Measuring α_s with jets

Kyle Devereaux Physics 290e Seminar, Fall 2023

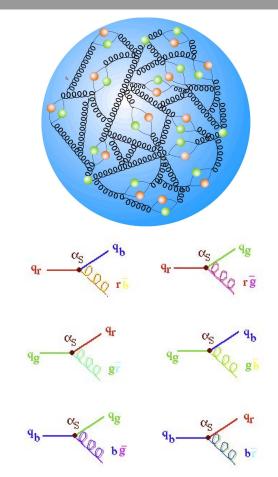
The strong coupling constant

- "Strength of interactions between quarks and gluons"
- ➤ Only free parameter in the QCD Lagrangian, besides quark masses

$$\mathcal{L} = \sum_{q} \bar{\psi}_{q,a} (i \gamma^{\mu} \partial_{\mu} \delta_{ab} - g_s)^{\mu} t_{ab}^{C} \mathcal{A}_{\mu}^{C} - m_{q} \delta_{ab}) \psi_{q,b} - \frac{1}{4} F_{\mu\nu}^{A} F^{A\mu\nu}$$

$$\alpha_{s} = \frac{g_{s}^{2}}{4\pi} \qquad \text{(from quark-gluon interaction)}$$
Strong coupling constant
$$QCD \text{ coupling constant}$$

- ightharpoonup QCD predictions often given in terms of $\alpha_s(\mu_R^2)$ for some (unphysical) renormalization scale μ_R
- $\sim \alpha_s(\mu_R^2 \simeq Q^2)$ = effective strength of strong interaction for the process
- \rightarrow $\mu_R^2 = M_Z^2$ (Z boson mass) often the choice



"Running" of the strong coupling constant

- $\sim \alpha_s$ varies ("runs") with energy scale of interaction, Q^2
- The Beta function gives the behavior of a coupling parameter 'g' as a function of interaction energy scale 'Q'

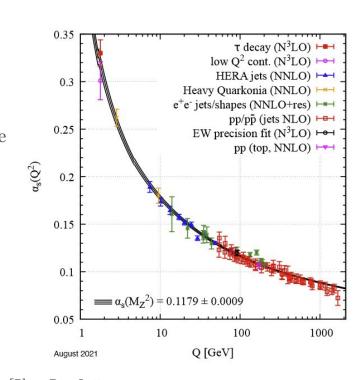
$$\beta(g) = \frac{\partial g}{\partial \ln Q}$$

- ➤ In some non-abelian gauge theories, the beta function can be negative
- ➤ "Asymptotic freedom" discovered for QCD in 1973 by David Gross, Frank Wilczek, David Politzer (2004 Physics Nobel)

$$\alpha_s(Q^2) = \frac{g_s^2(Q^2)}{4\pi} \approx \frac{1}{\beta_0 \ln(Q^2/\Lambda_{\rm QCD}^2)}$$

$$\mu_R^2 \frac{d\alpha_s}{d\mu_P^2} = \beta(\alpha_s) = -(b_0 \alpha_s^2 + b_1 \alpha_s^3 + b_2 \alpha_s^4 + \cdots)$$

- ➤ Low energies -> stronger coupling -> confinement
- ➤ High energies -> weaker coupling -> perturbative calculations



[Phys. Rev. Lett. 30, 1343, Phys. Rev. Lett. 30, 1346, Prog. Theor. Exp. Phys. 2022, 083C01 (2022)]

Examples of $\alpha_s(\mu_R^2)$ in QCD predictions

Fully inclusive cross-sections for $e^+e^- \to hadrons$

$$\frac{\sigma(e^+e^- \to \text{hadrons}, Q)}{\sigma(e^+e^- \to \mu^+\mu^-, Q)} \equiv R(Q) = R_{\text{EW}}(Q)(1 + \delta_{\text{QCD}}(Q))$$

$$\text{Correction for QCD effects}$$

$$\text{Expressed for arbitrary scale } \mu_R$$

$$\text{Independent of choice}$$

$$\delta_{\text{QCD}}(Q) = \sum_{n=1}^{\infty} \overline{c}_n \left(\frac{\mu_R^2}{Q^2}\right) \cdot \left(\frac{\alpha_s(\mu_R^2)}{\pi}\right)^n + \mathcal{O}\left(\frac{\Lambda^4}{Q^4}\right)$$

Deep-inelastic scattering for $ep \rightarrow e + X$

$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{2xQ^4} \left[(1 + (1-y)^2)F_2(x,Q^2) - y^2F_L(x,Q^2) \right]$$

$$F_2(x,Q^2) = x \sum_{n=0}^{\infty} \frac{\alpha_s^n(\mu_R^2)}{(2\pi)^n} \sum_{i=q,q} \int_x^1 \frac{dz}{z} C_{2,i}^{(n)}(z,Q^2,\mu_R^2,\mu_F^2) f_{i/p}(\frac{x}{z},\mu_F^2) + \mathcal{O}(\frac{\Lambda^2}{Q^2})$$

Structure function F_2 is a series in powers of $\alpha_s(\mu_R^2)$ Incalculable from first principles, but calculable when PDFs $f_{i/p}$ are known

Measuring α_s

- Not directly observable, or calculable from first principles
 - Theory prediction + experimentally measured observables needed

Experimental cross section

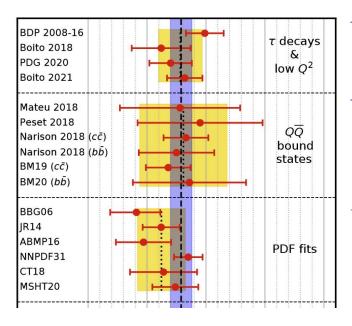
Theoretical cross section

α_s extraction plan

- 1) Pick your favorite observable
- 2) Get a pQCD prediction of it in terms of $\alpha_s(M_Z^2)$ as a free parameter, PDFs
- 3) Apply non-perturbative corrections
- 4) Go measure the observable in a collider experiment
- 5) Construct χ^2 comparison of data and theory, minimize it wrt $\alpha_s(M_Z^2)$
- 6) Evolve $\alpha_s(M_Z^2)$ with the renormalization group eqn to find $\alpha_s(Q^2)$

Uncertainties in both theory calculation and experiment propagate to central value

Manyyyy ways to extract...



