Generalized Parton Distributions (GPDs)

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Physics 290e

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Outline

- A review of electron-proton scattering
  - At different values of Q^2
- What are GPDs?
- How do we measure GPDs?
  - Deeply virtual Compton scattering (DVCS)
- Getting back what we started with

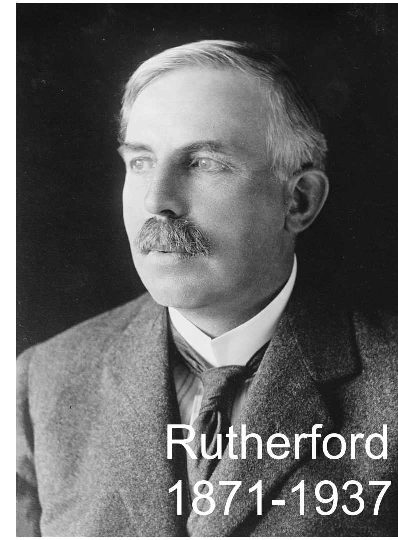
Electron-proton scattering

- $Q^2 << 1/r_p$
  - Electron recoils from point-like spinless object
- $Q^2 \sim 1/r_p$
  - Electron recoils from extended charged object with spin 1/2
- $Q^2 > 1/r_p$
  - Electron can resolve proton structure

Q^2 (virtuality of exchanged photon)

Rutherford Scattering

$Q^2 << 1/r_p$



$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16(p_e^2/2m_e)^2 \sin^4(\theta/2)}$$

- Scattering of charged point particles via Coulomb interaction
- Assume:
  - The electron is non-relativistic
  - The proton does not recoil and we can ignore proton spin
  - The proton is point-like

Mott Scattering

$Q^2 \sim 1/r_p$

$$\frac{d\sigma}{d\Omega} = \underbrace{\frac{\alpha^2}{4E_1^2 \sin^4(\theta/2)}}_{\text{Rutherford scattering with relativistic electron energy}} \overbrace{\frac{E_3}{E_1}}^{\text{proton recoil}} \left(\underbrace{\cos^2\frac{\theta}{2}}_{\text{Taking electron spin states into account}} - \overbrace{\frac{q^2}{2M_p^2}\sin^2\frac{\theta}{2}}^{\text{spin-spin interactions}} \right)$$

- Assume:
  - ~~The electron is non-relativistic~~
  - ~~The proton does not recoil and we can ignore proton spin~~
  - The proton is point-like

Rosenbluth Formula

$Q^2 \sim 1/r_p$

Mott scattering + terms describing proton's structure

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1 \sin^4(\theta/2)} \frac{E_3}{E_1}$$

$$\left\{ \left(F_1^2 - \frac{\kappa_p^2 q^2}{4M_p^2} F_2^2\right) \cos^2\frac{\theta}{2} - \left(F_1 + \kappa_p F_2\right) \frac{q^2}{2M_p^2} \sin^2\frac{\theta}{2} \right\}$$

- Assume:
  - ~~The electron is non-relativistic~~
  - ~~The proton does not recoil and we can ignore proton spin~~
  - ~~The proton is point-like~~

Elastic Form Factors

- All information about the proton's structure is contained in ***form factors*** F_1 and F_2

$$\left(F_1^2 - \frac{\kappa_p^2 q^2}{4M_p^2} F_2^2\right)$$

$$\left(F_1 + \kappa_p F_2\right)$$

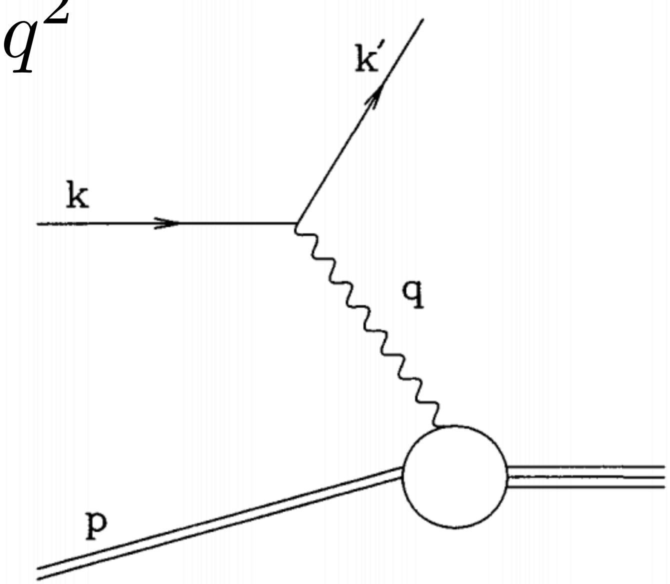
  – The form factors are functions of Q^2

