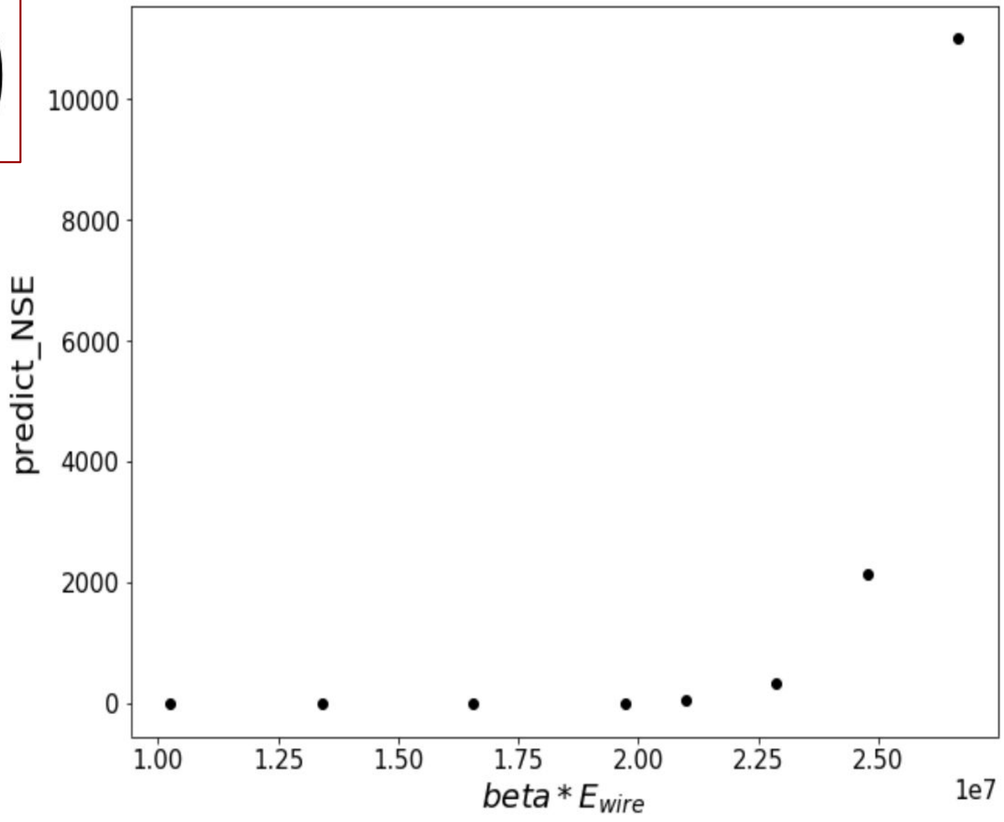
Fowler-Nordheim Theory: How to predict FN current

$$R_e(E) = \frac{A}{e}\frac{1.54 \times 10^{-6} E^2}{\phi} \exp\left(\frac{-6.83 \times 10^9 \phi^{3/2}}{E}\right)$$

$$\beta = \frac{E}{E_w} \qquad E_{wire} = E_{ext} \cdot \frac{p}{d},$$

In this plot, β = 120 and φ = 4.71 eV,
Monel wire (a copper-nickel alloy)

Before S1 region



Fowler-Nordheim Plot

$$j(E) = \frac{A_0 E^2}{\phi t^2(y)} \cdot \exp[\frac{-B_0 \cdot \phi^{3/2} \cdot v(y)}{E}]$$

- Beta is extracted from the slope:

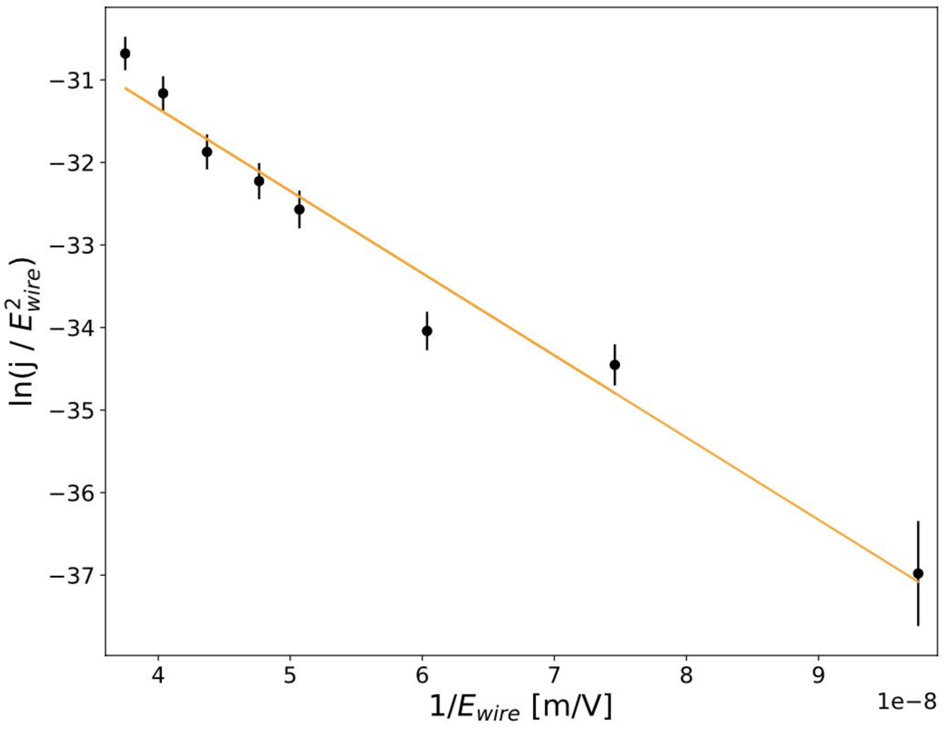
$$t = -6.83 \cdot 10^9 \cdot \phi_a^{3/2}$$

$$\beta = \frac{t}{m}$$

β : 420 +- 8

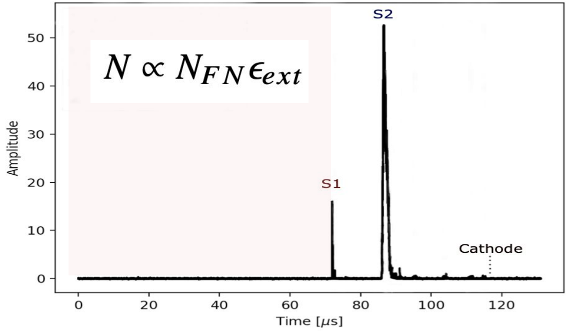
t -target slope (theoretical)
m - measure slope (from the data)

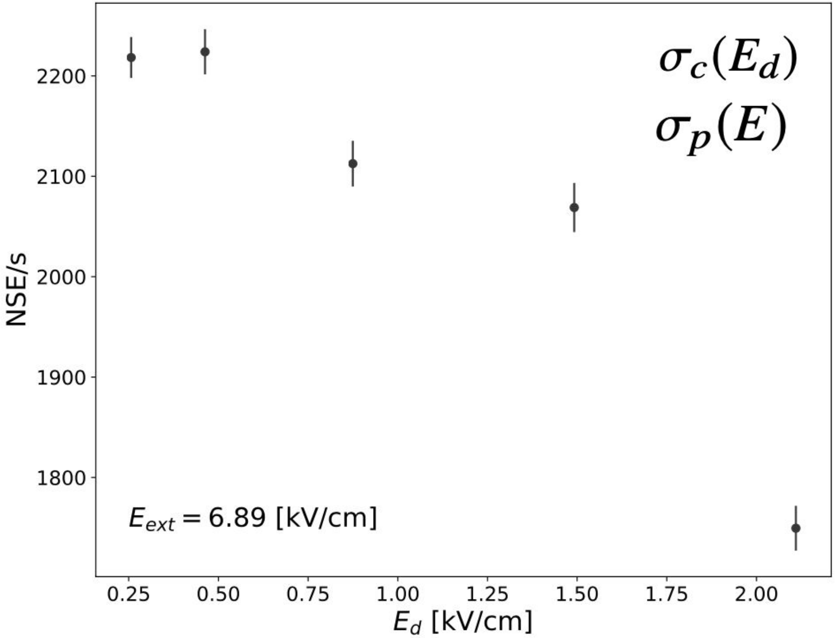
- Error bars are mainly coming from the extraction efficiency and the extraction field

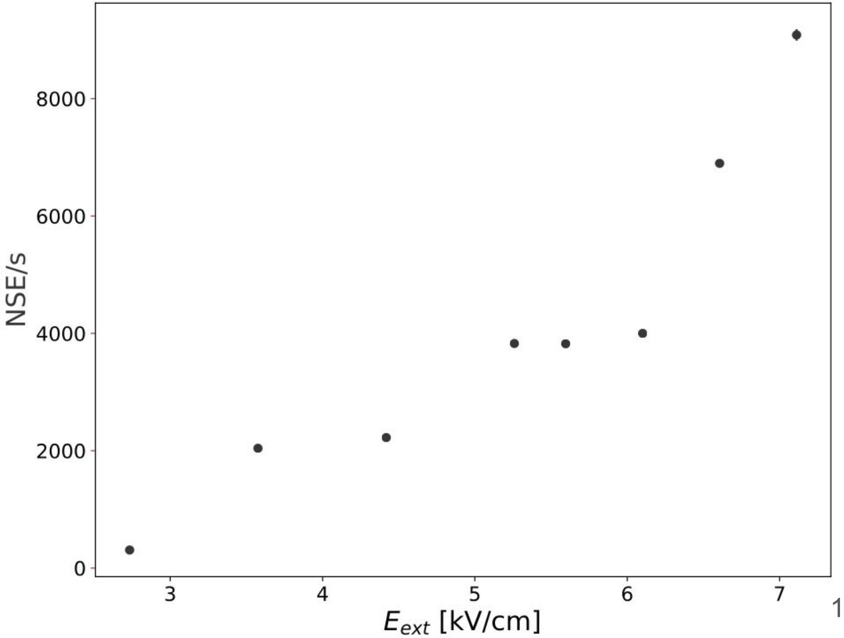


Single Electron Rate: Long timescale

- FN emission
- NSE rate scales as the product of FN emission and the electron extraction efficiency
- Sign of LXe impurities with field-dependent capturing cross section/field-dependent photoionization cross section