- Tau hadronic decays, spectral functions
- precise at M~ tau mass
- N₃LO predictions
- Heavy quarkonia decay predictions
- **NNLO**

Electron-positron annihilation measured around MZ peak Re-analyzed LEP2 with NNLO

- Analyzing structure functions at NNLO
- Combined with hadron collider data

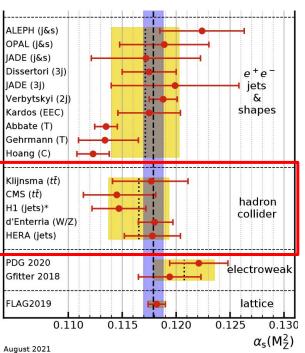
Jet observable and tt-bar production Hadron colliders, precise at mid-to-high energy scales

> LHC top quark and W mass measurements

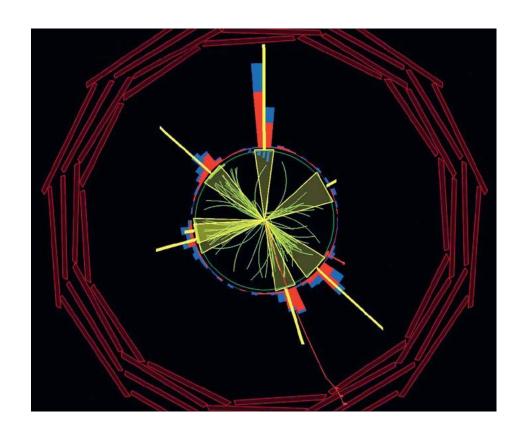
> > Lattice QCD prediction

 $\alpha_s(M_Z^2) = 0.1179 \pm 0.0009$.

(PDG world average)

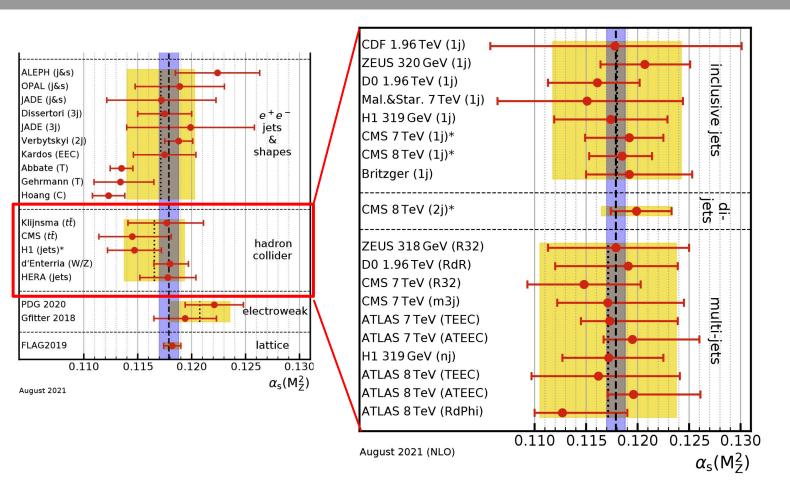


Why jets?

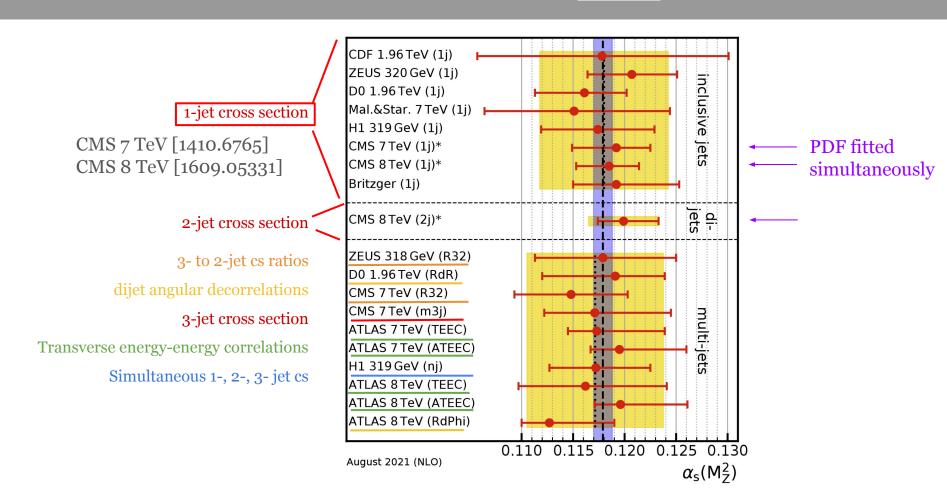


- Sensitive to pQCD effects in general
- Cross sections straight forward to measure in collider experiments
- Precise constraint on $\alpha_s(Q^2)$ at high energy scales O(100 GeV 10 TeV)
- Possible to fit PDF and $\alpha_s(M_Z^2)$ simultaneously
- NNLO calculation available for increasingly more jet observables
- Jet observables typically well described by pQCD calculations over full accessible range (as opposed to event shape observables)