- The proton also has anomalous magnetic moment $\kappa_p = 1.79$

Deep Inelastic Scattering

- Know p and k (from your beam/target)
- Measure k'
- This is enough to determine all of the following, with $Q^2 = -q^2$

$$
\begin{aligned}
M^2 &= p^2 \\
\nu &= p \cdot q = M(E' - E) \\
x &= \frac{Q^2}{2\nu} = \frac{Q^2}{2M(E - E')} \\
y &= \frac{q \cdot p}{k \cdot p} = 1 - E'/E \ ,
\end{aligned}
$$



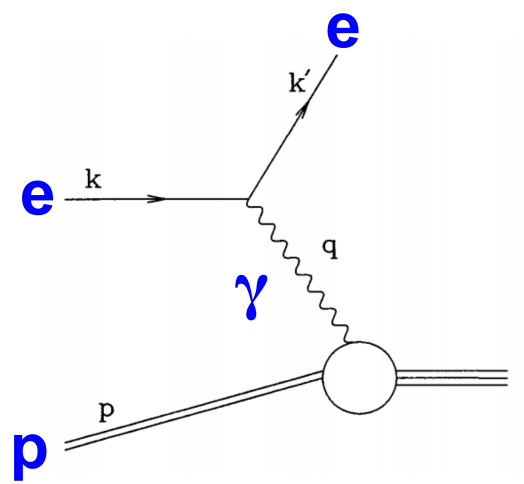
Bjorken x: $p_{parton} = x\ p_{proton}$

Deep Inelastic Scattering

- Charged lepton scattering

$$e + p \to e + X$$

$$\frac{d^2\sigma^{em}}{dx\,dy} = \frac{8\pi\alpha^2 ME}{Q^4}\Bigg[\left(\frac{1+(1-y)^2}{2}\right)2xF_1^{em} + (1-y)(F_2^{em} - 2xF_1^{em}) - (M/2E)xyF_2^{em}\Bigg]$$



Disclaimer: I don't care about weak interactions

Deep Inelastic Scattering

- Charged lepton scattering

$$e + p \rightarrow e + X$$

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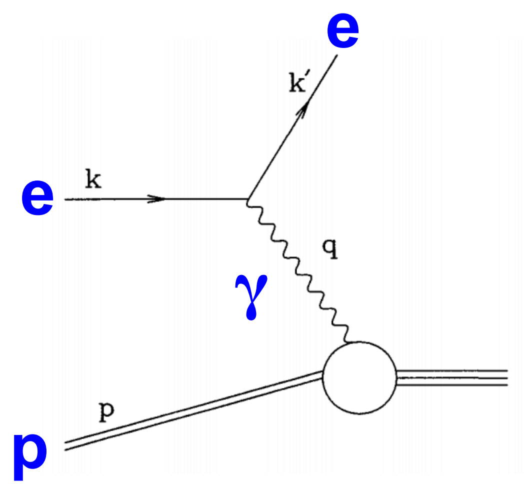


- All information about the proton's structure is contained in ***structure functions***

$$F_i(x, Q^2)$$

Bjorken limit

$Q^2 \to \infty$

- In this limit, the parton momentum is parallel to the proton momentum
  - Structure functions and PDFs are independent of Q^2
- The structure functions are sensitive to the quark PDFs by