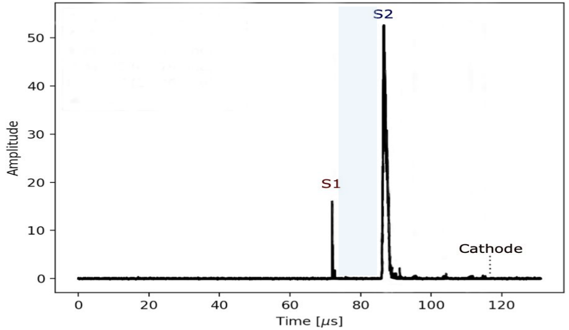


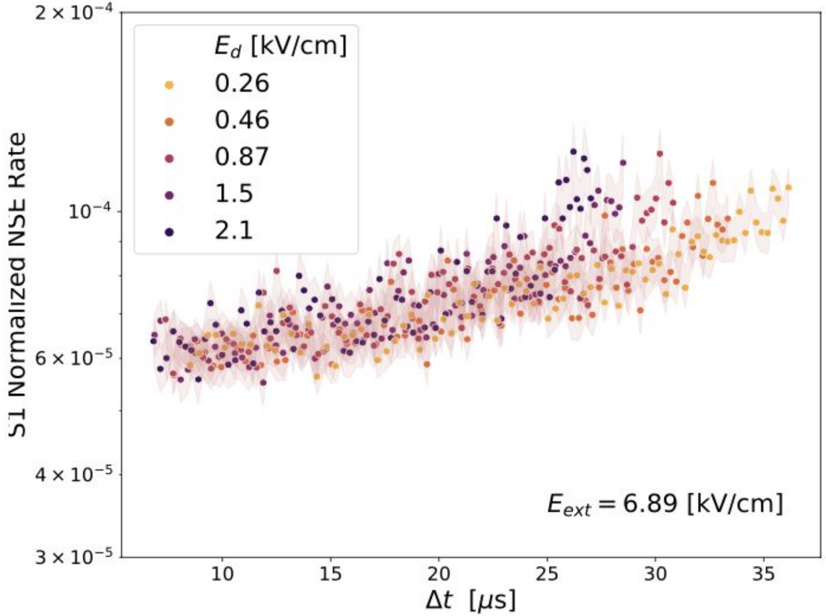


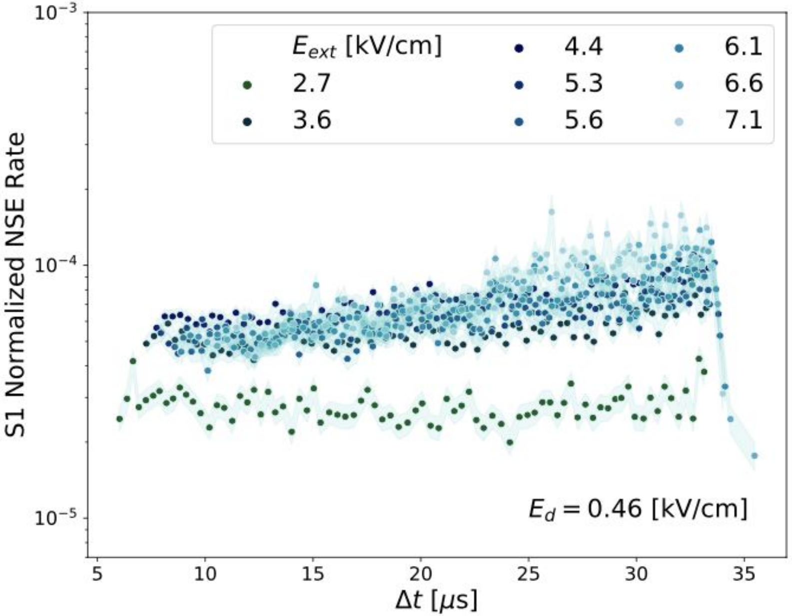


Single Electron Rate: Between S1 and S2

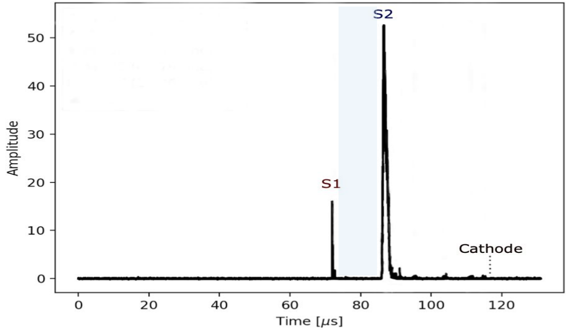
- FN emission
- Neutral impurities



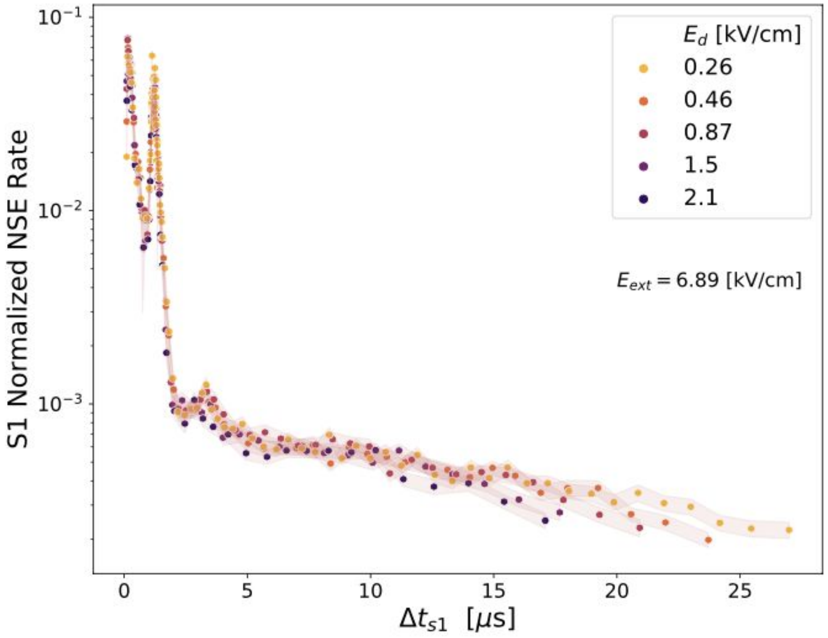


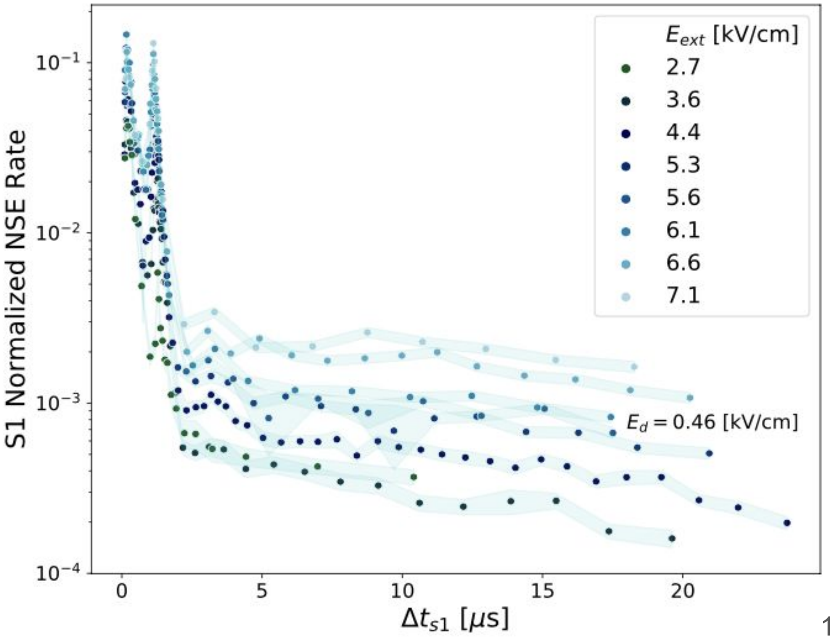


Single Electron Rate: Between S1 and S2



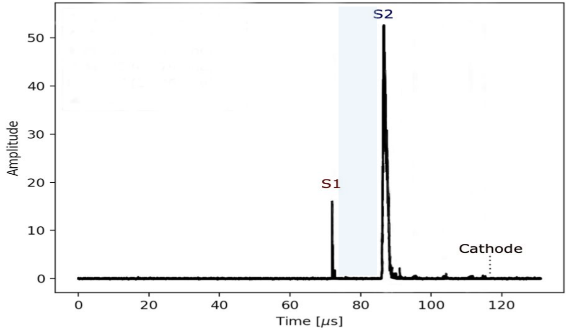
- FN emission
- Neutral impurities
- Field dependent impurity-related cross section

