Discovery of Neutrino Oscillations

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Motivation: 1957-1958

In 1957, B. Pontecorvo first proposed the idea of “neutrino oscillations” which was inspired by the neutral Kaon oscillation:

$$K^0 \leftrightarrow \bar{K}^0$$

However, only one neutrino type was known at that time and he proposed the oscillation between neutrinos and antineutrinos.

Despite of these limitations, the connection between neutrino oscillations and a nonzero neutrino mass was established.

From Pontecorvo B 1957 Sov. Phys. JETP 7 172.
Motivation: 1962

a) The weak neutrinos must be re-defined by a relation
\[
\begin{align*}
\nu_e &= \nu_1 \cos \delta - \nu_2 \sin \delta, \\
\nu_\mu &= \nu_1 \sin \delta + \nu_2 \cos \delta.
\end{align*}
\] (2.18)

The leptonic weak current (2.9) turns out to be of the same form with (2.1). In the present case, however, weak neutrinos are not stable due to the occurrence of a virtual transmutation $\nu_e \leftrightarrow \nu_\mu$ induced by the interaction (2.10). If the mass difference between $\nu_2$ and $\nu_1$, i.e. $|m_{\nu_2} - m_{\nu_1}| = m_{\nu_2}^*$ is assumed to be a few Mev, the transmutation time $T(\nu_e \leftrightarrow \nu_\mu)$ becomes $\sim 10^{-18}$ sec for fast neutrinos with a momentum of $\sim$Bev/c. Therefore, a chain of reactions such


In 1962, muon neutrino was discovered and people knew there were two types (e and $\mu$) of neutrinos.

In the same year, Z. Maki, M. Nakagawa and S. Sakata proposed the idea of neutrino mixing of the two flavors of neutrinos: weak neutrinos are related to neutrinos with definite masses.

(Later, the mixing matrix for 3 flavors of neutrinos was named PMNS matrix)
Main processes of neutrino production in the sun:

1. Proton-proton reaction (86%)
   \[ p + p \rightarrow d + e^+ + \nu_e \]

2. Beryllium-7 reaction (14%)
   \[ ^7\text{Be} + e^- \rightarrow ^7\text{Li} + \nu_e \]

3. Beta decay of Boron-8 (0.02%)
   \[ ^8\text{B} \rightarrow ^8\text{Be}^* + e^+ + \nu_e \]
Solar Neutrino Problem

In late 1960s, Raymond Davis and John Bahcall started the Homestake Experiment, the first experiment to detect neutrinos created by nuclear fusion in the sun.

An underground 380 m³ tank was filled with tetrachloroethylene (rich in chlorine) and was aimed to detect solar neutrinos with energy > 0.814 MeV through reaction

$$\nu_e + ^{37}\text{Cl} \rightarrow ^{37}\text{Ar} + e^-$$

by counting isotopes of Argon.

Expect: 7.5 SNU (unit of capture rate). Observed: ~2.56 SNU.
-> A deficit in electron neutrino flux!
Neutrino Oscillation of 2 Flavors

Since 1970, the phenomenological theory of neutrino mixing and oscillations in vacuum has been developed. The weak eigenstates are related to the mass eigenstates by a unitary matrix with a mixing angle:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}. $$

where the mass eigenstates propagate as plane waves:

$$|\nu_1(t)\rangle = |\nu_1\rangle e^{i(p_1 \cdot x - E_1 t)} = e^{-ip_1 \cdot x}, $$

$$|\nu_2(t)\rangle = |\nu_2\rangle e^{i(p_2 \cdot x - E_2 t)} = e^{-ip_2 \cdot x},$$

Idea: If we have two planes waves with different frequencies, there could be interference!
If the initial mass eigenstates have the same energy/momentum/velocity, the 2-flavor oscillation probability is

\[ P(\nu_e \rightarrow \nu_\mu) = \sin^2(2\theta) \sin^2\left(1.27 \frac{\Delta m^2[eV^2]}{E_\nu[GeV]} L[km]\right). \]