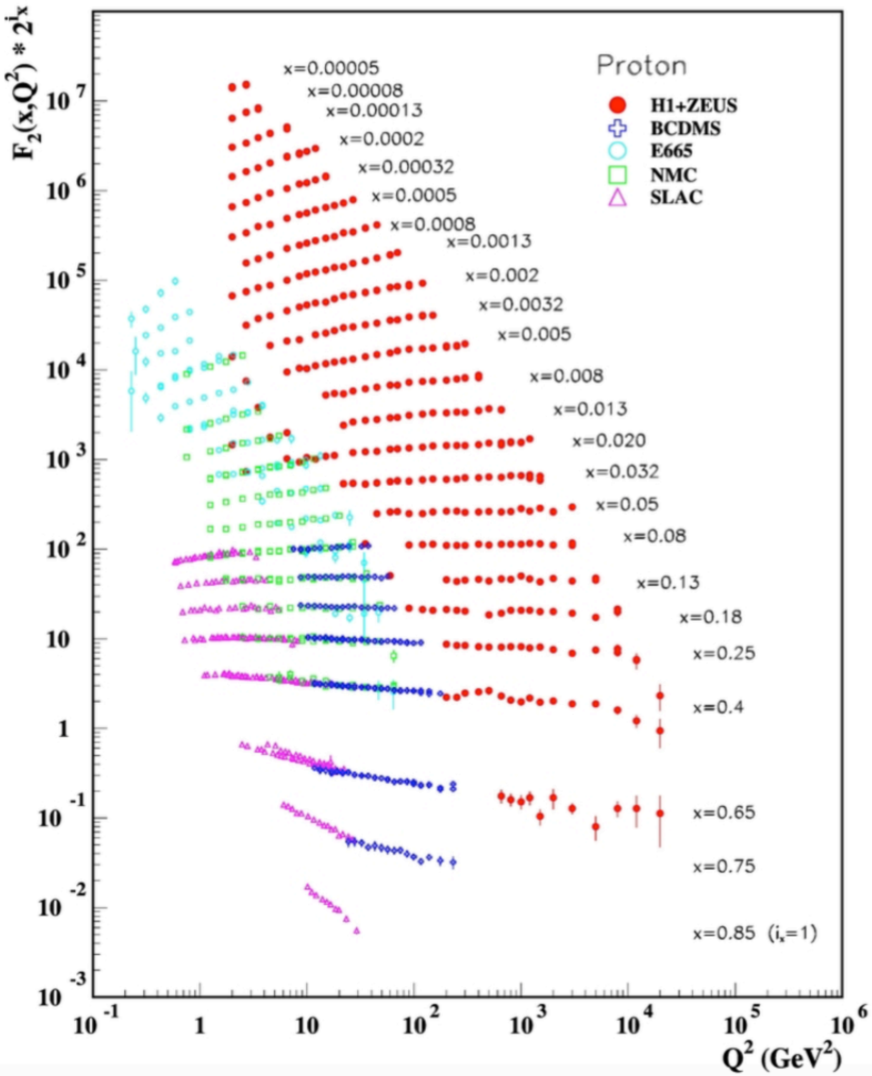
$$F_2^{em}(x) = 2xF_1^{em}(x) = \sum_{q,\bar{q}} e_q^2 xq(x)$$

Bjorken limit

$Q^2 \to \infty$

- No longer applies if we allow constituent quarks to emit a gluon
  - Gluon emission allows quarks to acquire momentum perpendicular to proton momentum
- Scaling violation: must consider dependence of structure functions (and PDFs) on Q^2
  - If we calculate the structure functions to ≥ first order in $\alpha_S \sim g^2$, PDFs are $q(x,Q^2)$

Summary of DIS Experiments



- Can see the dependence of the structure function F_2 on x and Q^2
- PDFs are extracted from cross section measurements
  - e.g. H1 and ZEUS at the ep collider HERA

Cool, but…

isn't this talk about GPDs?

Q^2 ↓

Form factors

$F_1(Q^2)$ & $F_2(Q^2)$

Structure functions

$F_1(x,Q^2)$ & $F_2(x,Q^2)$

Cool, but…

isn't this talk about GPDs?