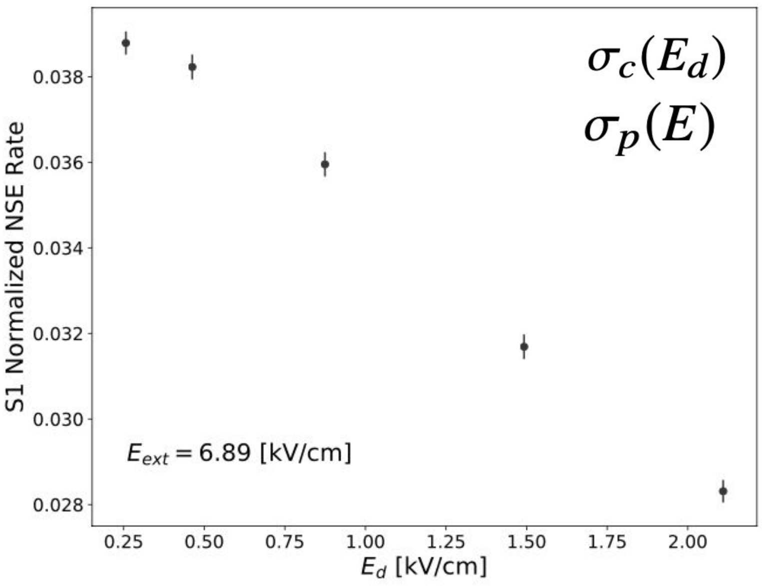


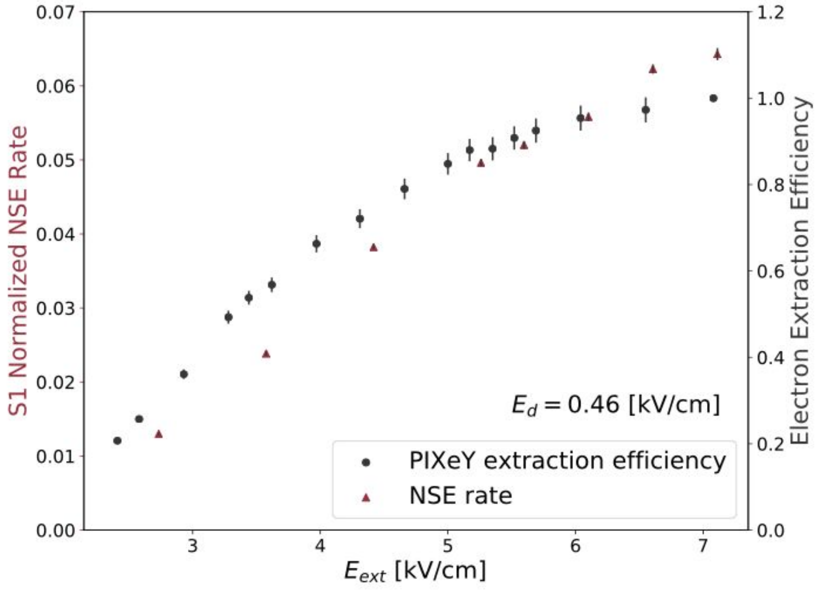


Single Electron Rate: Between S1 and S2

- FN emission
- Neutral impurities
- NSE rate scales linearly as the extraction efficiency (photoioniza is dominant)

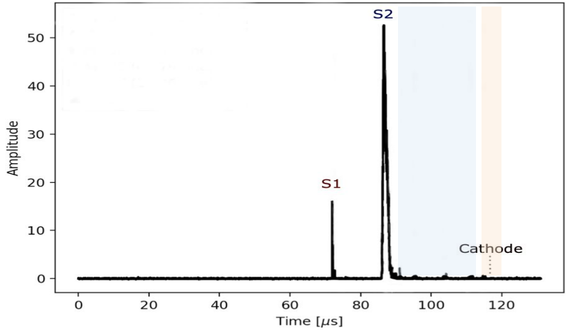


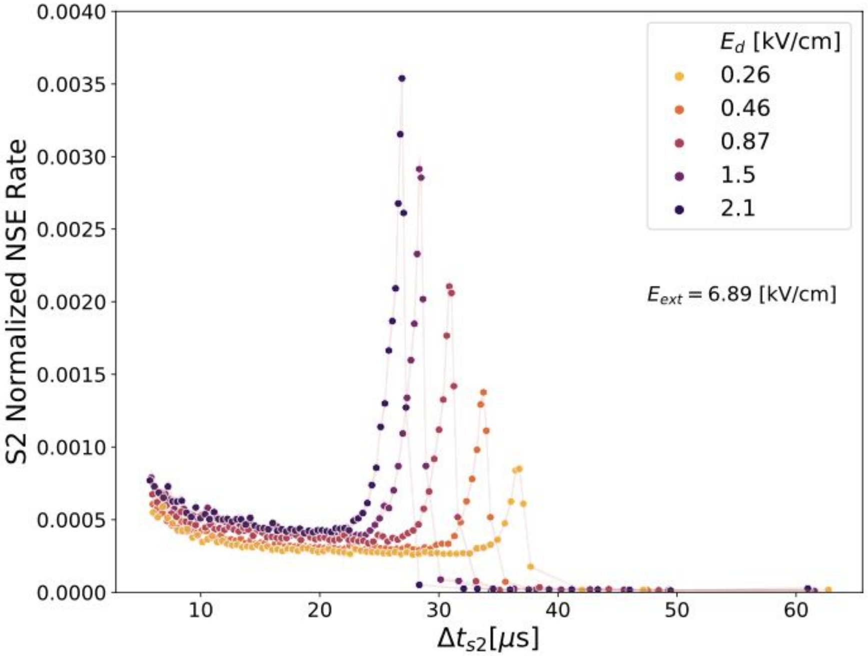


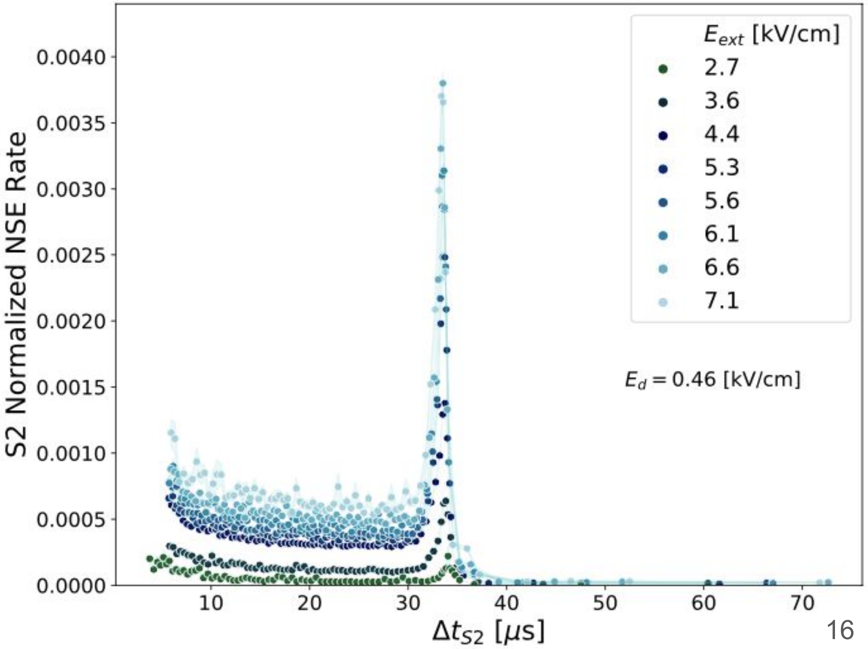


Single Electron Rate: After S2

- FN emission
- **Cathode spikes:**
    - Schottky potential decreased under E -> photoeffect due to S2 is enhanced
    - Quantum efficiency is growing as a function of E field