Smaller mass difference results in larger oscillation wavelength; the mixing angle \( \theta \) controls the amplitude of the oscillation probability:
- \( \theta = 0 \): no mixture;
- \( \theta = 45^\circ \): largest mixture.
Similarly, the three flavor eigenstates are related to the mass eigenstates by the unitary PMNS matrix:

$$
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} =
\begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} \\
U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}.
$$

The PMNS can be represented by 3 mixing angles $\theta_{12}$, $\theta_{13}$, $\theta_{23}$ and a phase factor $\delta$, which is non-zero if neutrino oscillations violate CP violation:

$$
U =
\begin{bmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} \\
U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{bmatrix}
= 
\begin{bmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{bmatrix}
\begin{bmatrix}
c_{13} & 0 & s_{13}e^{-i\delta} \\
0 & 1 & 0 \\
-s_{13}e^{i\delta} & 0 & c_{13}
\end{bmatrix}
\begin{bmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{bmatrix}
= 
\begin{bmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{bmatrix}
$$
Neutrino Oscillation of 3 Flavors

The oscillation probability can be described by 3 mixing angles, two mass-squared differences, the distance traveled and the neutrino energy.

\[
P_{\alpha \to \beta} = \delta_{\alpha \beta} - 4 \sum_{i > j} \text{Re} \left( U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \right) \sin^2 \left( \frac{\Delta m_{ij}^2 L}{4E} \right) \\
+ 2 \sum_{i > j} \text{Im} \left( U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \right) \sin \left( \frac{\Delta m_{ij}^2 L}{2E} \right),
\]

The part that is responsible for the oscillation is usually written as:

\[
1.27 \times \frac{\Delta m^2}{\text{eV}^2} \frac{L}{\text{km}} \frac{\text{GeV}}{E},
\]

where \(\Delta m^2\) is around the order of \(10^{-4}\) eV\(^2\).
From 1980s to 1990s, there were many experiments looking for neutrino oscillations with reactors/accelerators but did not find much evidence.

Reason: large neutrino $E$, small $L$, sensitive to $\Delta m^2$ around 0.1 to 1 eV. Need a long $L$ experiment!

In 1983, Kamiokande (Kamioka Nucleon Decay Experiment), a 3000-ton water Cherenkov detector located 1000m underground in Kamioka, Japan, was constructed. The cylindrical tank was filled with pure water and 1000 photomultiplier tubes (50 cm in diameter) were installed on the inner surfaces of the detector to detect Cherenkov photons. Its primary purpose was to search for proton decay but it could also detect neutrinos (dominant background).
Method

Principle: when neutrinos interact with water, charged particles are created. When a charged particle travels faster than the speed of light in water, Cherenkov photons are emitted and can be detected by photomultiplier tubes. These photons are emitted in a cone along the neutrino direction and forms a ring pattern on the walls.

In Kamiokande, $\nu_e$ and $\nu_\mu$ interactions can be differentiated due to different event topology ($\nu_e$ events are fuzzier due to EM shower)

Although proton decay was not observed (not surprising), Takaaki Kajita and his colleagues found a deficit of the observed atmospheric $\nu_\mu$ events.

Above: an illustration of a neutrino event in Kamiokande. Energy and direction of the neutrino can be reconstructed.

Atmospheric neutrinos originate from Pion decay when cosmic rays interact with the atmosphere. The ratio of $\nu_\mu$ to $\nu_e$ is expected to be around 2 for $\leq 1$ GeV neutrinos (v stands for both neutrino and antineutrino).

However, the observed number of $\nu_\mu$ with energy $\leq 1$ GeV is much smaller than predicted. This might indicate the existence of neutrino oscillations with a large mixing angle (for muon neutrinos).