Q^2

Form factors

$F_1(Q^2)$ & $F_2(Q^2)$

GPDs = Generalized Parton Distributions

higher-level objects that reduce to these if we take the right limits/averages

Structure functions

$F_1(x,Q^2)$ & $F_2(x,Q^2)$

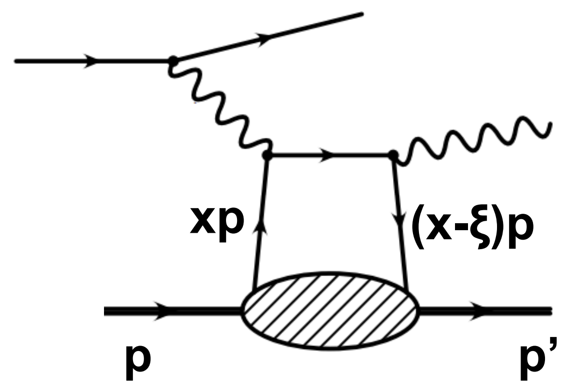
Generalized Parton Distributions

- Each parton flavor has two GPDs
  - $H^q(x,\xi,t,Q^2)$: for when the proton helicity is unchanged
  - $E^q(x,\xi,t,Q^2)$: for when the proton helicity flips
- To understand the variables the GPDs depend on, let's look at the main process useful for probing them
  - Deeply Virtual Compton Scattering (DVCS)

Deeply Virtual Compton Scattering

$$e + p \rightarrow e + \gamma + p$$

What variables to we use to describe the leading order DVCS diagram?



Q^2 = photon virtuality

Bjorken **x**

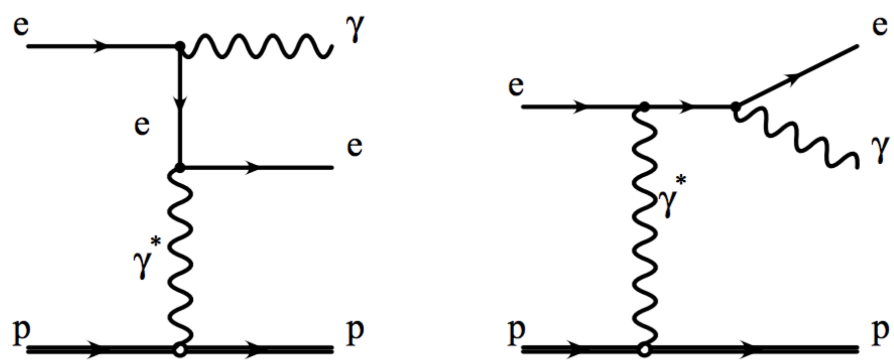
ξ tells you about the quark momentum carried away by γ

Mandelstam **t** = $(p - p')^2$

Background: Bethe-Heitler Process

Also $e + p \rightarrow e + \gamma + p$

- Here, a photon is emitted from the electron/ positron line



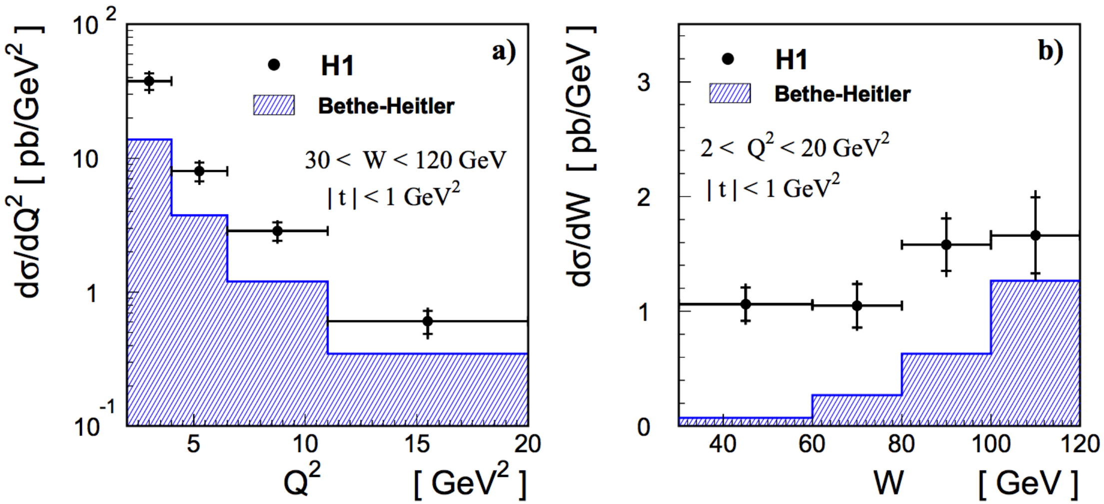
- BH contribution to the DVCS final state is known from QED and can be subtracted off
  - Interference vanishes when integrated over ϕ