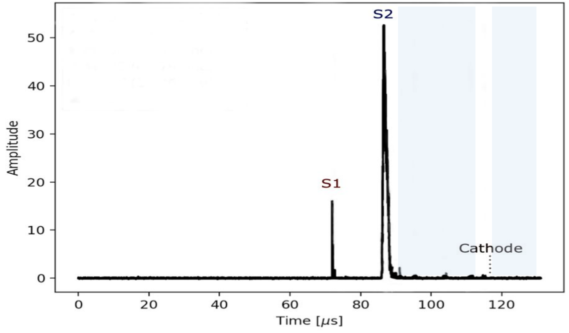


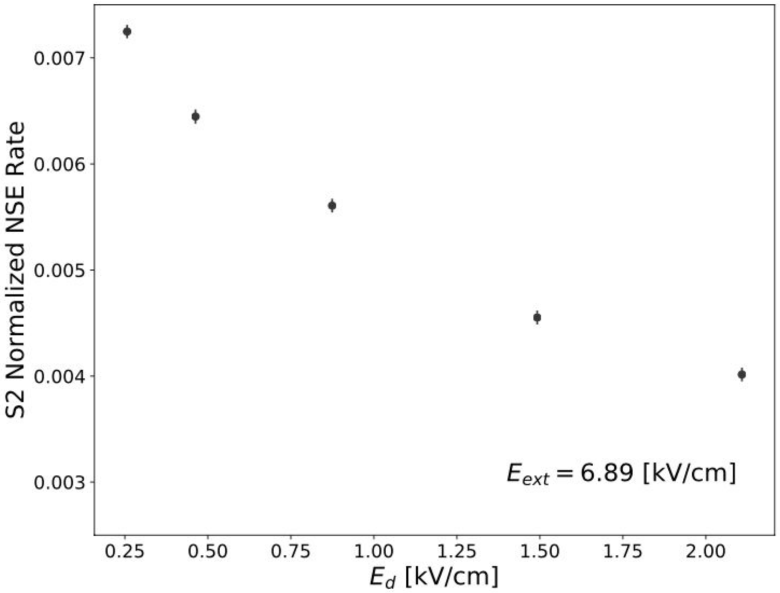


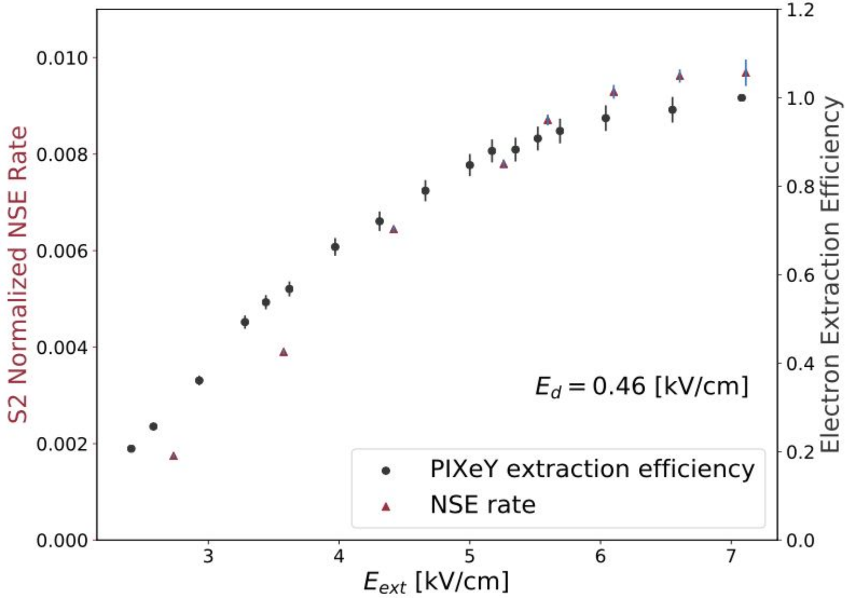


Single Electron Rate: After S2

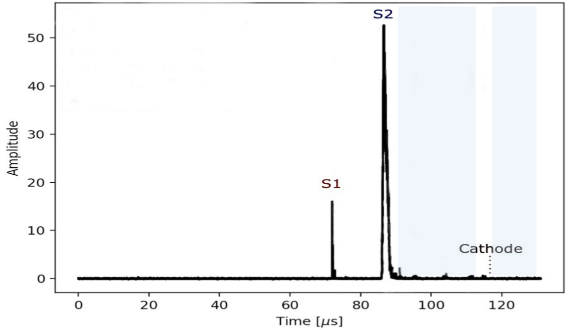
- Everything is similar to the region “between S1 and S2” except now more light -> more photoionization
- FN emission
- NSE rate scales linearly as the extraction efficiency (photoionization mechanism is dominant) -> SE are not trapped





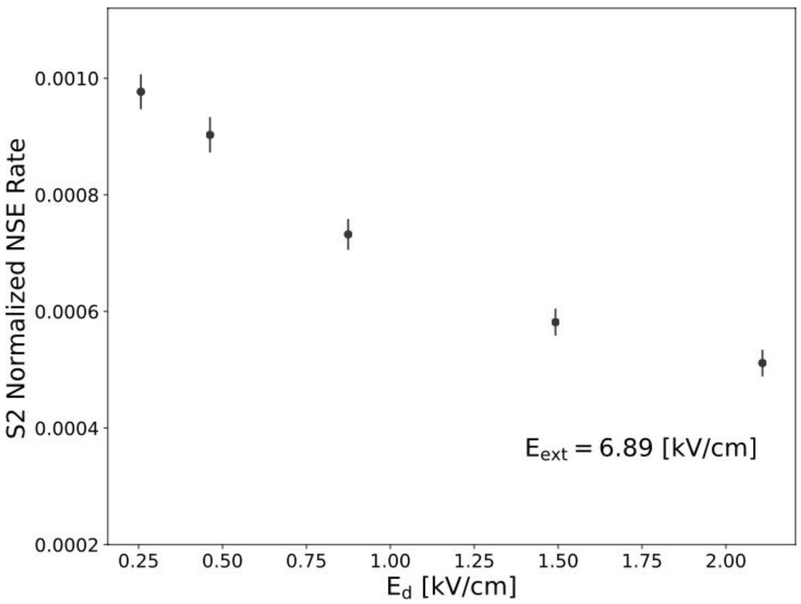


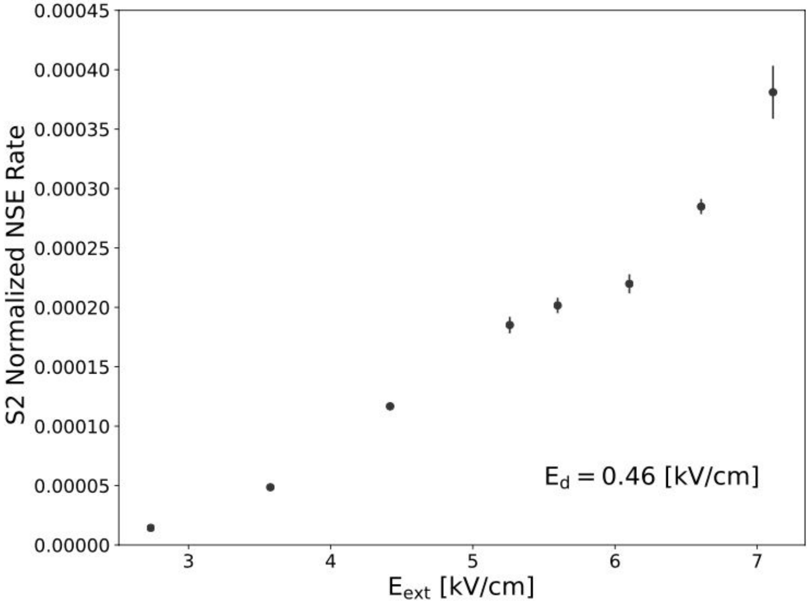
Single Electron Rate: After S2



- Everything is similar to the region "before the S1" more SE released spontaneously

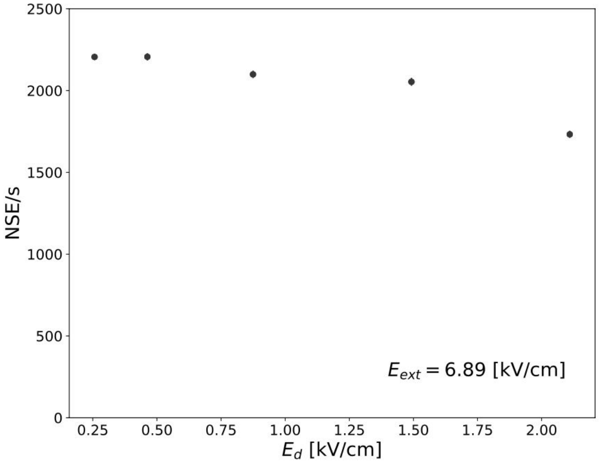
- FN emission

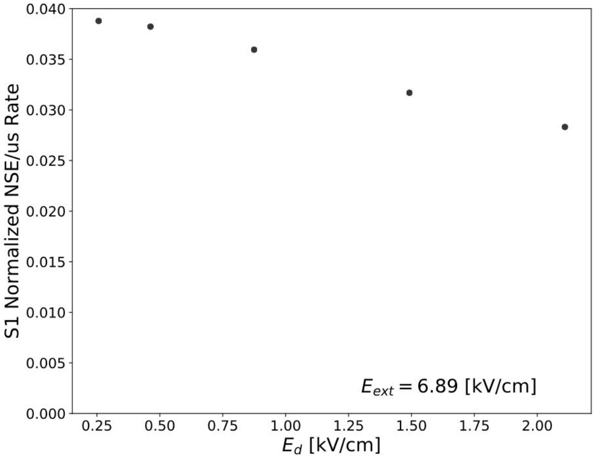


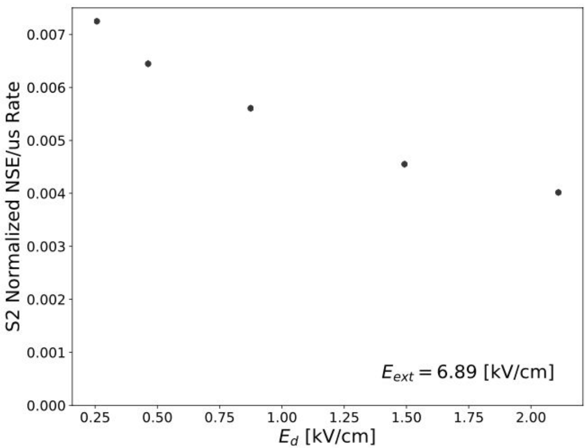


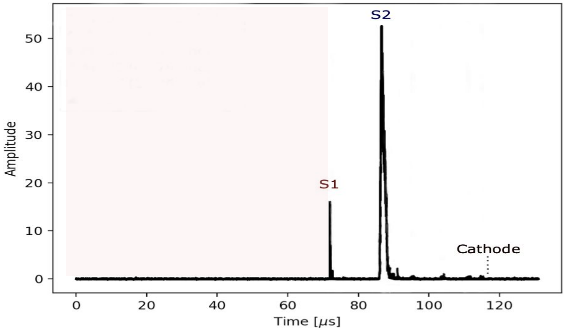
Discussion about the impurities in LXe

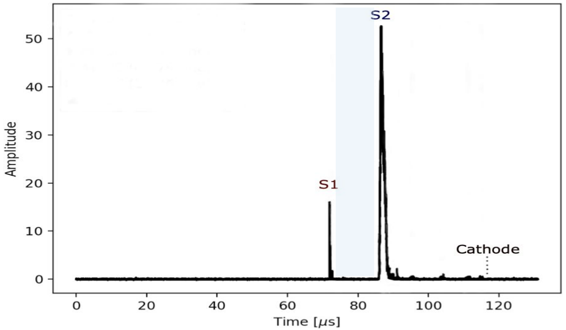
$$N \propto N_{FN}\epsilon_{ext} + \epsilon_{ext}\cdot S\cdot\left(1-\rho\int_0^\infty \sigma_c(E_d)\cdot f(v,E)\,dv\right)$$ **or** $$N \propto N_{FN}\epsilon_{ext} + \epsilon_{ext}\cdot S\cdot \rho_p\cdot\sigma_p(E_d)\cdot G$$