Because people at that time believed in small neutrino mixing angles, nobody trusted this result and a one-year examination was done on the data and the software. No errors were found and the observed data could not be explained by statistical fluctuation or systematics.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Monte Carlo Prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e$-like events</td>
<td>$93 \pm 9.6$</td>
<td>88.5</td>
</tr>
<tr>
<td>$\mu$-like events</td>
<td>$85 \pm 9.2$</td>
<td>144.0</td>
</tr>
</tbody>
</table>

Above: data from Kamiokande in 1988. The Irvine–Michigan–Brookhaven (IMB) experiment also observed a similar deficit several years later.

If not mentioned, figures are from: https://www.jstage.jst.go.jp/article/pjab/86/4/86_4_303/ _article.
up-down symmetry

A more powerful test of neutrino oscillation at Kamiokande is from the ‘up-down symmetry’ property of atm neutrinos.

Assuming that the CR flux is about the same at all positions on Earth (isotropic), the neutrino flux should be symmetry above and below the horizon.

The calculated atm neutrino flux (without oscillation) as a function of zenith angle at Kamiokande are shown to be symmetric especially for multi-GeV neutrinos. At lower energy, cosmic rays are bent by the geomagnetic field and the symmetry is not exact.
Angular distribution of neutrino events in Kamiokande. FC: fully contained events. PC: partially contained events (created muon can leave the detector).

The symmetry will be broken if neutrinos oscillate. If neutrinos have very small mass, the oscillation length could be ~1000 km. Dowgoing neutrinos usually only travel ~15 km after being produced and may not oscillate. Upgoing neutrinos (especially $\nu_\mu$) traveling through the Earth may be able to oscillate to another flavor before reaching the detector.

Kamiokande was upgraded to observe multi-GeV events (they also have better angular resolutions than sub-GeV events) and observed an asymmetry in 1994. Due to limited statistics at high energy, only 20 Multi-GeV muon events were observed. A deficit of upgoing $\nu_\mu$ was observed with a significance of 2.8 $\sigma$.

-> Need a larger detector!
Super-Kamiokande started operation in 1996 after 5-years construction.

Improvements from Kamiokande:
Dimension (d×H): 15×16 -> 39×41 m
Fiducial mass: 1000 -> 22500 tons
Number of PMTs: 1000 -> 11200

By 1998, 535 days of data were collected and ~5400 atm neutrino events were detected.
Left: a typical $\nu_e$ event. Right: a $\nu_\mu$ event.
Results

The figure on the left was presented at the *18th International Conference on Neutrino Physics and Astrophysics* (Neutrino 1998). The events here are multi-GeV (visible energy > 1.33 GeV) events.

A deficit of upgoing $\nu_\mu$ is observed with high statistical significance. Two other collaborations, Soudan-2 in Minnesota and MACRO in Italy, also observed a similar zenith-dependent deficit.

Conclusions: $\nu_\mu$ oscillate to other types of neutrinos, most likely to tau neutrinos. $\nu_e$ does not oscillate as long as the length traveled is smaller than the diameter of the Earth.
The analysis for the oscillation parameters were performed assuming $\nu_\mu$ to $\nu_\tau$ oscillation. 90% C.L. were presented.

Legend:
1. contained events from Super-Kamiokande
2. contained events from Kamiokande
3. upward through-going events from Super-Kamiokande
4. upward through-going events from Kamiokande
5. stop/through ratio analysis for upward-going muons from Super-Kamiokande

Contours from Kamiokande agrees with those from Super-Kamiokande.

“Atmospheric neutrino anomaly” is resolved by neutrino ($\nu_\mu$) oscillation!
In 1998, another analysis for the solar neutrino flux was performed at Super-K. The detector was only sensitive to $^8\text{B}$ $\nu_e$ flux between 6.5 MeV (background from radioactive decays in the PMT glass dominates below this threshold) and 20 MeV which can be detected through neutrino-electron elastic scattering.

Observed solar $\nu_e$ flux: $2.42 \times 10^6$/cm$^2$/s. Only 36% of expected from Standard Solar Model (SSM) BP95. However, it could be caused by either a wrong solar neutrino flux model or by neutrino oscillation.