Using jets to extract $\alpha_s(M_Z^2)$, NLO theory prediction



Using jets to extract $\alpha_s(M_Z^2)$



Extraction using inclusive single jet cs (CMS 2015, 2016)

1-jet cross section (CMS)

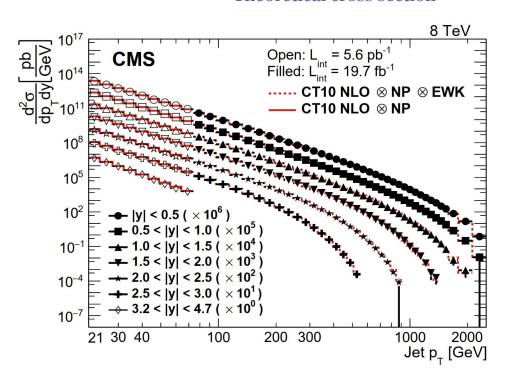
Experimental cross section

$$\frac{\mathrm{d}^2 \sigma}{\mathrm{d} p_{\mathrm{T}} \, \mathrm{d} y} = \frac{1}{\epsilon \cdot \mathcal{L}_{\mathrm{int}}} \frac{N_{\mathrm{jets}}}{\Delta p_{\mathrm{T}} \, \left(2 \cdot \Delta |y|\right)},$$

- Double-differential measurement
- Corrected for detector effects (unfolded at this step)
- ➤ Uncertainties
 - Jet energy scale
 - Luminosity
 - Unfolding
 - other uncorrelated effects

Two ingredients

Theoretical cross section



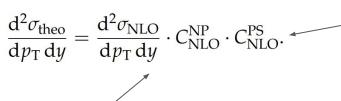
1-jet cs: theoretical cross section

1-jet cross section (CMS)

Two ingredients

Experimental cross section

Theoretical cross section

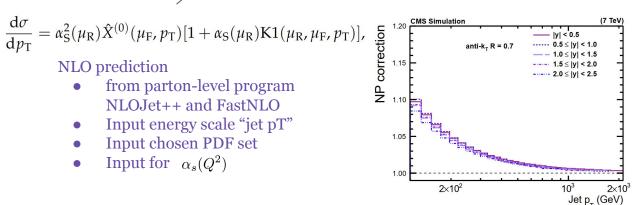


Correction terms

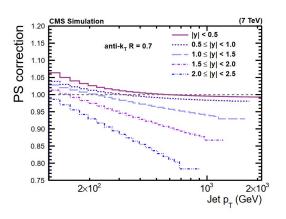
- Nonperturbative (hadronization)
- Parton shower
- Generated with POWHEG+PYTHIA6

$$C_{\text{NLO}}^{\text{PS}} = \frac{\sigma_{\text{NLO+PS}}}{\sigma_{\text{NLO}}}$$

Nonperturbative correction



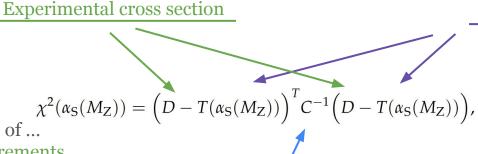
Parton shower correction



$\alpha_s(M_Z^2)$ extraction via χ^2 minimization

1-jet cross section (CMS)

Two ingredients



N samples of ...
D: measurements

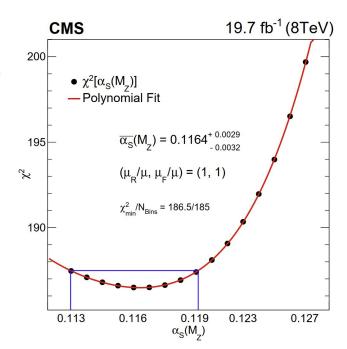
T: theoretical predictions

$$C = C^{\text{stat}} + C^{\text{unfolding}} + \sum C^{\text{JES}} + C^{\text{uncor}} + C^{\text{lumi}} + C^{\text{PDF}} + C^{\text{NP}},$$

Covariance matrix gives uncertainty

- The theoretical predictions are varied in $\alpha_s(M_Z^2)$ over the range 0.110-0.130 with steps of 0.001
- $\sim \alpha_s(M_Z^2)$ central value taken to be that which minimizes chi²

Theoretical cross section



$\alpha_s(M_Z^2)$ fit visualization

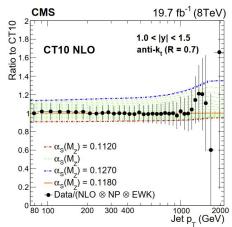
1-jet cross section (CMS)

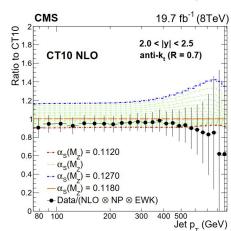
$$\left. \frac{d\sigma_{\rm exp}}{dp_T \, dy} \middle/ \frac{d\sigma_{\rm theo}}{dp_T \, dy} \right.$$

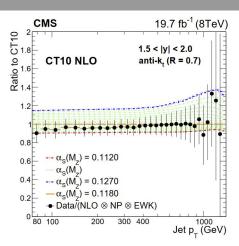
- For one PD set choice: C10
- Note large variation in performance for different PDF sets