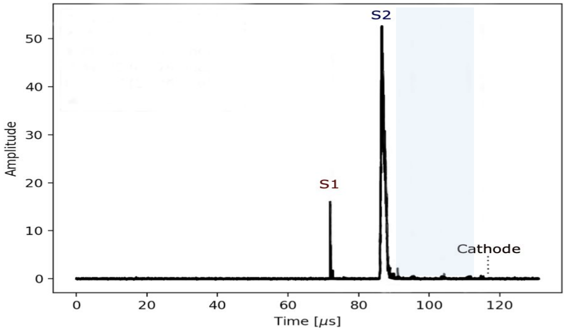






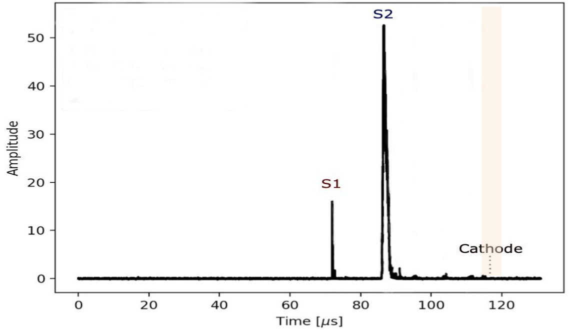


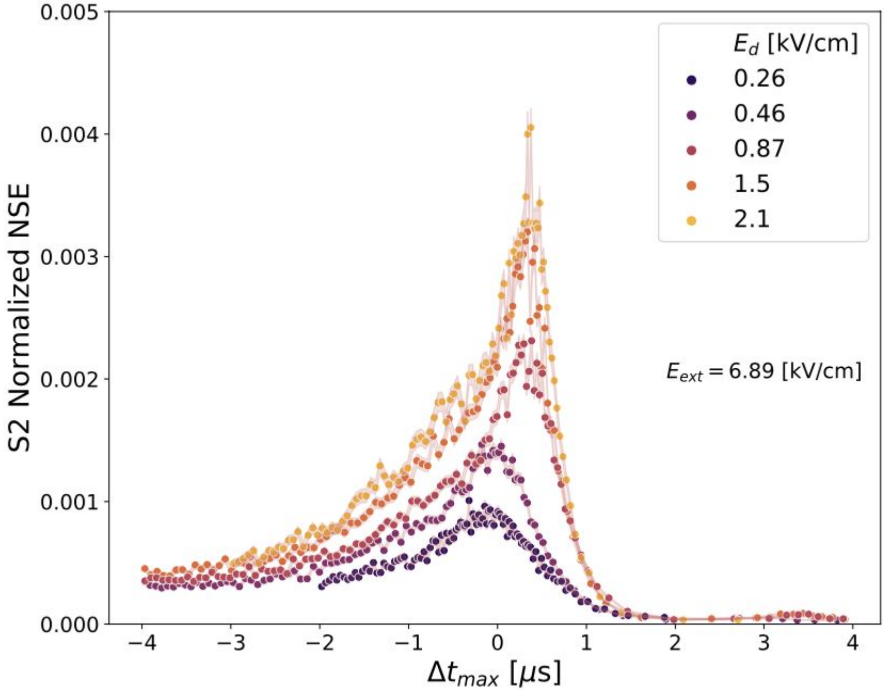


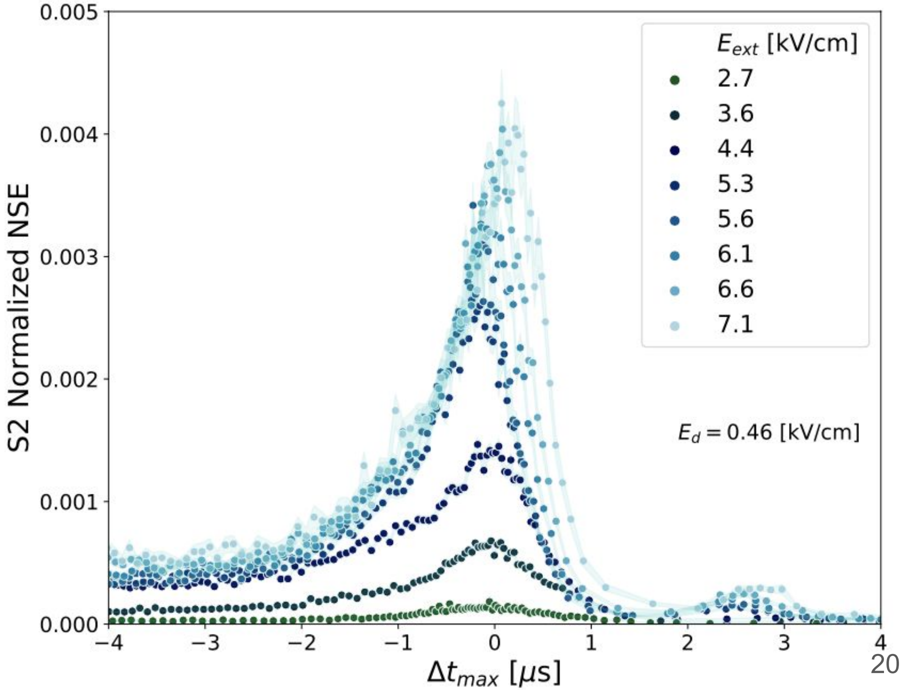


Cathode Spike

FN-theory + photoeffect







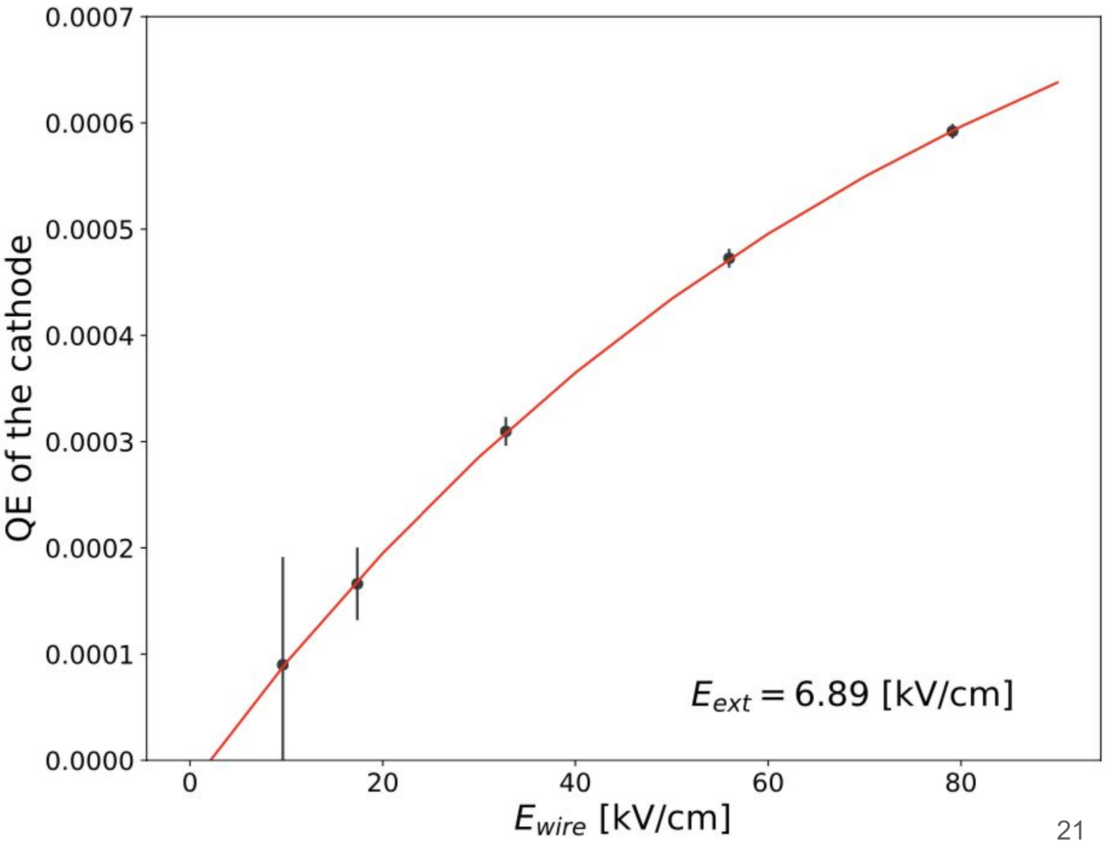
Cathode Spike: Quantum Efficiency

$$QE = \frac{N_e}{N_{ph}} \sim c - A \cdot \exp\frac{-E_{wire}}{E_c}$$

$$E_{wire} = E_{ext} \cdot \frac{p}{d},$$

A = 0.00096 is the maximum QE
Ec = characteristic field ~80 kV/cm
T
he QE includes the effect of the fraction of field lines going up