DVCS results from H1 at HERA

- Positron-proton collisions
- For DVCS, require exactly two calorimeter clusters
  - Outgoing positron & photon, but no hadrons
- Small background from inelastic collisions
  - Proton remnants not detected
- Measuring cluster angles and energies gives information about x, ξ, and t

DVCS results from H1 at HERA

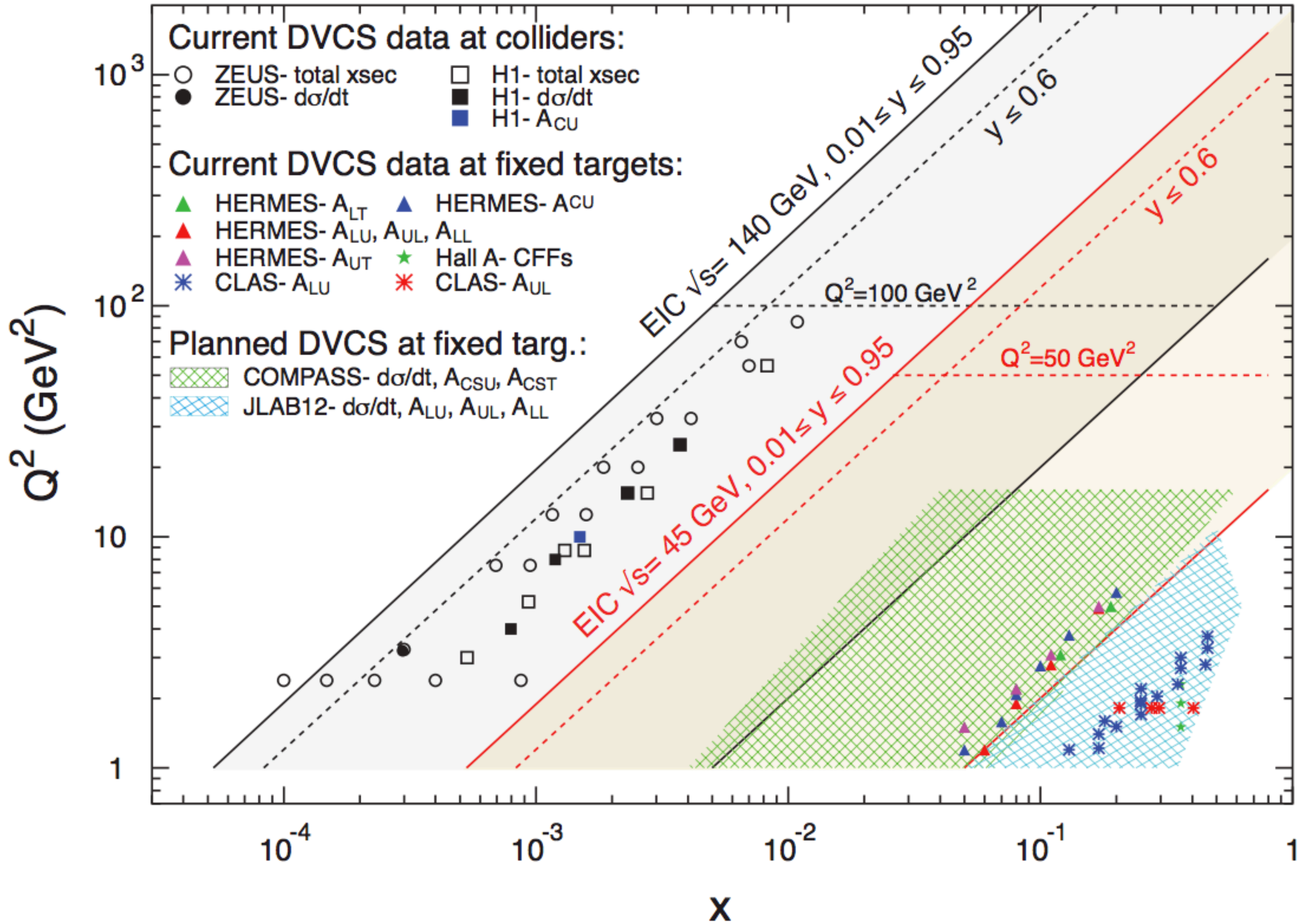
- Differential DVCS cross sections



- Q^2 and t are now familiar variables, but they introduce

$$W^2 = \frac{Q^2}{x}(1-x)$$

Summary of DVCS data



Reducing the GPDs

To elastic form factors

- Take the limit $\xi = 0$, $Q^2 = t$.
  - This gets us back to $e + p \rightarrow e + p$
- Integrate over x, weighting each GPD by the charge of the corresponding quark
  - Since elastic scattering is not sensitive to the parton structure of the proton