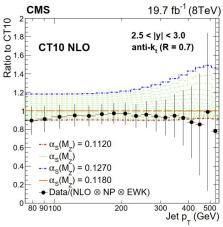
	y	$N_{ m bins}$	CT10	HERAPDF1.5	MSTW2008	NNPDF2.1	ABM11	NNPDF3.0
χ^2	0.0-0.5	37	49.2	66.3	68.0	58.3	136.6	62.5
	0.5 - 1.0	37	28.7	47.2	39.0	35.4	155.5	42.2
	1.0-1.5	36	19.3	28.6	27.4	20.2	111.8	25.9
	1.5 - 2.0	32	65.7	49.0	55.3	54.5	168.1	64.7
	2.0 - 2.5	25	38.7	32.0	53.1	34.6	80.2	36.0
	2.5 - 3.0	18	14.5	19.1	18.2	15.4	43.8	16.3

	PDF set	Refs.	Order	$N_{\rm f}$	$M_{\rm t}$ (GeV)	$M_{\rm Z}$ (GeV)	$\alpha_{\rm S}(M_{\rm Z})$	$\alpha_{\rm S}(M_{\rm Z})$ range
	ABM11	[41]	NLO	5	180	91.174	0.1180	0.110-0.130
(9 \	CT10	[36]	NLO	≤5	172	91.188	0.1180	0.112 – 0.127
$\alpha_s(M_Z^2)$	HERAPDF1.5	[40]	NLO	≤5	180	91.187	0.1176	0.114-0.122
o (<u>Z</u>)	MSTW2008	[37]	NLO	≤ 5	10^{10}	91.1876	0.1202	0.110-0.130
	NNPDF2.1	[38]	NLO	≤6	175	91.2	0.1190	0.114-0.124
	NNPDF3.0	[39]	NLO	≤5	175	91.2	0.1180	0.115-0.121



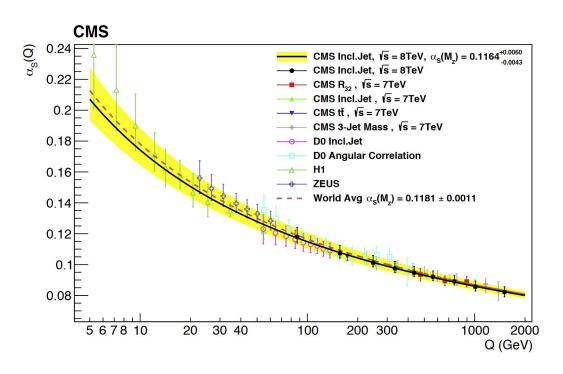






Running of $\alpha_s(M_Z^2)$

1-jet cross section (CMS)



Notained by evolving the fitted $\alpha_s(M_Z^2)$ values using the NLO renormalization group eqn

$$\alpha_{s}(Q^{2}) = \frac{1}{\beta_{0} \log x} \left[1 - \frac{\beta_{1}}{\beta_{0}^{2}} \frac{\log (\log x)}{\log x} \right]; \quad x = \frac{Q^{2}}{\Lambda^{2}},$$

$$\beta_{0} = \frac{1}{4\pi} \left(11 - \frac{2}{3} n_{f} \right); \quad \beta_{1} = \frac{1}{(4\pi)^{2}} \left(102 - \frac{38}{3} n_{f} \right),$$

- ➤ Extends HERA (H1, ZEUS) and Do results into TeV region
- ➤ Still using CT10 NLO PDF set
 - What about fitting pdfs simultaneously?

Simultaneous PDF fit

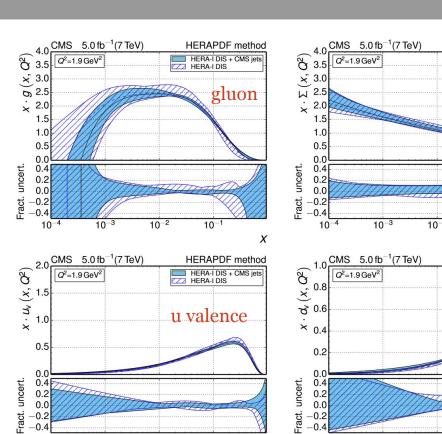
0.2 0.0

 10^{-4}

 10^{-3}

1-jet cross section (CMS)

- Extraction of PDFs from inclusive jet cs depends on
 - By fitting the PDFs taking $\alpha_s(M_Z^2)$ as a free parameter, both can be fit simultaneously
 - Less correlation between the gluon PDF and $\alpha_s(M_Z^2)$
- Performed using the HERAPDF method
- Jet measurement important since this can't be done in HERA-1 DIS measurements alone