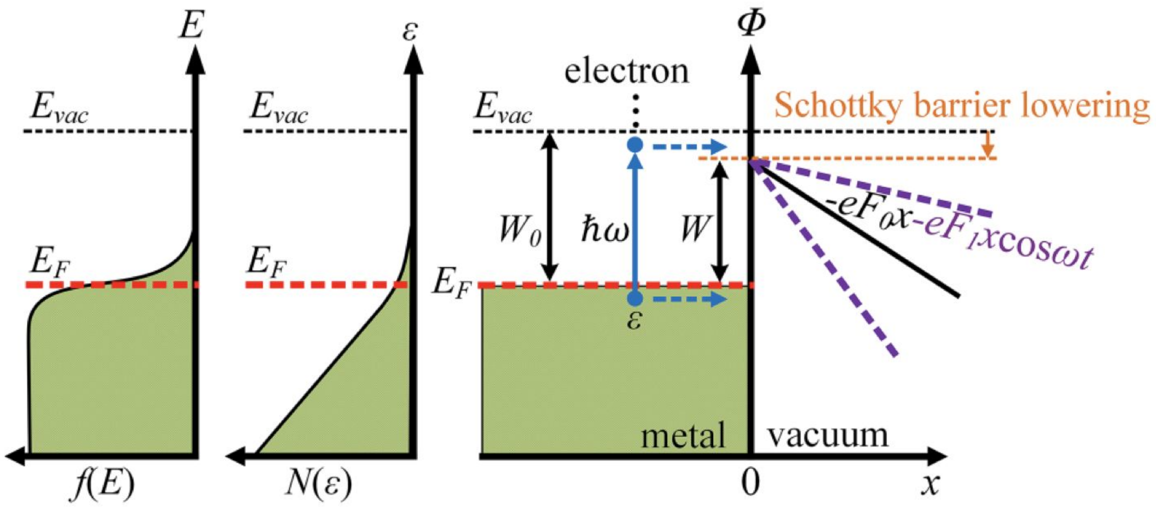
Ne = se-area /eee*g2
Nph = P*(S2 area) / g1, where g1 ~ 10%



Further Work for LZ

1) **FNP (Fowler-Nordheim Photoionization theory)** : temperature, predicting “right” coating function, time-dependent behavior

2) Predict LZ rate in LXe and GXe regions based on FN and FNP theories



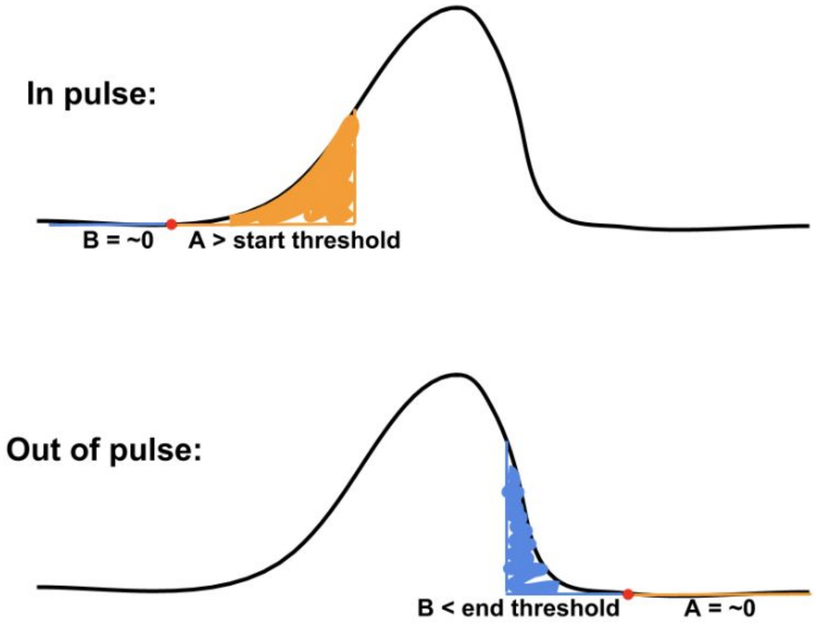
Backup

First pass

Step 1: How RT algorithm works

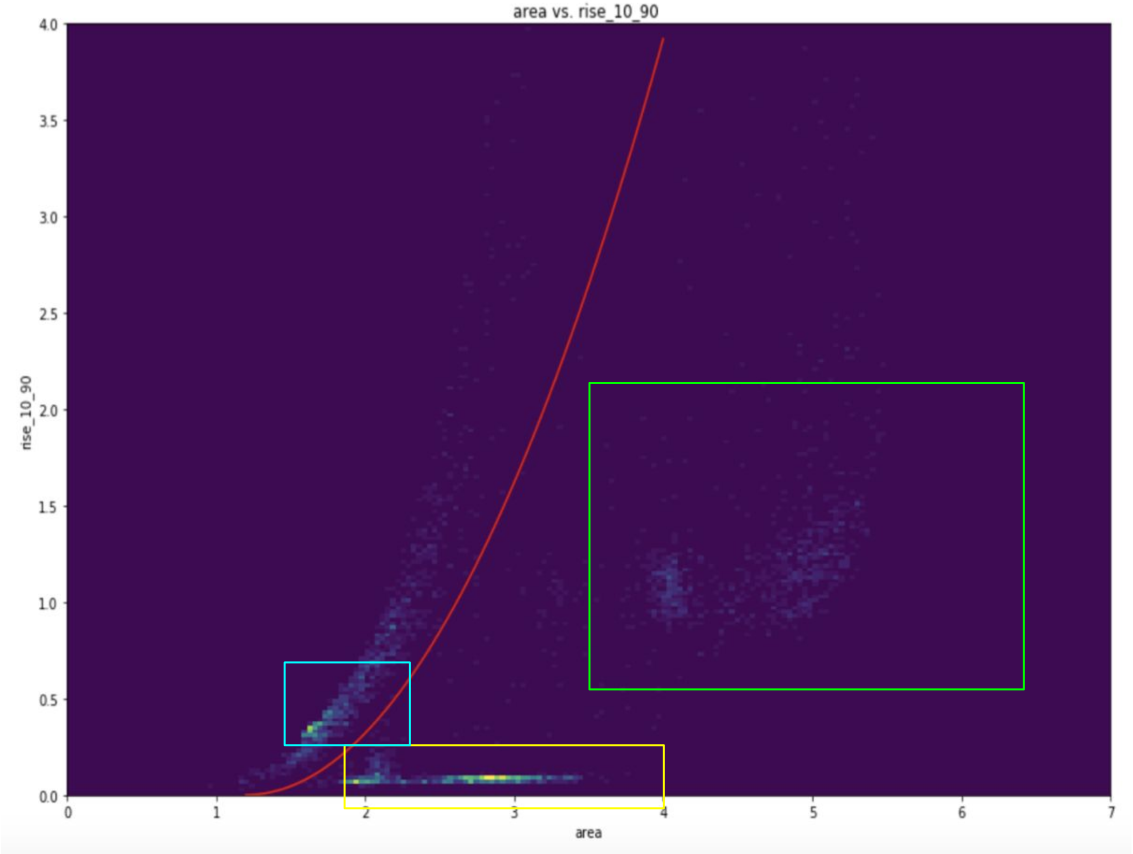
For each summed pod, track two rolling sums (call them A and B)

- When rolling sum A crosses above threshold (th1), declare *in pulse*
- Within rolling window A, do fine start search
- While *in pulse*, and rolling sum B crosses below threshold, declare *out of pulse*
- Create another threshold (th2) in order to get correct boundaries for the pulse and be above the noise level



First pass

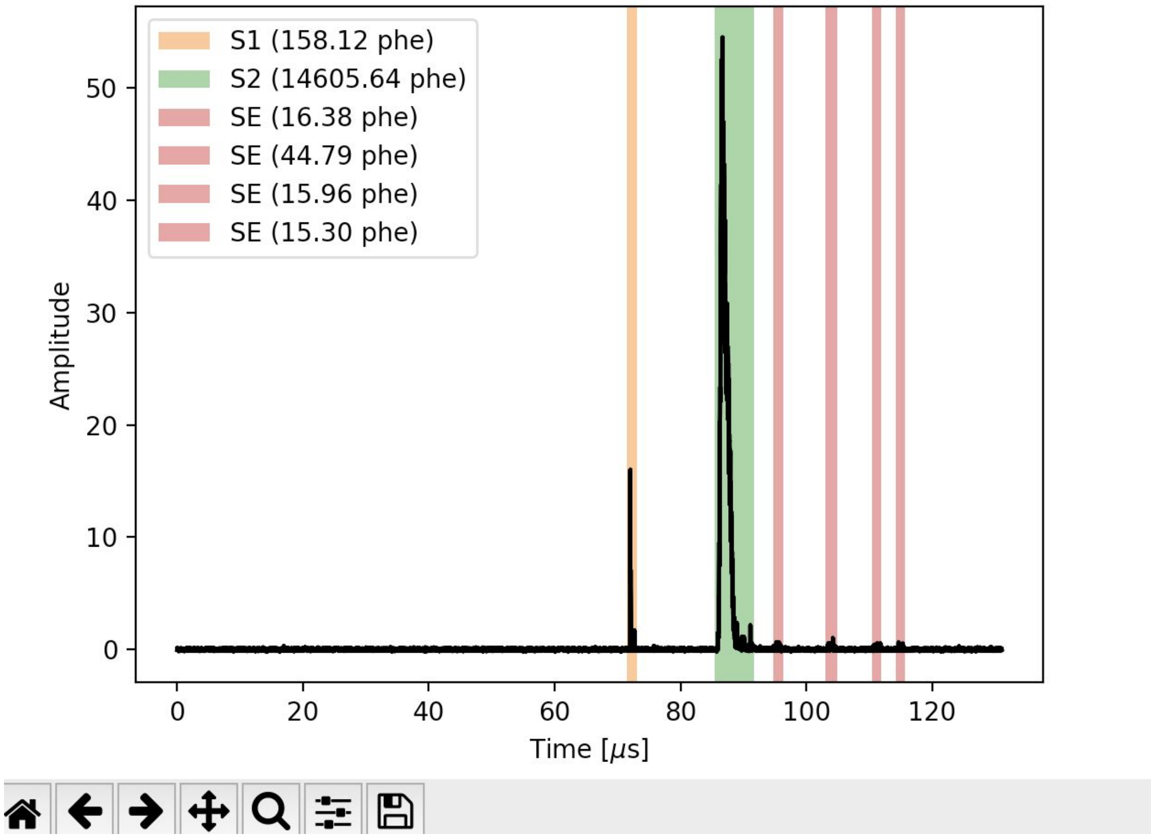
Step 2: Use the cuts to classify s1 and s2