--> Need an experiment that can detect both $\nu_e$ and total solar neutrino flux!
SNO is also a water Cherenkov detector located 2 km under the ground in Ontario, Canada. It uses 1000 tons of ultrapure heavy water ($\text{D}_2\text{O}$) and over 9000 8”-PMTs to detect all-flavor $^{8}\text{B}$ solar neutrinos. Deuteron has a low binding energy (2.2 MeV) comparing to $^{8}\text{B}$ neutrinos, which allows different reactions to be detected:

$$\nu_e + d \rightarrow p + p + e^- \quad \text{(CC)},$$
$$\nu_x + d \rightarrow p + n + \nu_x \quad \text{(NC)},$$
$$\nu_x + e^- \rightarrow \nu_x + e^- \quad \text{(ES)}.$$

CC is only sensitive to $\nu_e$. NC is equally sensitive to all flavors. ES is also sensitive to all flavors but less sensitive to $\nu_\mu$ and $\nu_\tau$. Electrons created from CC are isotropic and point back to the sun in ES. Different sensitivities of different channels allows the determination of the electron and non-electron component of solar neutrino flux.

Fun fact: the PMT support structure (PSUP) was engineered and built by scientists from LBL.

Photo from wikipedia.
Sudbury Neutrino Observatory (SNO)

SNO is also a water Cherenkov detector located 2 km under the ground in Ontario, Canada. It uses 1000 tons of ultrapure heavy water (D\textsubscript{2}O) and over 9000 8''-PMTs to detect all-flavor \(^8\)B solar neutrinos. Deuterium has a low binding energy (2.2 MeV) comparing to \(^8\)B neutrinos, which allows different reactions to be detected:

\[
\begin{align*}
\nu_e + d &\rightarrow p + p + e^- \quad \text{(CC)}, \\
\nu_x + d &\rightarrow p + n + \nu_x \quad \text{(NC)}, \\
\nu_x + e^- &\rightarrow \nu_x + e^- \quad \text{(ES)}. 
\end{align*}
\]

CC rate \(\propto \phi(\nu_e)\).
NC rate \(\propto \phi(\nu_e) + \phi(\nu_\mu) + \phi(\nu_\tau)\).
ES rate \(\propto \phi(\nu_e) + 0.154 \left[ \phi(\nu_\mu) + \phi(\nu_\tau) \right]\).
Results

SNO began taking data since 1999 and reported their findings in 2002. The measured $^8$B solar neutrino fluxes for each channel are (in units $10^6$ cm$^{-2}$ s$^{-1}$)

\[ \phi_{CC}^{SNO} = 1.76^{+0.06}_{-0.05}^{\text{stat}}^{+0.09}_{-0.09}^{\text{syst}}, \]
\[ \phi_{ES}^{SNO} = 2.39^{+0.24}_{-0.23}^{\text{stat}}^{+0.12}_{-0.12}^{\text{syst}}, \]
\[ \phi_{NC}^{SNO} = 5.09^{+0.44}_{-0.43}^{\text{stat}}^{+0.46}_{-0.43}^{\text{syst}}. \]

which translates to

\[ \phi_e = 1.76^{+0.05}_{-0.05}^{\text{stat}}^{+0.09}_{-0.09}^{\text{syst}}, \]
\[ \phi_{\mu\tau} = 3.41^{+0.45}_{-0.45}^{\text{stat}}^{+0.48}_{-0.48}^{\text{syst}}, \]

The non-electron neutrino flux is $5.6\sigma$ above 0. The total flux is consistent with SSM:

\[ \phi_{SSM} = 5.05^{+1.01}_{-0.81}. \]
Results

In other words, the total neutrino flux detected is consistent with the solar model prediction (model is not quite wrong).

However, instead of $\nu_e$ alone, the solar neutrino flux is consist of all three flavors of neutrinos.

-> “...strong evidence for flavor transformation consistent with neutrino oscillations.”

-> The solar neutrino problem is finally resolved by neutrino oscillations!

The plot above shows the constraints on fluxes from SNO data. The measured total flux is consistent with SSM. The bands intersect at the best fit values.

Thanks!