 10^{-1}

Х

0.0

 10^{-3}

HERAPDF method

HERA-I DIS + CMS jets
HERA-I DIS

sea quark

10-1

HERAPDF method

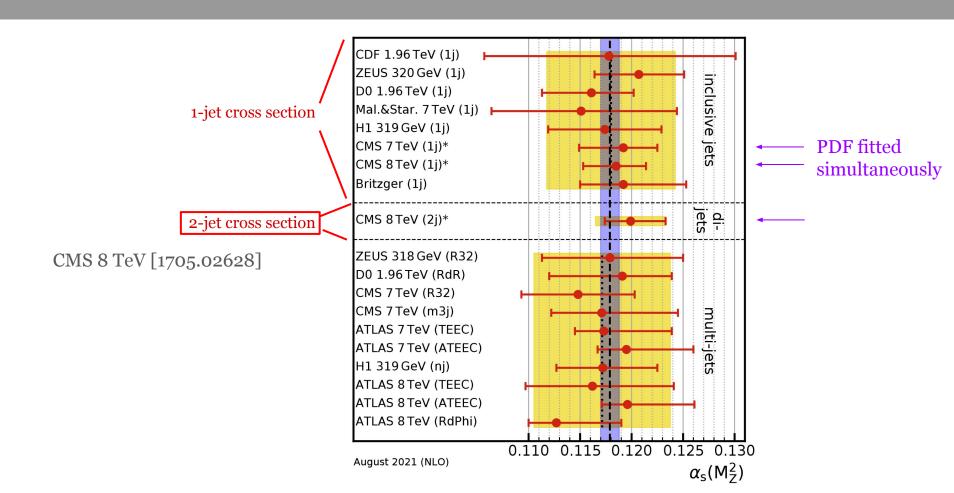
HERA-I DIS + CMS jets
HERA-I DIS

d valence

 10^{-1}

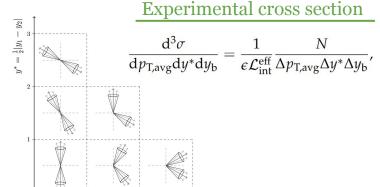
Х

Extraction using dijet cs (CMS 2017)

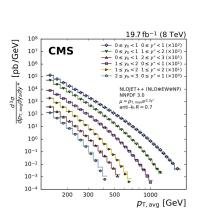


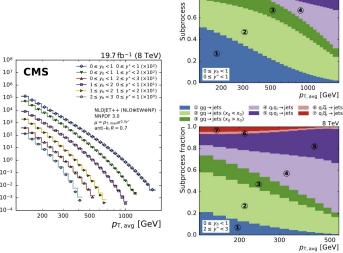
Extraction using dijet cs (CMS 2017)

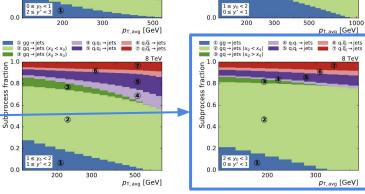




 $y_b = \frac{1}{2}|y_1 + y_2|$







o.6

0.2

0.6

0.2

200 300

① $gg \rightarrow jets$ ② $gq \rightarrow jets$ ② $gq \rightarrow jets$ ② $gq \rightarrow jets$ ② $gq \rightarrow jets$

p_{T. avg} [GeV]

② gq → jets (x_g < x_q) ③ qq → jets $(x_a > x_a)$

- Triple-differential measurement
- y* and yb parameterize jet orientations
- Main idea: more sensitive probe to PDFs in highly boosted regime
 - For large yb, ~80% of cross section has gluon participating in interaction
 - Higher sensitivity to gluon PDF

Extraction using dijet cs (CMS 2017)

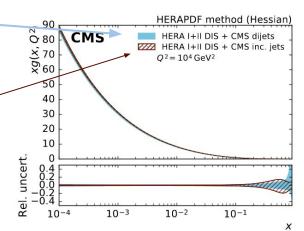
2-jet cross section (CMS)

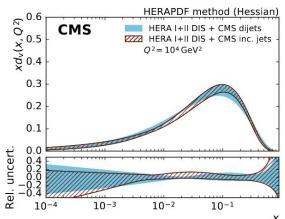
$$\alpha_S(M_{\rm Z}) = 0.1199\,\pm 0.0015\,({\rm exp})\,{}^{+0.0031}_{-0.0020}\,({\rm theo})$$

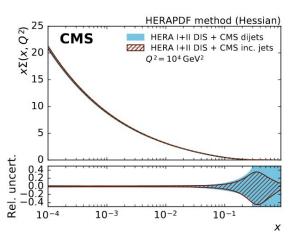
1-jet cross section (CMS)

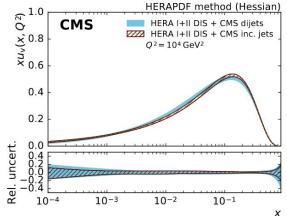
$$\alpha_{\rm S}(M_{\rm Z}) = 0.1164^{+0.0060}_{-0.0043},$$

- Slightly tighter constraint on $\alpha_s(M_Z^2)$
- When dijets included, increased gluon PDF at high-x
- Uncertainties of the PDF, esp the gluon PDF, is significantly reduced compared to inc. jet

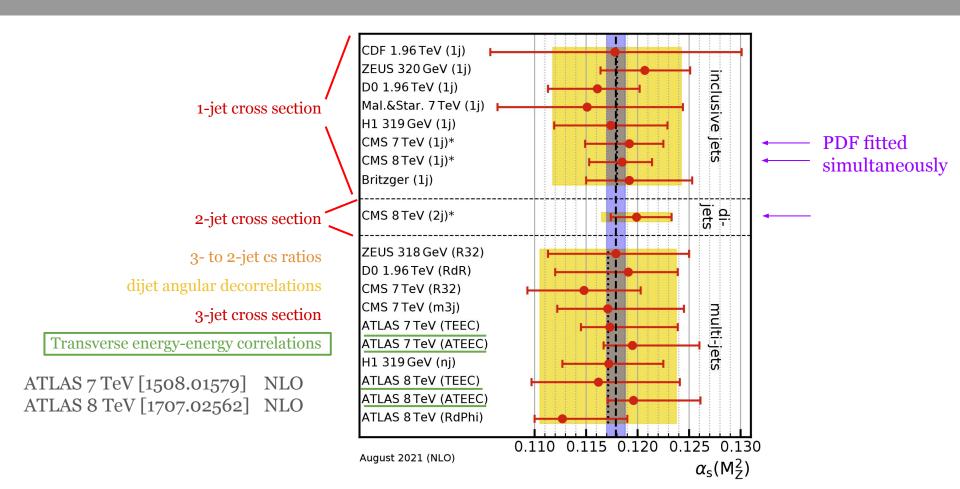








Extraction using transverse energy-energy correlators (ATLAS 2015, 2017)