- Print out id's events for the noted areas
- Visual inspect their waveforms
- Add classifying function to the rolling thunderer algorithm, so it can plot s1 and s2, and potential SE pulses

Green- s2
Yellow - s1
Blue - SE

Example of the First Pass output:



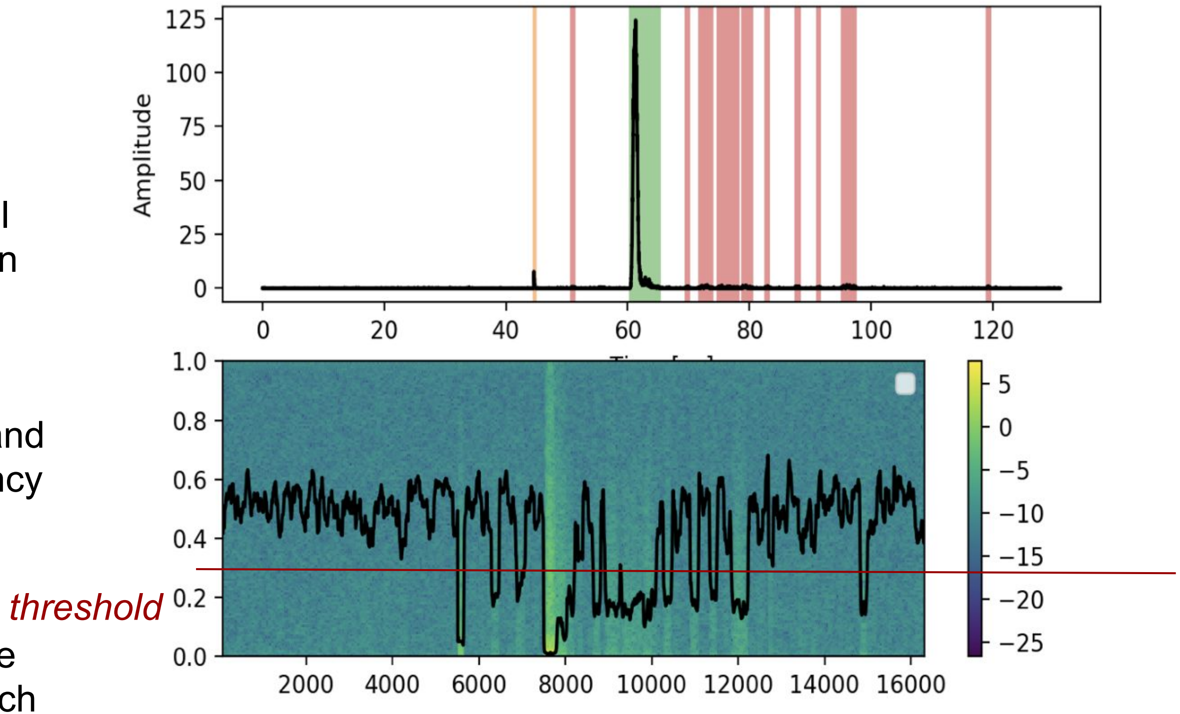
Second pass:

Step 1: Using spectrogram to find small signals

- Spectrogram is a numpy function which “converts” the original signal into frequency space (time domain to frequency domain for each window)

- Physical signals, such as s1, s2, and SE are tend to be at lower frequency range (below threshold)

- Noise won’t be too visible in the frequency space, since white noise contains all of the frequencies which are equally distributed
- An additional condition for SE pulses: pulse area > 18



The SE signals are the ones which below the threshold and are not s1 and s2

Summary on events selection

So far, the described algorithm for SE detection is able:

- Find all of the pulses on the waveform. Precision can be tuned by threshold and window parameters

- Classify s1, s2 signals

- Reject false detections

- Find small pulses and classify single electron pulses

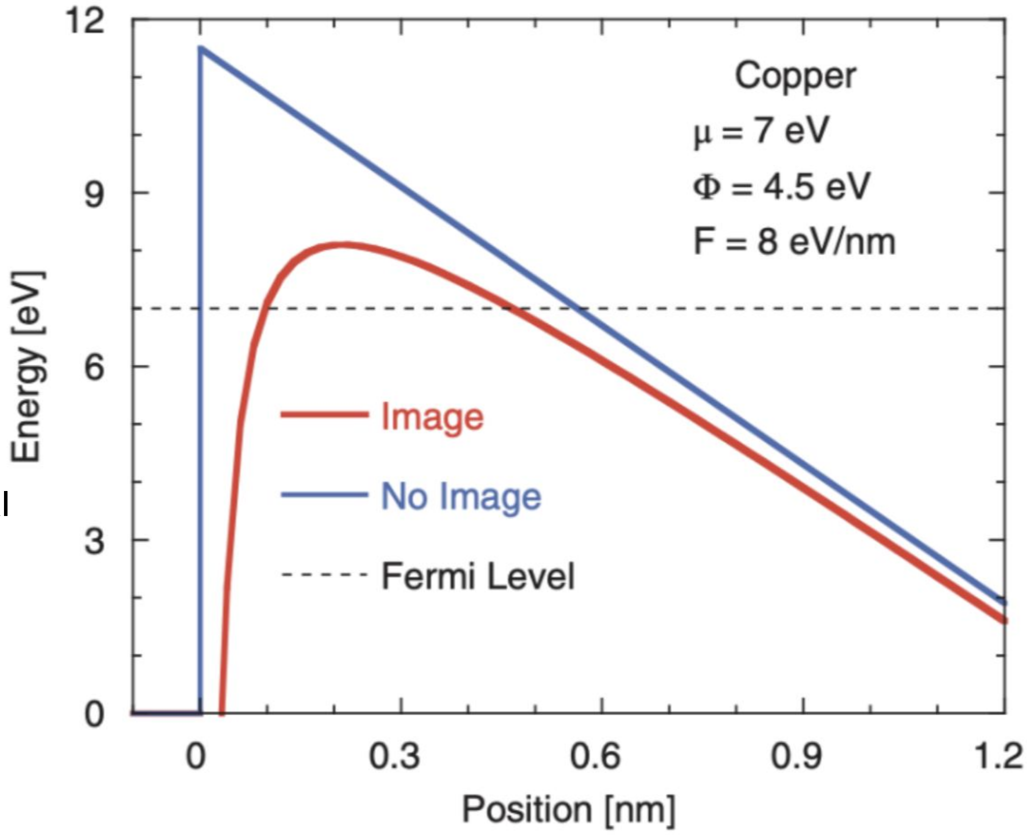
- It works well on different data sets (16 data sets were tested so far)

More about FN Barrier

$$V_{image} = \mu - F \cdot x - \frac{Q}{x} + \Phi$$

V near the metal surface
(force F due to charge q on the image)

Coulomb potential from the image charge inside the metal



FN Error Bars

1. **NSE** = Single electron area/(g2*extraction efficiency) (per event)
   a. We sum all the single electrons per event and take average
2. **Main error sources:** single electron area, extraction efficiency (EE).
3. Propagation error :

$f = AB$	$\sigma_f^2 \approx f^2 \left[\left(\frac{\sigma_A}{A}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2 + 2\frac{\sigma_{AB}}{AB}\right]$ [9][10]	$\sigma_f \approx \lvert f\rvert \sqrt{\left(\frac{\sigma_A}{A}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2 + 2\frac{\sigma_{AB}}{AB}}$
$f = \frac{A}{B}$	$\sigma_f^2 \approx f^2 \left[\left(\frac{\sigma_A}{A}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2 - 2\frac{\sigma_{AB}}{AB}\right]$ [11]	$\sigma_f \approx \lvert f\rvert \sqrt{\left(\frac{\sigma_A}{A}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2 - 2\frac{\sigma_{AB}}{AB}}$

4. Assume that the errors from the EE and SE area are not correlated, then

$$\sigma_f \approx |f| \sqrt{\left(\frac{\sigma_A}{A}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2}$$

5. Remember that we are drawing FN plot
   - Y = j/ E_wire[i]^2
   - X = 1/E_wire[i] , where E_wire = E_ext * (wire pitch/ wire diameter)
   - Remember we will need log(Y) for FN plot and the errors for log :

$f = a\ln(bA)$	$\sigma_f^2 \approx \left(a\frac{\sigma_A}{A}\right)^2$ [12]	$\sigma_f \approx \left\lvert a\frac{\sigma_A}{A}\right\rvert$

$$\sigma_{lnY} = \left\lfloor \frac{\sigma_A}{A} \right\rfloor, \ \sigma_A = error = \sqrt{\left(\frac{\delta NSE}{NSE}\right)^2 + \left(\frac{\delta EE}{EE}\right)^2}$$

Since |f| = |A| cancels out

Anatomy of Current Density: The Supply Function

1. Consider a collection of electrons with the probability of an energy state governed by Fermi-Dirac statistics

$$|\psi(t)\rangle = e^{-i\hat{H}t/\hbar}|\psi(0)\rangle = \sum_j e^{iE_jt/\hbar}\boxed{C_j}|\psi_j\rangle$$

2. $|C_j|^2$ is the probability for the state with E_j to be occupied:

$$\rho = \langle\psi(t)|\psi(t)\rangle = \langle\psi(0)|\psi(0)\rangle = \sum_j |C_j(E_j)\psi_j(x)|^2$$

3. The goal is to find the density variation near the surface of metal:

$$J_q = \frac{q}{2\pi}\int\left(\frac{\hbar k_x}{m}\right)dk_x \boxed{\left\{\frac{1}{(2\pi)^2}\int f_{FD}(E_k)dk_y dk_z\right\}}$$

The transverse velocities are integrated over in FD distribution: supply function

4. Let T -> 0 and explore FD function in this limit, then {} becomes:

$$\frac{1}{2\pi}\int_0^{\sqrt{k_F^2-k_x^2}} k_\perp dk_\perp = \frac{1}{4\pi}\left(k_F^2 - k_x^2\right)$$

$$\boxed{\rho_{FD}(T \to 0) = \frac{2}{3}N_c\mu_o^{3/2} = \frac{k_F^3}{3\pi^2}}$$

This fundamentally refers to the concept of chemical potential

Anatomy of Current Density: The Supply Function

1. Max Fermi momentum occurs at j = N+1: $C_j^2 = \frac{(N+1)^2 - j^2}{(N+1)^2}$ (This is typically solved numerically)

2. Taking 1. into account, the density looks like this:

$$\rho(x) \propto \sum_{j=1}^{N} C_j^2 \begin{cases} \left|\left(e^{ik_j x} + r(k_{-j})e^{ik_{-j}x}\right)\right|^2 & (x < 0) \\ \left|t(k_j)e^{i\kappa_j x}\right|^2 & (x \geqslant 0) \end{cases}$$

-> for lower momentum than Fermi k , the density is not zero! tunneling occurs

The electron emission is purely quantum mechanical phenomenon!