$$\sum_q e_q \int dx H^q(x, 0, Q^2, Q^2) = F_1(Q^2)$$

$$\sum_q e_q \int dx E^q(x, 0, Q^2, Q^2) = F_2(Q^2)$$

Reducing the GPDs

To PDFs

- Throw away the GPDs E^q
  - It doesn't make sense to talk about proton helicity flip in DIS
- Take the limit $\xi = 0$
- Fourier transform t (transverse momentum info) to b (transverse position info)
- Integrate over b

$$\int db \tilde{H}^q(x, 0, b, Q^2) = q(x, Q^2)$$

Bigger scale: nuclear GPDs

When should we treat the nucleus as a bag of nucleons vs. as a bag of partons?

- GPDs and DVCS become very useful here
- One goal of a future EIC is to determine how nuclear GPDs are built up
  - Summing over nucleon GPDs? Convolving nucleon GPDs with other functions? Calculating nuclear PDFs?

Summary & Conclusions

- GPDs provide a high-level description of proton structure that simplifies to the form factors and structure functions
- GPDs are probed through Deeply Virtual Compton Scattering
  - At ep colliders and a future EIC
- Understanding proton GPDs is important for describing the structure of larger nuclei

References

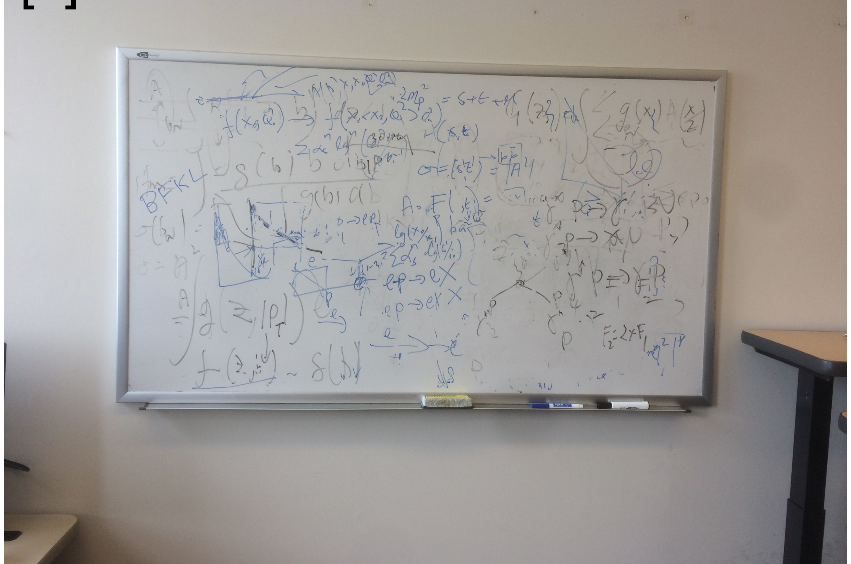
[1] http://www.hep.phy.cam.ac.uk/~thomson/lectures/partIIIparticles/Handout5_2009.pdf

[2] http://www.hep.phy.cam.ac.uk/~thomson/lectures/partIIIparticles/Handout6_2009.pdf

[3] https://arxiv.org/pdf/1212.1701.pdf

[4] https://arxiv.org/pdf/hep-ex/0107005.pdf

[5] Ian



Backup

PDFs at hadron-hadron colliders

- If we measure PDFs in ep and pp collisions, do we expect them to agree?
  - Do strong interactions between hadrons distort the PDFs?
- These interactions give corrections ~ powers of m^2/E_{CM}^2
  - Ok to neglect these at high energies
- So PDFs will be the same in ep and high energy pp experiments