Extraction using transverse energy-energy correlators (ATLAS 2015, 2017)

TEEC (ATLAS)

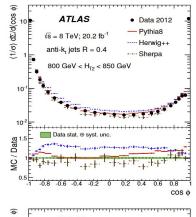
Two ingredients

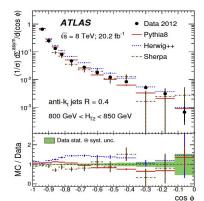
Experimental cross section

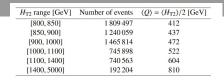
$$\frac{1}{\sigma} \frac{\mathrm{d}\Sigma}{\mathrm{d}\cos\phi} = \frac{1}{N} \sum_{A=1}^{N} \sum_{ij} \frac{E_{\mathrm{T}i}^{A} E_{\mathrm{T}j}^{A}}{\left(\sum_{k} E_{\mathrm{T}k}^{A}\right)^{2}} \delta(\cos\phi - \cos\phi_{ij}), \quad (\text{TEEC})$$

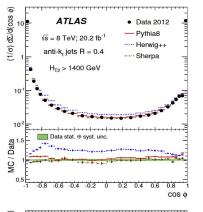
$$\frac{1}{\sigma} \frac{\mathrm{d}\Sigma^{asym}}{\mathrm{d}\cos\phi} \equiv \left. \frac{1}{\sigma} \frac{\mathrm{d}\Sigma}{\mathrm{d}\cos\phi} \right|_{\phi} - \left. \frac{1}{\sigma} \frac{\mathrm{d}\Sigma}{\mathrm{d}\cos\phi} \right|_{\pi-\phi}. \tag{ATEEC}$$

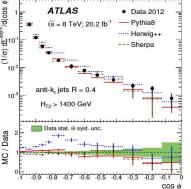
- ➤ Multijet observable
- TEECs are a generalization of EECs (as used in electron-positron collisions) to hadron colliders
- Energy-weighted angular distribution of jet pairs in an event
- Analyzed quickly after the NLO prediction was made available











TEEC: theoretical cross section

TEEC (ATLAS)

Experimental cross section

Two ingredients

Theoretical cross section

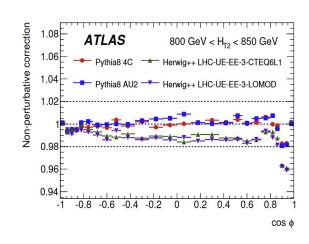
$$\frac{1}{\sigma} \frac{d\Sigma}{d\phi} = \frac{\sum_{a_i,b_i} f_{a_1/p}(x_1) f_{a_2/p}(x_2) \otimes \hat{\Sigma}^{a_1 a_2 \to b_1 b_2 b_3}}{\sum_{a_i,b_i} f_{a_1/p}(x_1) f_{a_2/p}(x_2) \otimes \hat{\sigma}^{a_1 a_2 \to b_1 b_2}},$$

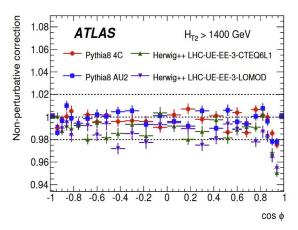
NLO prediction

- Convolved with NNLO PDF sets
- Numerator: PDFs convolved with 2->3 partonic subprocess at NLO
- Denominator: PDFs convolved with 2->2 subprocesses

Non-perturbative corrections

- Bin-by-bin correction calculated from ratio of MC and TEEC distributions
- Hadronization and underlying event turned on/off
- PYTHIA, HERWIG, SHERPA compared

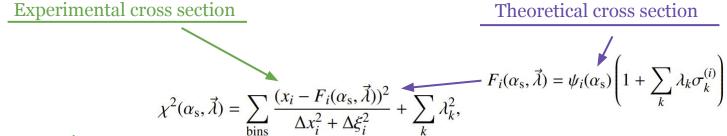




$\alpha_s(M_Z^2)$ extraction via χ^2 minimization

TEEC (ATLAS)

Two ingredients



x_i : measurements

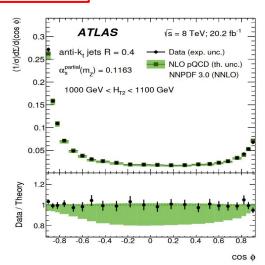
F_i: theoretical predictions weighed with nuisance parameters

- ➤ Minimization done in 74-dimensional space
 - 1 parameter for alpha_s
 - 73 parameters for nuisance variables lambda (1 per source of uncertainty)
- Psi found by fitting TEEC (ATEEC) prediction to data in each $(H_{T2}, \cos \phi)$ bin to a second-degree polynomial

$\alpha_s(M_Z^2)$ fit visualization

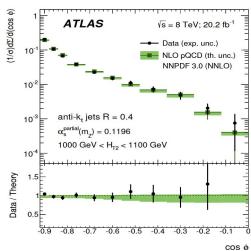
TEEC (ATLAS)