Anatomy of Current Density: Simple Harmonic Oscillator

1. Another solution for SHO : the temperature-dependent Wigner domain f(x, t, k)

$$V(x) = Kx^2/2$$

2. Number and current densities then:

$$\rho(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x,k,t)\,dk$$

$$J(x,t) = \frac{q}{2\pi}\int_{-\infty}^{\infty} \frac{\hbar k}{m} f(x,k,t)\,dk$$

$$f(x,k,t)dxdk = f(x',k',t')dx'dk'$$

# of particles is conserved

Contains all QM info

3. How the whole system behaves in time? $x' = x + \hbar k dt/m \quad k' = k + F dt/\hbar$

4. The volume is changing of the phase space; treat probability as liquid:

$$dx'dk' = \begin{vmatrix} \partial_x x' & \partial_x k' \\ \partial_k x' & \partial_k k' \end{vmatrix} dxdk = \begin{vmatrix} 1 & 0 \\ \hbar dt & 1 \end{vmatrix} dx\,dk = dx\,dk$$

Anatomy of Current Density: Simple Harmonic Oscillator

5. Consider F to be the linearl field , and for a single particle:

$$\partial_t f(x,k,t) = -\frac{\hbar k}{m}\partial_x f(x,k,t) + \frac{F}{\hbar}\partial_k f(x,k,t)$$

solution: $$f(x,k,t+dt) = f\left(x - \frac{\hbar k}{m}dt, k - \frac{F}{\hbar}dt, t\right)$$

Longer times: $$f(x,k,t) = f\left(x - \frac{\hbar k}{m}t + \frac{F}{2m}t^2, k - \frac{F}{\hbar}t, 0\right)$$

6. If electric field F is constant in time:

$$f(x,k,t) = f\left(x - \frac{\hbar k}{m}t + \frac{F}{2m}t^2, k - \frac{F}{\hbar}t, 0\right)$$

We want for df/dt the particles to be following the contour lines of a surface plot of the distribution function f(x, k)
-> Wigner's function is constant along each trajectory with constant energy.

7. How the density should be expressed in terms of Wigner function then?what about time evolution?

$$\frac{\partial}{\partial t}f(x,k,t) = -\frac{\hbar k}{m}\frac{\partial}{\partial x}f(x,k,t) + \int_{-\infty}^{\infty} \boxed{V(x,k-k')} f(x,k',t)\, dk'$$

Non-local nature

Extraction Field (kV/cm)	Single Electron Size (phe)	Single Electron Width (μs)
2.7	11.15 ± 3.18	2.34 ± 0.37
3.6	15.26 ± 3.17	2.15 ± 0.26
4.4	18.91 ± 3.14	1.91 ± 0.18
5.3	23.75 ± 3.20	1.84 ± 0.18
5.6	25.62 ± 4.20	1.82 ± 0.19
6.1	27.65 ± 4.87	1.77 ± 0.19
6.6	28.32 ± 5.44	1.73 ± 0.19
7.1	29.14 ± 5.76	1.69 ± 0.18

Table 1. The signal magnitudes and widths for single electrons observed in $^{83\mathrm{m}}$Kr events.

Anatomy of Current Density: Simple Harmonic Oscillator

8. Why Wigner function is important? It gives the insights on nature of tunneling including the tunneling resonance! Wigner potential can be treat as the SHO:

$$V(x+y) - V(x-y) = \sum_{n=0}^{\infty} \frac{2y^{2n+1}}{(2n+1)!} \left(\frac{\partial}{\partial x}\right)^{2n} V(x) \qquad \int_{-\infty}^{\infty} y^n e^{2iky} dy = \pi \left(-\frac{i}{2}\frac{\partial}{\partial k}\right)^n \delta(k)$$

$$\frac{\partial}{\partial t} f(x,k,t) = -\frac{\hbar k}{m} \partial_x f + \frac{1}{\hbar}\left(\partial_x V\right)\partial_k f \quad + \frac{1}{\hbar}\sum_{n=1}^{\infty} \frac{(-1)^n}{2^{2n}(2n+1)!}\left(\partial_x^{2n+1} V\right)\left(\partial_k^{2n+1} f\right)$$

The equation is solved as if V is from SHO except for real SHO all the higher orders of the derivatives vanish

9. For the TIME-DEPENDENT SHO:

$$\hat{O} \equiv -\frac{\hbar k}{m}\frac{\partial}{\partial x} + \frac{1}{\hbar}\left(\partial_x V\right)\frac{\partial}{\partial k}$$

And the function in real SHO dependents only on the combination of kinetic terms

10. **Consequences**: the particles trajectories are the circles on Wigner domain and the solutions are similar to real SHO! (this will be very important once we start talking about resonant tunneling which will (hopefully) explain time-dependent nature of SLAC data

Anatomy of Current Density

The electrons with energies are closer to Fermi level have higher probability to tunnel through the barrier in the metal

- Velocity of the electrons
- Distribution function of *emitted* electrons
- The lower limit of the integral is 0 because the emission theory was studied mainly into vacuum and no electrons are incident from the vacuum.. Which is not true for us and concerns me now (has to be -inf for our case)
- Argument of J(F, T) implies that the transmission probability is field dependent (F) and the supply function is temperature dependent

Genesis for all possible emissions:

Momentum domain:

$$J(F,T) = \frac{q}{2\pi}\int_0^\infty \boxed{\frac{\hbar k}{m}}\,\boxed{D(k)f(k)}\,dk$$

Energy domain:

Transmission probability

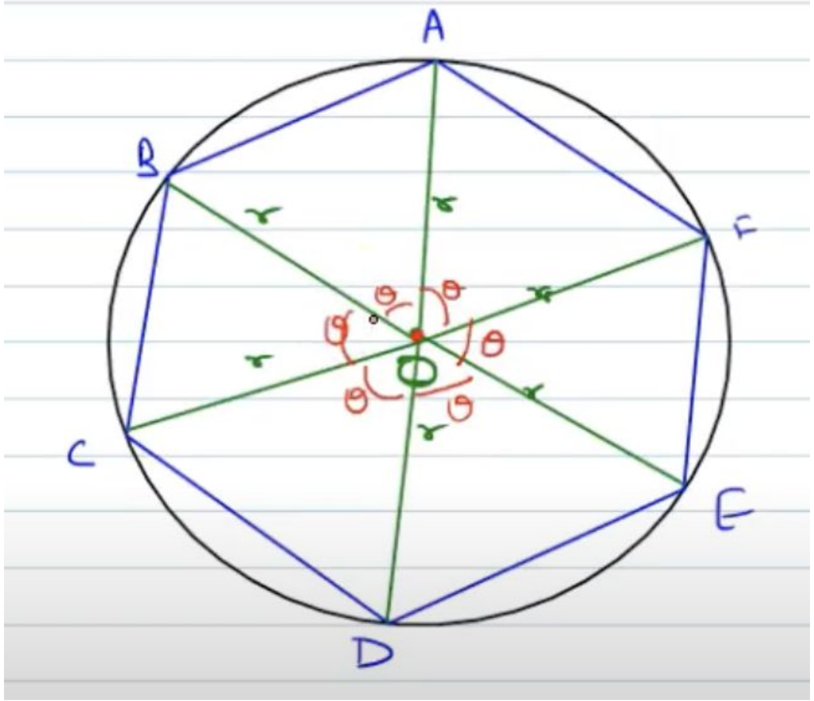
$$J(F,T) = \frac{q}{2\pi\hbar}\int_0^\infty \boxed{D(E)}f(E)\,dE$$

$$\hbar^{-1}dE = (\hbar k/m)dk$$

Extra work for me when working on FN emission for LZ

Area of the emitter A

Boulton claims the widest point of the detector is 18.4 cm
https://arxiv.org/pdf/1705.08958.pdf



Theta = 60, so ABO is equilateral -> AB = a = 9.2 cm

Formula for the area of a hexagon with the side a:

$$A = \frac{3\sqrt{3}}{2}a^2$$

Plugging in a = 9.2 cm -> 220 cm^2 = 0.022 m^2

So, total area of a hexagon 0.022 m^2. However, taking into account 250 wires with a wire pitch (3 mm), diameter (80 um), **total area covered by the wires: 250* 80 (um)*13.8 (cm) = 0.00276 m^2 = 2.76*10^(-3) m^2**

Quantum Efficiency: Cathode

The cathode spike region:

$$QE = \frac{N_e}{N_{ph}}$$

- N_e - number of electrons coming from the grids
  $$N_e = \frac{SE-area}{g2 \cdot \epsilon}$$
  $SE - area$ is the single electrons area in phe, and ϵ is the electron extraction efficiency
- N_{ph} - number of photons hitting the grids
  $$N_{ph} = \frac{S2-area}{g1}$$