$\langle Q \rangle ({\rm GeV})$	$\alpha_{\rm s}(m_{\rm Z})$ value (NNPDF 3.0)	$\chi^2/N_{\rm dof}$
412	0.1209 ± 0.0036 (exp.) $^{+0.0085}_{-0.0031}$ (scale) ± 0.0013 (PDF) ± 0.0004 (NP)	10.6 / 10
437	0.1211 ± 0.0026 (exp.) $^{+0.0064}_{-0.0014}$ (scale) ± 0.0015 (PDF) ± 0.0010 (NP)	6.8 / 10
472	0.1203 ± 0.0028 (exp.) $^{+0.0060}_{-0.0013}$ (scale) ± 0.0016 (PDF) ± 0.0002 (NP)	8.8 / 10
522	0.1196 ± 0.0025 (exp.) $^{+0.0054}_{-0.0010}$ (scale) ± 0.0017 (PDF) ± 0.0004 (NP)	10.9 / 10
604	0.1176 ± 0.0031 (exp.) $^{+0.0058}_{-0.0008}$ (scale) ± 0.0020 (PDF) ± 0.0005 (NP)	6.4 / 10
810	0.1172 ± 0.0037 (exp.) $^{+0.0053}_{-0.0009}$ (scale) ± 0.0022 (PDF) ± 0.0001 (NP)	9.8 / 10

ATEEC



		cos o

$\langle Q \rangle (\text{GeV})$	$\alpha_{\rm s}(m_{\rm Z})$ value (NNPDF 3.0)	$\chi^2/N_{\rm dof}$
412	0.1209 ± 0.0036 (exp.) $^{+0.0085}_{-0.0031}$ (scale) ± 0.0013 (PDF) ± 0.0004 (NP)	10.6 / 10
437	0.1211 ± 0.0026 (exp.) $^{+0.0064}_{-0.0014}$ (scale) ± 0.0015 (PDF) ± 0.0010 (NP)	6.8 / 10
472	0.1203 ± 0.0028 (exp.) $^{+0.0060}_{-0.0013}$ (scale) ± 0.0016 (PDF) ± 0.0002 (NP)	8.8 / 10
522	0.1196 ± 0.0025 (exp.) $^{+0.0054}_{-0.0010}$ (scale) ± 0.0017 (PDF) ± 0.0004 (NP)	10.9 / 10
604	0.1176 ± 0.0031 (exp.) $^{+0.0058}_{-0.0008}$ (scale) ± 0.0020 (PDF) ± 0.0005 (NP)	6.4 / 10
810	$0.1172 \pm 0.0037 \text{ (exp.)} ^{+0.0053}_{-0.0009} \text{ (scale)} \pm 0.0022 \text{ (PDF)} \pm 0.0001 \text{ (NP)}$	9.8 / 10

 $\alpha_{\rm s}(m_{\rm Z}) = 0.1162 \pm 0.0011 \text{ (exp.)} ^{+0.0076}_{-0.0061} \text{ (scale)} \pm 0.0018 \text{ (PDF)} \pm 0.0003 \text{ (NP)},$

 $\alpha_{\rm s}(m_Z) = 0.1196 \pm 0.0013 \text{ (exp.)} ^{+0.0061}_{-0.0013} \text{ (scale)} \pm 0.0017 \text{ (PDF)} \pm 0.0004 \text{ (NP)},$

How to improve?

Majority of error stems from theoretical side, missing terms in the NLO prediction, PDF fits

NNLO required to make better progress...

scale values Q, derived again with the FASTNLO framework, are identical within about 1 GeV for different PDFs. To emphasise that theoretical uncertainties limit the achievable precision, Tables $\boxed{6}$ and $\boxed{7}$ present for the six bins in p_T the total uncertainty as well as the experimental, PDF, NP, and scale components, where the six experimental uncertainties are all correlated.

Not until recently did NNLO calculations for many observables become available

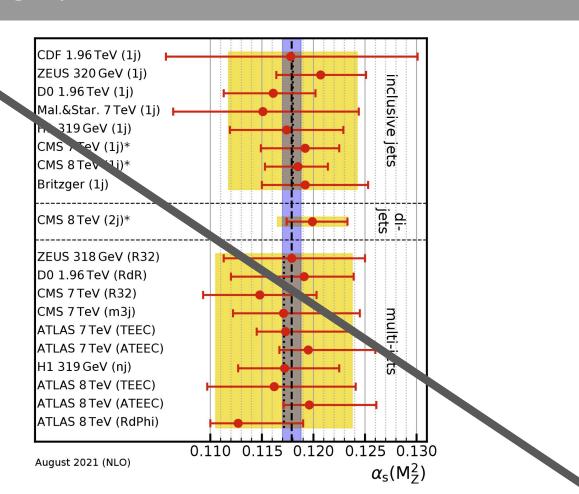
Moving beyond NLO: NNLO

All these measurements use NLO theory predictions

NNLO is now the standard (only NNLO considered for PDG)

Transverse energy-energy correlations

ATLAS 7 TeV [1508.01579] NLO ATLAS 8 TeV [1707.02562] NLO ATLAS 13 TeV [2301.09351] NNLO!!



TEEC: theoretical cross section NNLO

TEEC (ATLAS)

Two ingredients

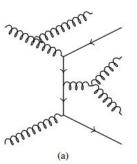
Experimental cross section

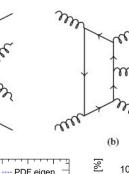
NNLO prediction

- Scattering amplitudes calculated in OpenLoops2,
 FivePointAmplitudes, and PentagonFunctions++
- Calculated with O(1E13) events
- Real-real, real-virtual, virtual-virtual terms included
- NNLO PDF sets used

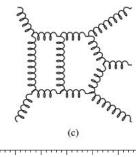
Corrections

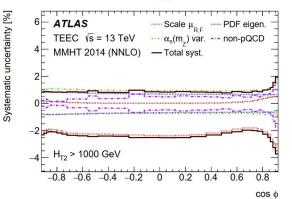
- Scale uncertainties reduced three fold going NLO to NNLO
- PDF uncertainties ~1% range (extrapolate from NLO result)
- Nonperturbative corrections dominant corrected with PYTHIA8, HERWIG simulation for hadronization and UE effects

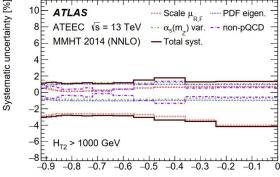




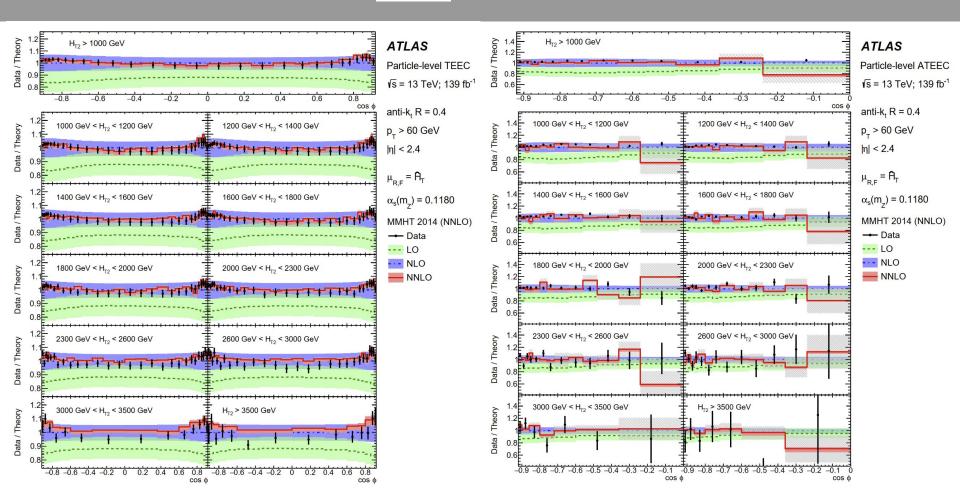
Theoretical cross section







NNLO $\alpha_s(M_Z^2)$ fit visualization



Comparison NNLO with NLO

TEEC (ATLAS)

NLO

TEEC

$$\alpha_{\rm s}(m_Z) = 0.1162 \pm 0.0011 \text{ (exp.)} ^{+0.0076}_{-0.0061} \text{ (scale)} \pm 0.0018 \text{ (PDF)} \pm 0.0003 \text{ (NP)},$$

ATEEC

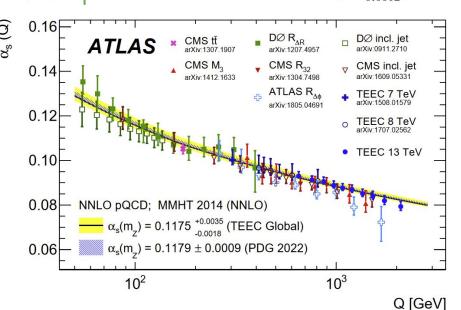
$$\alpha_s(m_Z) = 0.1196 \pm 0.0013 \text{ (exp.)} ^{+0.0061}_{-0.0013} \text{ (scale)} \pm 0.0017 \text{ (PDF)} \pm 0.0004 \text{ (NP)},$$

- Theoretical bounds significantly improved
- Extends previous results further into TeV region

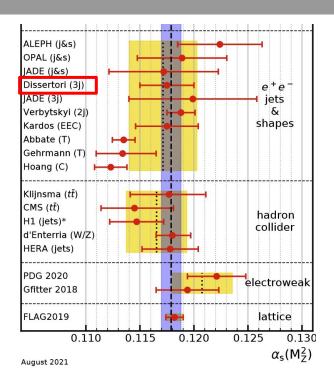
NNLO

$$\alpha_{\rm s}(m_Z) = 0.1175 \pm 0.0006 \,({\rm exp.})^{+0.0034}_{-0.0017} \,({\rm theo.})$$

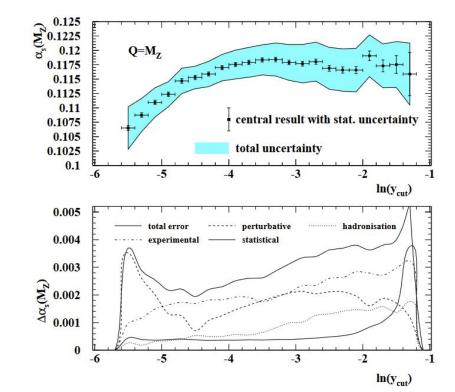
$$\alpha_s(m_Z) = 0.1185 \pm 0.0009 \text{ (exp.)}_{-0.0012}^{+0.0025} \text{ (theo.)}.$$

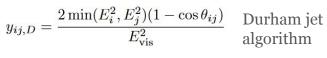


Using jets to extract $\alpha_s(M_Z^2)$, LEP 3-jet rate



- ► LEP electron-positron annihilation re-analyzed, ALEPH data
- ➤ NNLO calculation available for 3-jet production, recently available
- Measured at the Z peak





 $y_{ij,D} < y_{\text{cut}}$: jet resolution parameter

[0910.4283]

Summary

- $\sim \alpha_s(M_Z^2)$ is a significant parameter in the standard model, but relatively poorly known
- > Jets measurements are powerful for constraining the strong coupling constant $\alpha_s(M_Z^2)$ at high energy scales
- Possible to fit PDFs simultaneously with $\alpha_s(M_Z^2)$, obtaining better constraints on both
- ➤ In general good, pQCD predictions for inclusive and dijet production are good, but NNLO calculations are needed for further progress

Thank you

